INTERNATIONAL LIBRARY OF TECHNOLOGY

A SERIES OF TEXTBOOKS FOR PERSONS ENGAGED IN THE ENGINEERING PROFESSIONS AND TRADES OR FOR THOSE WHO DESIRE INFORMATION CONCERNING THEM. FULLY ILLUSTRATED AND CONTAINING NUMEROUS PRACTICAL EXAMPLES AND THEIR SOLUTIONS

HYDROSTATICS
PNEUMATICS
HYDRAULICS
ELEMENTARY CHEMISTRY
HEAT
ENTROPY AND STEAM
STEAM-ENGINE MECHANISM
STEAM-ENGINE INDICATORS AND DIAGRAMS
SIMPLE NON-CONDENSING STEAM ENGINES
COMPOUND AND CONDENSING ENGINES
STEAM TURBINES

SCRANTON:
INTERNATIONAL TEXTBOOK COMPANY
6B

PRINTED IN THE UNITED STATES

BURR PRINTING HOUSE
FRANKFORT AND JACOB STREETS
NEW YORK

287

It 6 B

T-6 B-6

PREFACE

The International Library of Technology is the outgrowth of a large and increasing demand that has arisen for the Reference Libraries of the International Correspondence Schools on the part of those who are not students of the Schools. As the volumes composing this Library are all printed from the same plates used in printing the Reference Libraries above mentioned, a few words are necessary regarding the scope and purpose of the instruction imparted to the students of—and the class of students taught by—these Schools, in order to afford a clear understanding of their salient and unique features.

The only requirement for admission to any of the courses offered by the International Correspondence Schools, is that the applicant shall be able to read the English language and to write it sufficiently well to make his written answers to the questions asked him intelligible. Each course is complete in itself, and no textbooks are required other than those prepared by the Schools for the particular course selected. The students themselves are from every class, trade, and profession and from every country; they are, almost without exception, busily engaged in some vocation, and can spare but little time for study, and that usually outside of their regular working hours. The information desired is such as can be immediately applied in practice, so that the student may be enabled to exchange his present vocation for a more congenial one, or to rise to a higher level in the one he now pursues. Furthermore, he wishes to obtain a good working knowledge of the subjects treated in the shortest time and in the most direct manner possible.

In meeting these requirements, we have produced a set of books that in many respects, and particularly in the general plan followed, are absolutely unique. In the majority of subjects treated the knowledge of mathematics required is limited to the simplest principles of arithmetic and mensuration, and in no case is any greater knowledge of mathematics needed than the simplest elementary principles of algebra, geometry, and trigonometry, with a thorough, practical acquaintance with the use of the logarithmic table. To effect this result, derivations of rules and formulas are omitted, but thorough and complete instructions are given regarding how, when, and under what circumstances any particular rule, formula, or process should be applied; and whenever possible one or more examples, such as would be likely to arise in actual practice—together with their solutions—are given to illustrate and explain its application.

In preparing these textbooks, it has been our constant endeavor to view the matter from the student's standpoint, and to try and anticipate everything that would cause him trouble. The utmost pains have been taken to avoid and correct any and all ambiguous expressions—both those due to faulty rhetoric and those due to insufficiency of statement or explanation. As the best way to make a statement, explanation, or description clear is to give a picture or a diagram in connection with it, illustrations have been used almost without limit. The illustrations have in all cases been adapted to the requirements of the text, and projections and sections or outline, partially shaded, or full-shaded perspectives have been used, according to which will best produce the desired results. Half-tones have been used rather sparingly, except in those cases where the general effect is desired rather than the actual details.

It is obvious that books prepared along the lines mentioned must not only be clear and concise beyond anything heretofore attempted, but they must also possess unequaled value for reference purposes. They not only give the maximum of information in a minimum space, but this information is so ingeniously arranged and correlated, and the

indexes are so full and complete, that it can at once be made available to the reader. The numerous examples and explanatory remarks, together with the absence of long demonstrations and abstruse mathematical calculations, are of great assistance in helping one select the proper formula, method, or process and in teaching him how and when it should be used.

The earlier papers contained in this volume are elementary in character and deal with the fundamental principles relating to the action of liquids and gases under pressure, and the flow of fluids through pipes, channels, and orifices. The nature of heat, its production, its transformation into work, and its effects on solids, liquids, and gases, are discussed at length, and this is followed by a paper on the properties of steam. The latter portion of the volume deals with steam engines and steam turbines, and describes and illustrates the types now in general use. The action of steam in simple and compound engines is clearly discussed, and the section dealing with the indicator and indicator diagrams gives, in simple and concise form, instruction in the methods of determining the horsepower, steam consumption, and faults in valve setting from indicator diagrams. The information contained in this volume should be of great value to all persons interested in the construction, manner of working, and designing of steam engines and steam turbines.

The method of numbering the pages, cuts, articles, etc. is such that each subject or part, when the subject is divided into two or more parts, is complete in itself; hence, in order to make the index intelligible, it was necessary to give each subject or part a number. This number is placed at the top of each page, on the headline, opposite the page number; and to distinguish it from the page number it is preceded by the printer's section mark (§). Consequently, a reference such as § 16, page 26, will be readily found by looking along the inside edges of the headlines until § 16 is found, and then through § 16 until page 26 is found.

INTERNATIONAL TEXTBOOK COMPANY

CONTENTS

HYDROSTATICS

PROPERTIES OF LIQUIDS

1. Perfect and Viscous Liquids.—**Hydrostatics** treats of liquids at rest under the action of forces. A **liquid body** is one whose molecules change their relative positions easily. In some liquids, as water, alcohol, ether, etc., a change of form takes place almost instantly when the liquid is acted on by a force; thus, any one of these liquids will almost instantly assume a level surface under the action of gravity. Other liquids change form gradually, as, for example, tar, pitch, or molasses.

A **perfect liquid** is one whose particles can move on each other with the greatest freedom and without friction; that is, a liquid that instantly changes form under the action of a disturbing force. There is no such thing as an absolutely perfect liquid, but for practical purposes water, mercury, alcohol, and ether, at ordinary temperatures, may be treated as perfect liquids.

Liquids that do not change their form readily are prevented from doing so by the property called *viscosity*. All liquids are more or less viscous, and the degree of viscosity varies greatly. The liquids with great viscosity, as tar and pitch, are usually called **viscous liquids.**

In that which follows, the liquid that will be dealt with chiefly is water. The principles stated will, however, apply to other perfect liquids, and approximately to viscous liquids.

2. Compressibility of Liquids.—Liquids are only slightly compressible. A pressure of 15 pounds per square inch will compress water only about $\frac{1}{20000}$ of its volume.

For this reason, in all practical problems, water is generally regarded as incompressible.

3. Heaviness of Liquids.—The weight of a cubic unit of a liquid is called its **heaviness.** The weight, in pounds per cubic foot, of a substance is taken as the standard for heaviness in the United States and England. For example, the heaviness of distilled water at the temperature of maximum density, 39.1° F., is 62.425 pounds, and of ordinary seawater, 64.05 pounds. In engineering calculations, the heaviness of fresh water is generally taken as 62.5 pounds.

FLUID PRESSURE

TRANSMISSION OF PRESSURE

4. Pascal's Law.—Fig. 1 shows two cylindrical vessels of the same size. Vessel *a* is fitted with a wooden block of the same size as the cylinder; the vessel *b* is filled with water, whose depth is the same as the length of the wooden block in *a*. Both vessels are fitted with air-tight pistons *p*, whose areas are each 10 square inches.

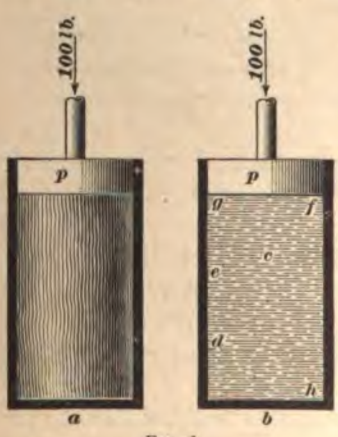


FIG. 1

Suppose, for convenience, that the weights of the cylinders, pistons, block, and water are neglected, and that a force of 100 pounds is applied to both pistons. The pressure per square inch is $100 \div 10 = 10$ pounds. In the vessel *a*, this pressure is transmitted without loss to the bottom of the vessel, and it is easy to see that there will be no pressure on the sides. In the vessel *b*, an entirely different result is obtained. The pressure on the bottom is the same as in the other case—that is,

10 pounds per square inch—but owing to the fact that the molecules of the water are perfectly free to move, this pressure of 10 pounds per square inch is transmitted in every direction with the same intensity; that is to say, the pressure at any point, *c*, *d*, *e*, *f*, *g*, *h*, etc. due to the force of 100 pounds, is exactly the same and is 10 pounds per square inch.

The truth of this statement may be proved, experimentally, by means of the apparatus shown in Fig. 2. Let the area of the piston *a* be 20 square inches; of *b*, 7 square inches; of *c*, 1 square inch; of *d*, 6 square inches; of *e*, 8 square inches; and of *f*, 4 square inches.

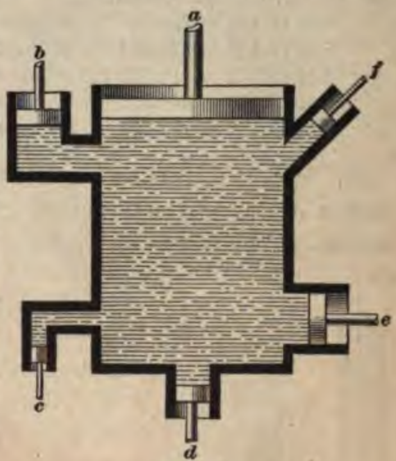


Fig. 2

If the pressure due to the weight of the water is neglected and a force of 5 pounds is applied at *c*, whose area is 1 square inch, a pressure of 5 pounds per square inch will be transmitted in all directions, and in order that there shall be no movement a force of $6 \times 5 = 30$ pounds must be applied at *d*, 40 pounds at *e*, 20 pounds at *f*, 100 pounds at *a*, and 35 pounds at *b*.

If a force of 99 pounds is applied at *a*, instead of 100 pounds, the piston *a* will rise, and the other pistons *b*, *c*, *d*, *e*, and *f* will move inwards; but if the force applied at *a* is 100 pounds, they will all be in equilibrium. Should 101 pounds be applied at *a*, the pressure per square inch will be $101 \div 20 = 5.05$ pounds, which will be transmitted in all directions; and, since the pressure due to *c* is only 5 pounds per square inch, it is evident that the piston *a* will move downwards, and the pistons *b*, *c*, *d*, *e*, and *f* will be forced outwards.

The whole may be summed up in the following law:

The pressure per unit of area exerted anywhere on a mass of liquid is transmitted undiminished in all directions, and acts

with the same intensity on all surfaces in a direction at right angles to those surfaces.

This law was first discovered by Pascal, and is the most important in hydrostatics. Its meaning should be thoroughly understood.

EXAMPLE.—If the area of the piston e in Fig. 2 is 8.25 square inches, and a force of 150 pounds is applied to it, what forces must be applied to the other pistons to keep the water in equilibrium, assuming that their areas are the same as those just given?

SOLUTION.—The intensity of pressure is $150 \div 8.25 = 18.182$ lb. per sq. in., nearly. Then,

$$\left.\begin{array}{l} 20 \times 18.182 = 363.64 \text{ lb.} = \text{force required to balance } a \\ 7 \times 18.182 = 127.274 \text{ lb.} = \text{force required to balance } b \\ 1 \times 18.182 = 18.182 \text{ lb.} = \text{force required to balance } c \\ 6 \times 18.182 = 109.092 \text{ lb.} = \text{force required to balance } d \\ 4 \times 18.182 = 72.728 \text{ lb.} = \text{force required to balance } f \end{array}\right\} \text{Ans.}$$

5. Application of Pascal's Law.—Let the area of the piston a, Fig. 3, be 1 square inch and that of b 40 square inches. According to Pascal's law, 1 pound placed on a will balance 40 pounds placed on b.

FIG. 3

Suppose that a moves downwards 10 inches; 10 cubic inches of water will then be forced into the tube b. This will be distributed over the entire area of the tube b in the form of a cylinder, whose cubic contents must be 10 cubic inches, whose base has an area of 40 square inches, and whose height must therefore be $\frac{10}{40} = \frac{1}{4}$ inch; that is, a movement of the piston a of 10 inches causes a movement of the piston b of $\frac{1}{4}$ inch.

This is an example of the familiar principle of work: *The applied force multiplied by the distance through which it moves is equal to the resistance multiplied by the distance through which it moves.*

6. The Hydraulic Press.—The foregoing principle is made use of in the hydraulic press shown in Fig. 4. As the lever a is depressed, the plunger b is forced down on the

water in the cylinder *c*. The water is forced through the bent tube *d* into the cylinder in which the large plunger *e* works, and causes the plunger to rise, thus lifting the platform *f*, and compressing the bales that lie on it.

Let the area of the plunger *b* be 1 square inch and that of *e* 100 square inches. Also, assume the length of the lever between the hand and fulcrum to be 10 times the length between the fulcrum and plunger *b*. If the end of the lever is

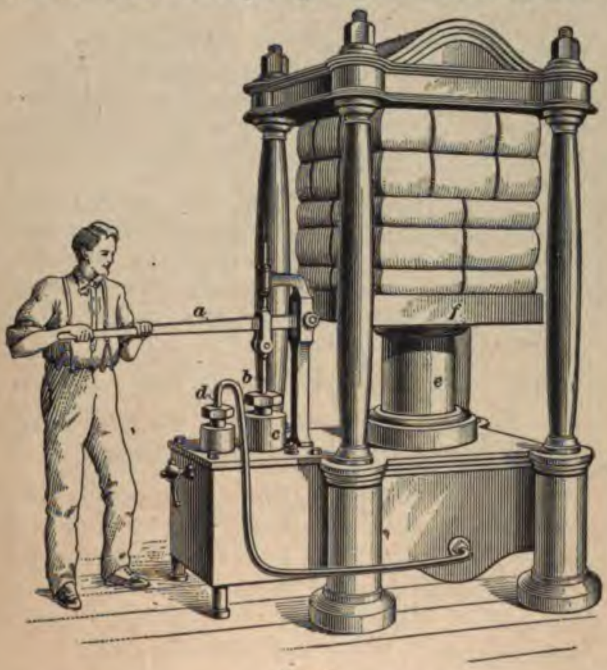


FIG. 4

depressed 10 inches, the plunger *b* is depressed one-tenth of 10 inches = 1 inch, and since 1 cubic inch of water is displaced and the area of plunger *e* is 100 square inches, it is raised $\frac{1}{100}$ inch. If p represents the force applied to the lever a, and q represents the pressure on the platform f, then $p \times 10$ inches $= q \times \frac{1}{100}$ inch. From this, $q = 1{,}000\,p$ or $p = \frac{1}{1000}\,q$. A force of 40 pounds applied by the hand will therefore cause a pressure of $40 \times 1{,}000 = 40{,}000$ pounds

to be exerted by the piston e. But if the average movement of the hand per stroke is 10 inches, and since a 10-inch movement of the hand lever moves the plunger e, $\frac{1}{100}$ inch, it will require $1 \div \frac{1}{100} = 100$ strokes to raise the platform 1 inch. It is here seen that what is gained in pressure is lost in speed.

Applications of Pascal's law are seen also in hydraulic machines used for forcing locomotive drivers on their axles, in punching plates, in bending rails, and in testing the strength of boiler shells.

EXAMPLE.—In a hydraulic punch, the plunger that carries the punch is $4\frac{1}{2}$ inches in diameter and the forcing plunger is $\frac{3}{8}$ inch in diameter. The latter plunger is joined to a lever 3 inches from the fulcrum, and the hand pressure is applied 36 inches from the fulcrum. The resistance to the punch is 40,000 pounds. What pressure must be exerted on the end of the lever?

SOLUTION.—When the end of the lever is depressed 12 in., the forcing plunger moves $12 \times \frac{3}{36} = 1$ in. The ratio of the areas of the two plungers is $(4\frac{1}{2})^2 : (\frac{3}{8})^2 = \frac{81}{4} \div \frac{9}{64} = 144$; hence, when the smaller plunger moves 1 in., the larger moves $\frac{1}{144}$ in. The hand pressure p exerted through 12 in. causes the resistance of 40,000 lb. to move $\frac{1}{144}$ in. Hence, $p \times 12 = 40{,}000 \times \frac{1}{144}$, or

$$p = \frac{40{,}000}{12 \times 144} = 23.15 \text{ lb. Ans.}$$

PRESSURES ON SURFACES

7. Pressure on the Bottom of a Vessel.—*The pressure on the flat horizontal bottom of a vessel due to the weight of the contained liquid is independent of the shape of the vessel and is equal to the weight of a prism of the liquid whose base has the same area as the bottom of the vessel and whose height is the distance between the bottom and the upper surface of the liquid.*

The truth of this principle may be shown by the following example: In Fig. 5, the pressure on the bottom of the vessel a is, of course, equal to the weight of the water it contains. If the area of the bottom of the vessel b and the depth of the liquid contained in it are the same as in the vessel a, the pressure on the bottom of b will be the same as on the bottom of a. Suppose that the bottoms of the vessels a and b are 6 inches square, and the part cd in the vessel b is 2 inches square, and that both vessels are filled with

water. Then, since the weight of 1 cubic inch of water is $62.5 \div 1{,}728 = .03617$ pound and the volume of vessel *a* 6 in. $\times$ 6 in. $\times$ 24 in. $= 864$ cubic inches, the weight of the water in *a* is $864 \times .03617 = 31.25$ pounds. Hence, the total pressure on the bottom of the vessel *a* is 31.25 pounds. The weight of water contained in the part *e c* of vessel *b* is $6 \times 6 \times 10 \times .03617 = 13.02$ pounds; hence, the pressure on the bottom due to this weight is 13.02 pounds. The weight of the part contained in *c d* is $2 \times 2 \times 14 \times .03617 = 2.0255$ pounds.

Imagine a thin partition to be placed horizontally at the bottom of the narrow part *c d*. The pressure on this partition would be 2.0255 pounds, the weight of the water above it; and, the area being 4 square inches, the pressure per square inch would be $2.0255 \div 4 = .5064$ pound. It is evident, therefore, that the upper horizontal layer of water in *e c*, just like the imaginary partition, is subjected to a downward pressure of .5064 pound per square inch.

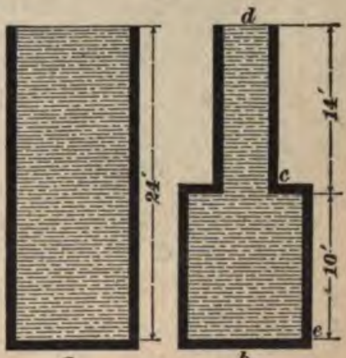


FIG. 5

According to Pascal's law, this pressure is transmitted equally in all directions; therefore, every square inch of the large part of the vessel *b* is subjected to a pressure of .5064 pound, owing to the body of water above *c*. The horizontal area of the part *e c* is $6 \times 6 = 36$ square inches, and the total pressure due to the weight of the water in the small part is $.5064 \times 36 = 18.23$ pounds. Hence, the total pressure on the bottom of *b* is $13.02 + 18.23 = 31.25$ pounds, the same result as in the case of the vessel *a*. In both cases, the pressure is uniformly distributed over the bottom, and its intensity, or the pressure per square inch, is $31.25 \div 36 = .868$ pound.

If an additional pressure of 10 pounds per square inch were applied to the upper surface of both vessels, the total

pressure on their bottoms would be $31.25 + (6 \times 6 \times 10) = 31.25 + 360 = 391.25$ pounds. In case this pressure were obtained by means of a weight placed on a piston, as shown in Figs. 1 and 2, the weight for the vessel *a* would be $6 \times 6 \times 10 = 360$ pounds, and for the vessel *b* it would be $2 \times 2 \times 10 = 40$ pounds.

8. Pressure Due to a Given Head.—The pressure of water at any given level in a tank or reservoir is due to the weight of the column of water above that level. This column of water is known as the **head** and is measured vertically from the level of the surface of water to the given level in the tank or reservoir. For example, suppose that a vessel, as shown in Fig. 6, is filled with liquid, and imagine a horizontal partition or thin piston placed at *m n*. The water above produces a downward pressure on the partition, just as if it were the bottom of a vessel, and, as shown in Art. 7, this pressure depends only on the area of the partition, and its depth below the surface of the liquid. The pressure per square inch on this partition is the weight of a prism of the liquid whose base is 1 square inch and whose height is the distance from *m n* to the surface.

FIG. 6

The horizontal layer of the liquid at *m n* may now be considered as the partition or piston, and the downward pressure per square inch on this layer will be the same as with the thin piston first assumed.

The distance from any horizontal layer of a body of liquid to the surface of the liquid is termed the head for that layer.

Let h = head, in feet, for any horizontal layer;

p = pressure per square inch on the layer, in pounds;

w = weight of a column of liquid 1 foot long and 1 square inch in cross-section.

Then, $$p = wh \qquad (1)$$

A column 1 foot long and 1 square inch in cross-section contains 12 cubic inches. Since water weighs .03617 pound per cubic inch, for water $w = .03617 \times 12 = .434$ pound.

Hence, when the liquid is water,

$$p = .434\,h \qquad (2)$$

EXAMPLE.—The depth of water in a stand pipe is 80 feet. (*a*) What is the pressure, per square inch, on the bottom? (*b*) What is the pressure, per square inch, on a layer 65 feet from the surface?

SOLUTION.—

(*a*) $p = .434\,h = .434 \times 80 = 34.72$ lb. per sq. in. Ans.

(*b*) $p = .434 \times 65 = 28.21$ lb. per sq. in. Ans.

9. Head Required for Given Pressure.—Using the symbols of Art. **8**, and solving formula **1** for h,

$$h = \frac{p}{w} \qquad (1)$$

When the liquid is water,

$$h = \frac{p}{.434} = 2.304\,p \qquad (2)$$

EXAMPLE.—What must be the height of water in a stand pipe to give a pressure of 80 pounds per square inch on the bottom?

SOLUTION.—The required head is

$$h = 2.304 \times 80 = 184.32 \text{ ft.} \quad \text{Ans.}$$

10. Upward and Lateral Pressure.—So far, only downward pressure has been discussed. Upward pressure and lateral pressure will now be considered. Let the vessel shown in Fig. 7 be filled with liquid to the level *a*. The part of the liquid in *a b* acts on the layer at *b* and produces over that surface an intensity of pressure of $w\,h_1$ pounds per square inch, where h_1 is the head on the layer at *b*. According to Pascal's law, this pressure per unit

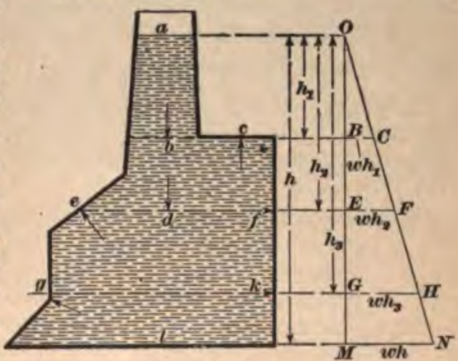


FIG. 7

area acts on all the bounding surfaces below the level b; hence, the liquid $a\,b$ exerts a pressure of $w\,h_1$ pounds per square inch at c, e, f, and k, at right angles to the surfaces. At e and f, however, there is additional pressure due to the weight of liquid between the level $e\,f$ and the level b; but at c, the liquid below the level b can exert no additional pressure; hence, the total upward pressure per unit of area at c is $w\,h_1$ pounds, the same as the downward pressure on the layer at b.

Consider now the layer of liquid $e\,f$ at a distance h_2 below the surface a. The pressure on this layer due to the weight of the liquid above is $w\,h_2$ pounds per square inch; and by Pascal's law, this pressure is transmitted to all parts of the bounding surface below the level $e\,f$, just as though the layer $e\,f$ were a solid piston. The liquid below $e\,f$ can exert no pressure at the points e and f; hence, at these points the pressure per unit area is the same as the downward pressure on the layer $e\,f$; namely, $w\,h_2$ pounds per square inch. The same reasoning shows that the lateral pressure per unit area at the points g and k is $w\,h_3$, where h_3 is the head for the layer $g\,k$. The following important law, therefore, is a direct consequence of Pascal's law:

The pressure per unit area at any point of a surface, whether downwards, upwards, lateral, or oblique, depends only on the depth of the point below the surface of the liquid. The magnitude of this pressure is given by formula **1** of Art. **8**, $p = w\,h$.

EXAMPLE.—In Fig. 7, suppose the depth of the various layers below the level a to be as follows: b, 10 feet; $e\,f$, 17 feet; $g\,k$, 25 feet; and l, 30 feet. The liquid is water. What are the pressures per square inch at points c, e, f, g, k, and l?

SOLUTION.—Using formula **2** of Art. **8**, in each case,

$$\left.\begin{array}{l}
\text{Pressure at } c \text{ is } .434 \times 10 = 4.34 \text{ lb. per sq. in.}\\
\text{Pressure at } e \text{ is } .434 \times 17 = 7.378 \text{ lb. per sq. in.}\\
\text{Pressure at } f \text{ is } .434 \times 17 = 7.378 \text{ lb. per sq. in.}\\
\text{Pressure at } g \text{ is } .434 \times 25 = 10.85 \text{ lb. per sq. in.}\\
\text{Pressure at } k \text{ is } .434 \times 25 = 10.85 \text{ lb. per sq. in.}\\
\text{Pressure at } l \text{ is } .434 \times 30 = 13.02 \text{ lb. per sq. in.}
\end{array}\right\}\text{Ans.}$$

The varying pressure per unit area for different points below the surface of a liquid may be determined, graphically,

as follows: In Fig. 7, the vertical line OM is drawn from the upper level a to the lower level l, and from M a length MN is laid off horizontally to represent, to some scale, the pressure per square inch at l. Points O and N are joined by a straight line and horizontal lines are drawn through c, f, and k to cut OM and ON. Then, the horizontal intercepts BC, EF, and GH represent the pressures per unit area at their respective depths to the same scale that MN represents the pressure per unit area on the surface l.

11. Pressure Due to External Load.—If the surface of the liquid is subjected to a pressure, this pressure, according to Pascal's law, is transmitted undiminished to all points of the enclosing vessel and must be added to the pressure due to the weight of the liquid. For example, suppose the surface a, Fig. 7, to be subjected to a pressure of 30 pounds per square inch. In the example of Art. **10**, the pressure at c due to the head of water is 4.34 pounds per square inch. To this is added the 30 pounds per square inch, giving as the total pressure $4.34 + 30 = 34.34$ pounds per square inch.

The pressure at e and f is $7.378 + 30 = 37.378$ pounds per square inch.

The pressure at g and k is $10.85 + 30 = 40.85$ pounds per square inch.

The pressure at l is $13.02 + 30 = 43.02$ pounds per square inch.

Let G = total load on surface of liquid;
A = area of surface loaded in square inches;
$p_o = \frac{G}{A}$ = pressure per square inch on surface;
p = pressure per square inch at a point h feet below surface of liquid.

Then,
$$p = wh + p_o = wh + \frac{G}{A} \qquad (1)$$

When the liquid is water,
$$p = .434\,h + p_o = .434\,h + \frac{G}{A} \qquad (2)$$

EXAMPLE 1.—A vessel filled with ordinary sea-water has a circular bottom 13 inches in diameter. A column of ordinary sea-water 12 inches

high by 1 square inch in cross-section weighs .445 pound. The top of the vessel is fitted with a piston 3 inches in diameter, on which is laid a weight of 75 pounds; what is the pressure per square inch on the bottom, if the depth of the water is 18 inches?

SOLUTION.—The head is 18 in. = 1.5 ft. Using formula **1**,

$$p = wh + \frac{G}{A} = .445 \times 1.5 + \frac{75}{3 \times 3 \times .7854} = 11.28 \text{ lb. per sq. in. Ans.}$$

EXAMPLE 2.—In a vertical boiler, the water level is 5 feet above the top of the firebox and the steam pressure is 65 pounds per square inch; what is the pressure per square inch on the top of the firebox?

SOLUTION.—Here p_0 = 65 lb. per sq. in. By formula **2**,

$$p = .434\, h + p_0 = .434 \times 5 + 65 = 67.17 \text{ lb. per sq. in. Ans.}$$

EXAMPLE 3.—A suspended vertical cylinder is tested for the tightness of its heads by filling it with water. A pipe whose inside diameter is ¼ inch and whose length is 20 feet is screwed into a hole in the upper head and then filled with water; what is the pressure per square inch on each head if the cylinder is 5 feet long?

SOLUTION.—For the upper cylinder head, h = 20 ft.; and for the lower, h = 25 ft. From formula **2** of Art. **8**,

$$p = .434\, h = .434 \times 20 = 8.68 \text{ lb. per sq. in., on the upper head}$$
$$p = .434 \times 25 = 10.85 \text{ lb. per sq. in., on the lower head. Ans.}$$

EXAMPLE 4.—In example 3, if the pipe is fitted with a piston weighing ¼ pound, and a 5-pound weight is laid on it, what is the pressure per square inch on the upper head?

SOLUTION.—The area of the surface subjected to the load of 5¼ lb. is $(\frac{1}{4})^2 \times .7854 = .0491$ sq. in. The head h is 20 ft. Using formula **2**,

$$p = .434 \times 20 + \frac{5\frac{1}{4}}{.0491} = 115.6 \text{ lb. per sq. in. Ans.}$$

12. Surface Level of Liquids.—Since the pressure on the bottom of a vessel due to the weight of the liquid is dependent only on the height of the liquid, and not on the shape of the vessel, it follows that if a vessel has a number of radiating tubes, as shown in Fig. 8, the water in all tubes will be at the same level, no matter what may be the shape of the tubes. For, if the water were higher in one tube than in the others, the downward pressure at the bottom due to the height of the water in this tube would be greater than that due to the height of the water in the other tubes. Consequently, the upward pressure in the other tubes would also be greater; the equilibrium would be destroyed, and water

would flow from this tube into the vessel, and rise in the other tubes until it stood at the same level in all, when equilibrium would be restored. This principle is expressed in the familiar saying, "water seeks its level."

This explains why city water reservoirs are located on high elevations, and why water leaving the hose nozzle spouts

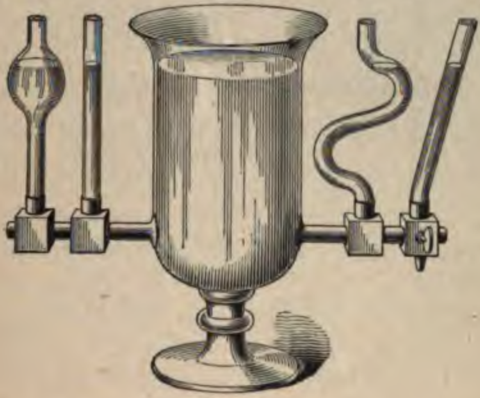


FIG. 8

so high. If there were no frictional and air resistance, the water would spout to a height equal to the level of the water in the reservoir. If a vertical pipe with a length equal to the vertical distance between the nozzle and the level of the water in the reservoir were attached to the nozzle, the water would just reach the end of the pipe; and, if the top of the pipe were lowered slightly, the water would flow over. Fountains, canal locks, and artesian wells are examples of the working of this principle.

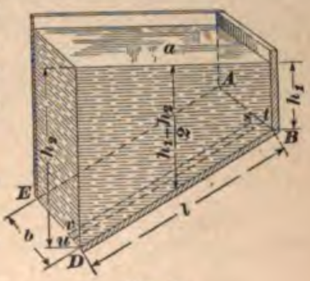


FIG. 9

13. Total Pressure on a Flat Surface.—Suppose $ABDE$, Fig. 9, to be a rectangular plane surface sustaining liquid pressure. In practice, such surfaces occur in dams, sluice gates, tanks with sloping sides, etc. Let the edges AB and DE be horizontal, and let the surface be

inclined at any angle with the horizontal. The depth of the edge AB below the liquid level a is h_1; hence, the pressure per square inch at this edge is $w h_1$. Likewise, the depth of the edge DE being h_2, the pressure at this edge is $w h_2$. Consider a strip of the surface $s t u v$, 1 inch wide and parallel to the edge BD. The pressure at the end $s t$ being $w h_1$ pounds per square inch, and that at the other end, $u v$, being $w h_2$ pounds per square inch, the average pressure per square inch is $\frac{w h_1 + w h_2}{2}$. Let the length of the edge BD be l inches; then the area of the strip is l square inches; and the total pressure on the strip is $l \frac{w h_1 + w h_2}{2}$ pounds. If b denotes the width DE of the surface, in inches, there will be b strips like $s t u v$, each 1 inch wide, and the total pressure P on the surface is

$$P = b l \frac{w h_1 + w h_2}{2} = b l w \frac{h_1 + h_2}{2} \qquad (1)$$

Now, $b \times l$ is the area of the surface and $\frac{h_1 + h_2}{2}$ is the depth of the center of gravity G of the surface below the liquid level. Hence, *the total liquid pressure on a rectangular plane surface is the product of the area of the surface and the pressure per unit area due to the head of the liquid above the center of gravity*. Sometimes the following statement is used: *The total liquid pressure is equal to the weight of a prism of liquid whose base is the area of the surface and whose height is the distance of the center of gravity of the surface below the liquid level.*

It can be proved that this law holds good for all flat surfaces whatever their outline. For a flat plate, only part of whose surface sustains liquid pressure, the law holds good for the part below the liquid level.

Let A = area in square inches, of a plane surface sustaining liquid pressure;

h_0 = head, in feet, of liquid above center of gravity;

w = weight of a column of liquid 1 foot long and 1 square inch in cross-section;

P = total pressure on surface, in pounds.

Then,
$$P = A w h_0 \qquad (2)$$

EXAMPLE.—A vertical sluice gate, Fig. 10, is $3\frac{1}{2}$ feet wide, and 5 feet of it is below the water level; what is the total pressure on the gate?

SOLUTION.—The area sustaining liquid pressure is $3\frac{1}{2} \times 5 \times 144 = 2{,}520$ sq. in. The center of gravity of the submerged part is $5 \div 2 = 2\frac{1}{2}$ ft. below the water surface; hence, by formula **2**, $P = A w h_0$
$= 2{,}520 \times .434 \times 2\frac{1}{2} = 2{,}734.2$ lb. Ans.

FIG. 10

14. Resolved Pressure in a Given Direction.—Let $BCDE$, Fig. 11, represent a rectangular plane area and p the normal pressure on it per unit of area. The force p may then be resolved into the rectangular components f_1 and f_2. Let m denote the angle between p and its component f_1; then $f_1 = p \cos m$. Suppose, now, that a plane is passed through the edge BC perpendicular to the force f_1, and let D and E be projected on this plane, giving the points D' and E'. The rectangular area $BCD'E'$ is the projection of the area $BCDE$ on the plane perpendicular to f_1. Evidently, angle $EBE' = m$, since p is perpendicular to BE and f_1 is perpendicular to BE'; then $BE' = BE \cos m$, and area $BCD'E'$ = area $BCDE \times \cos m$.

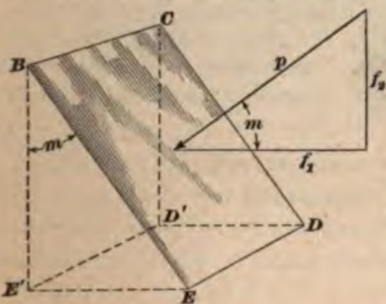


FIG. 11

Now, f_1 is the pressure per unit of area in its direction; hence, if A denotes the area of $BCDE$, the total pressure on this area in the direction of f_1 is

$$f_1 A = p \cos m \times A = p A \cos m$$

But, $A \cos m$ is the area of $BCD'E'$, the projection of $BCDE$ on the plane perpendicular to f_1. Hence, *the pressure exerted by a fluid in any direction on a plane surface, is equal to the weight of a prism of the fluid whose base is the area of the projection of the surface on a plane at right angles to the direction considered, and whose height is the*

depth of the center of gravity of the surface below the level of the liquid.

EXAMPLE.—The earthwork dam, Fig. 12, sustains a water pressure along the face AB, inclined at 60° to the horizontal; the distance AB in contact with water is 18 feet, and the length of the dam is 60 feet. What is the total pressure on the dam in a horizontal direction and in a vertical direction?

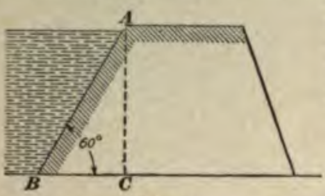


FIG. 12

SOLUTION.—The projection of AB on a vertical plane is AC, and the length of AC is $AB \times \sin 60° = 18 \times .86603 = 15.589$ ft. The length being 60 ft., the projection on the vertical plane of the oblique surface sustaining pressure is $15.589 \times 60 = 935.34$ sq. ft. The center of gravity of AB, and also of AC, lies at a distance of $15.589 \div 2 = 7.794$ ft. below the water level. The prism of water with a base of 935.34 sq. ft. and a height of 7.794 ft. weighs $935.34 \times 7.794 \times 62.5 = 455{,}627$ pounds, which is the total pressure in a horizontal direction. Ans.

The projection BC has a length $AB \times \cos 60° = 18 \times .5 = 9$ ft., and the area of the projection of the oblique surface on the horizontal plane is, therefore, $9 \times 60 = 540$ sq. ft. A prism of water with this base and with a height of 7.794 ft. weighs $540 \times 7.794 \times 62.5 = 263{,}050$ pounds, which is the total downward pressure of the water on the dam.

15. Pressures on Curved and Irregular Surfaces.—If the surface sustaining fluid pressures is not plane, but is curved or irregular, the total pressure on it in any direction is the same as the total pressure would be on the projection

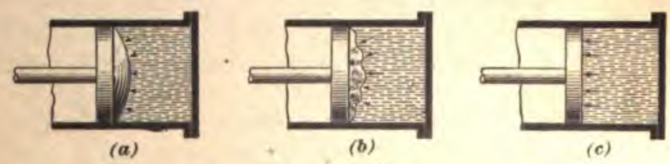


FIG. 13

of the surface on a plane at right angles to the given direction. To illustrate this statement, consider the three pistons shown in Fig. 13. In Fig. 13 (*a*) is shown a piston whose end has the form of a segment of a sphere; in Fig. 13 (*b*)

is shown one of irregular form; and in Fig. 13 (*c*) is shown one with a flat surface. In each case, the projection of the surface sustaining pressure, on a plane perpendicular to the piston rod, is the circular cross-section of the cylinder, and if the pressure per unit area is the same in the three cylinders, the thrust on the piston rod is the same. This law applies to all fluids, liquid or gaseous, and it assumes that the pressure per unit area is the same at all points of the surface.

In the case of a submerged curved or irregular surface, where the pressure per unit area varies with the depth below the surface, this law does not hold except for a projection on a vertical plane.

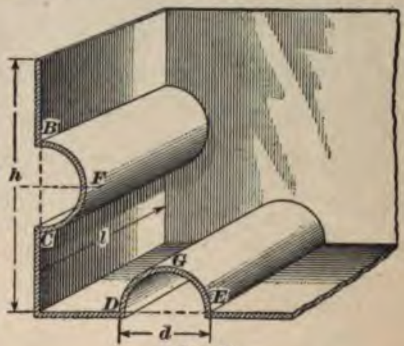


FIG. 14

Suppose, for example, that the side wall of a vessel, Fig. 14, has a cylindrical part BFC. Then the total horizontal pressure on this part is equal to the total horizontal pressure on the vertical plane surface BC, supposing BC to be substituted for the curved surface. This pressure may be found by the use of formula **2**, Art. **13**, using for A the area of the projection, not that of the curved surface; that is, the pressure is equal to the product of the area of the projection and the pressure per square inch due to the head above the center of gravity of the curved surface. Let the diameter BC be 10 inches, the length of the cylinder 15 inches, and the depth of the center of gravity below the surface, 30 inches = $2\frac{1}{2}$ feet. The projected area is $10 \times 15 = 150$ square inches, and the total pressure is $P = AWh_o = 150 \times .434 \times 2\frac{1}{2} = 162.75$ pounds.

For the cylindrical part DGE in the bottom of the vessel, this law is not true. To get the total vertical pressure, the projected area on the bottom, that is, the diameter DE multiplied by the length, must be multiplied by the pressure per

square inch due to a certain head. This head, however, is neither that above the center of gravity of the curved surface, nor that above the bottom. If d denotes the diameter DE of the cylinder, l its length, and h the head of water above the bottom, then the total downward pressure on the cylindrical part DGE is equal to the volume of the water resting on the curved surface, multiplied by the weight of water per unit volume. The total volume above the curved surface is equal to the volume $h\,d\,l$ above the flat side of the cylindrical part minus the volume $\frac{\pi d^2 l}{2 \times 4}$ of the cylindrical part. The volume, per unit of length, resting on the curved surface, is, therefore, $\left(h\,d - \frac{\pi d^2}{8}\right)$. If h is in feet and d in inches, the total downward pressure, per inch of length of the cylindrical part, is $.434\left(h\,d - \frac{\pi d^2}{8}\right)$. If h_1 represents the average head, in feet, of the water above the curved surface, then $.434\,h_1\,d$ is also the vertical pressure per inch of length of the cylindrical part, and therefore $.434\,h_1\,d = .434\left(h\,d - \frac{\pi d^2}{8}\right)$. Hence,

$$h_1 = h - \frac{\pi d}{8}$$

That is, the head required to give the true total pressure is $h - \frac{\pi d}{8}$, which is quite different from that above the center of gravity.

The whole subject of the pressure in any direction on a curved or irregular surface may be summarized as follows:

1. If the external pressure is so great that the pressure due merely to the weight of the fluid may be neglected in comparison, the total pressure in any direction is equal to the product of the projection of the surface on a plane perpendicular to that direction, and the pressure per unit of area.

2. When the pressure is due wholly or in part to the weight of the fluid, as in the case of surfaces submerged in liquids, the total pressure in any direction, except the horizontal

direction, cannot in general be determined except for regularly curved surfaces, such as those of spheres, cylinders, and cones, and for these the calculations are difficult and must be usually made by higher mathematics.

3. The total horizontal pressure on any surface, however, is easily found. It is precisely the same as the horizontal pressure on the projection of the given surface on a vertical plane.

16. Fluid Pressures in Cylinders.—Let a cylinder contain, under pressure, a fluid which may be a liquid, as in city water mains and stand pipes, or a gas, as in the case of pipes carrying steam, compressed air, etc. The pressure per square inch is assumed to be the same for all points; that is, the slight increase of pressure on the lower half of the pipe due to the weight of the fluid contained in it is neglected. Consider the upper half of a short section of a pipe or cylinder, as $AMBE$, Fig. 15. This is a curved surface whose under side is subjected to a uniform pressure of, say p pounds per square inch. Now, according to Art. **15**, the total pressure on this surface in a vertical direction is the same as if the pressure acted on the projection of the surface on a horizontal plane. If d denotes the inner diameter of the cylinder, and l the length BE of the given section, then dl is evidently the area of this projection; hence, the total vertical pressure is $P = pdl$.

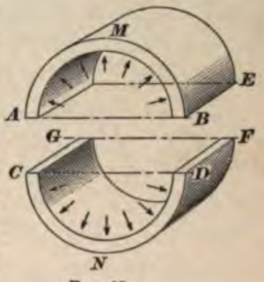


FIG. 15

The lower half $CNDFG$, likewise, is a curved surface subjected to a pressure of p pounds per square inch. It has the same projected area as the upper half; hence, the total vertical pressure on it, acting downwards, is likewise $P = pdl$.

In the figure, the two halves are shown separated; the same reasoning holds good, however, when they are united. The two forces, one of which tends to force the upper half upwards and the other to force the lower half downwards, are equal and opposite, and their combined tendency is to separate the two halves by tearing them apart along the

lines DF and CG. This tendency is resisted by the tenacity of the material comprising the cylinder. Evidently, the imaginary plane cutting the cylinder into halves may have any direction; and the force tending to separate the halves will be the same.

EXAMPLE 1.—The pressure in a water main is 60 pounds per square inch and the inner diameter of the main is 8 inches; what is the total force tending to separate one half from the other in a section of pipe 10 feet long?

SOLUTION.—Here $p = 60$, $d = 8$ in., and $l = 10$ ft. $= 120$ in.

$$P = p\,d\,l = 60 \times 8 \times 120 = 57{,}600 \text{ lb. Ans.}$$

EXAMPLE 2.—A section of stand pipe is 4 feet long and 10 feet in diameter, and its upper edge is 70 feet below the level of the water; what is the force tending to separate one half from the other?

SOLUTION.—The center of gravity of the section is 2 ft. below the upper edge, and therefore $70 + 2 = 72$ ft. below the water level. The pressure due to this head is $p = .434 \times 72$. The projected area of one half is $4 \times 10 = 40$ sq. ft. $= 5{,}760$ sq. in. The total pressure is therefore $.434 \times 72 \times 5{,}760 = 179{,}988.48$ lb. Ans.

17. Fluid Pressure in a Sphere.—For a hollow sphere filled with fluid under pressure, the same principle applies as in the case of the cylinder.

Let d = inner diameter of sphere in inches;
p = pressure per square inch;
P = total pressure in one direction tending to separate one half of sphere from the other.

The projection of one half of the sphere on the plane cutting it into two halves is a circle whose diameter is d and whose area is $\frac{1}{4}\pi d^2$; therefore,

$$P = \tfrac{1}{4}\pi d^2 p$$

EXAMPLE.—A spherical boiler 8 feet in diameter contains steam and water having a pressure of 45 pounds per square inch; what is the force tending to separate one half from the other?

SOLUTION.— $P = \frac{1}{4}\pi d^2 p = .7854 \times 96^2 \times 45 = 325{,}721$ lb. Ans.

EXAMPLES FOR PRACTICE

1. The diameter of the plunger of a hydraulic press used in an engineering establishment is 12 inches. Water is forced into the cylinder of the press by means of a small pump having a plunger

whose diameter is $\frac{3}{4}$ inch and stroke 4 inches. What pressure is exerted by the large plunger when the force acting on the small plunger is 125 pounds? Ans. 32,000 lb.

2. If the small plunger in example 1 makes 96 working strokes per minute: (*a*) how long will it take the large plunger to move 8 inches? (*b*) what is the velocity ratio?

Ans. $\begin{cases} (a) & 5\frac{1}{3} \text{ min.} \\ (b) & 256:1 \end{cases}$

3. A vertical pipe, 88 feet high, is filled with water. (*a*) What is the pressure per square inch on the bottom? (*b*) If the diameter of the pipe is 8 inches, what is the total pressure tending to burst a section $2\frac{1}{2}$ inches high, whose center of gravity is 21 feet from the bottom?

Ans. $\begin{cases} (a) & 38.2 \text{ lb. per sq. in., nearly} \\ (b) & 581.56 \text{ lb.} \end{cases}$

4. A hollow sphere 8 inches in diameter is connected to a pipe in which a head of water 10 feet above the center of gravity of the sphere is maintained, and the upper surface of the water is subjected to a pressure of 8 pounds per square inch; what is the total pressure tending to rupture the sphere on a plane passing through its center? Ans. 620 lb., nearly

BUOYANT EFFECT OF LIQUIDS

IMMERSION AND FLOTATION

18. Buoyant Effort.—In a mass of liquid at rest, suppose a part of the liquid $m\,m$, Fig. 16, to become rigid without changing its form. Having the same density as before, the part will evidently remain at rest and will be held in equilibrium. Let the weight of the rigid part be denoted by B, the total downward pressure by P, and the total upward pressure by P_1. That the part may remain at rest, the upward force P_1 must balance the downward forces P and B; that is,

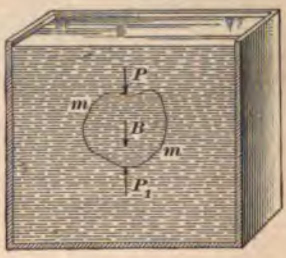


FIG. 16

$$P_1 = P + B; \text{ or, } P_1 - P = B$$

The difference between the total upward and downward pressures, $P_1 - P$, or the amount by which P_1 exceeds P, is called the **buoyant effort,** which acts upwards. Since

$P_1 - P = B$, it is evident that the buoyant effort is equal to B, the weight of the rigid mass $m\,m$ of the liquid.

Suppose that a solid body is immersed in the liquid, taking the place of the part $m\,m$, which is considered rigid, and let the weight of this body be denoted by G. Evidently, the body will be subjected to the same vertical pressures P and P_1; therefore, since the vertical forces acting on the body are G and P downwards and P_1 upwards, the net vertical force downwards is

$$P + G - P_1 = G - (P_1 - P) = G - B$$

B is equal to the buoyant effort and is the weight of the liquid displaced by the body. Hence, *when a solid body is immersed in a liquid, a buoyant effort equal to the weight of the liquid displaced acts upwards and opposes the action of gravity. The weight of the body, as shown by a scale, is decreased by an amount equal to the buoyant effort, that is, the weight of liquid displaced.* This is called the **principle of Archimedes**, because it was first stated by him.

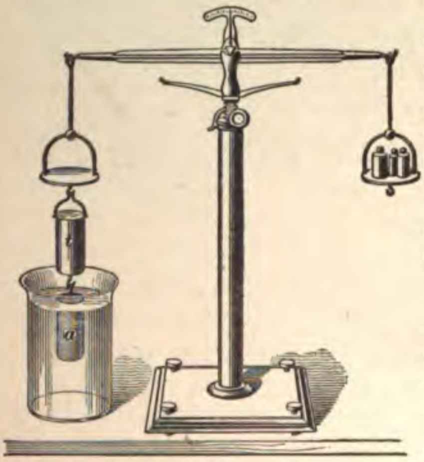


FIG. 17

Archimedes's principle may be experimentally demonstrated with the beam scales shown in Fig. 17. From one scale pan suspend a hollow cylinder of metal t, and below that a solid cylinder a of the same size as the hollow part of the upper cylinder. Put weights in the other scale pan until they exactly balance the two cylinders. If a be immersed in water, the scale pan containing the weights will descend, showing that a has lost some of its weight. Now, fill t with water, and the volume of water that can be poured into t will equal that displaced by a. The scale pan that contains the weights will gradually rise until t is filled, when the scales balance again.

If a body immersed in a liquid has the same weight as the liquid it displaces, then $G = B$, the net vertical force $G - B$ is zero, and the body will remain at rest at any depth below the surface.

If the body is heavier than the liquid it displaces, then G is greater than B, the net vertical force $G - B$ is downwards, and the body will sink to the bottom.

If, on the other hand, the body is lighter than the liquid it displaces, B is greater than G, the net vertical force $B - G$ is upwards, and the body will rise to the surface.

An interesting experiment in confirmation of the above facts may be performed as follows: Drop an egg into a glass jar filled with fresh water. The mean density of the egg being a little greater than that of the water, the egg will fall to the bottom of the jar. Then, dissolve salt in the water, stirring it so as to mix the fresh and salt water. The salt water will presently become denser than the egg and the egg will rise. Now, if fresh water is poured in until the egg and the water have the same density, the egg will remain in any position in which it may be placed below the surface of the water when the latter is perfectly at rest.

19. Floating Bodies.—A body lighter than an equal volume of a liquid rises to the surface when immersed in the liquid, and floats. For equilibrium, the buoyant effort represented by B must be just equal to the weight G of the body. But since B is the weight of the liquid displaced, the following principle results: *The weight of the liquid displaced by a floating body is equal to the weight of the body.*

The depth at which a body floats in a liquid depends on the relative weights of equal volumes of the body and the liquid. If the body is nearly as heavy as the liquid, it will sink until it displaces nearly its own volume; if very light compared with the liquid, the larger part of the body will be above the liquid surface. For example, ice has a specific gravity of about .9; hence, about one-tenth of an iceberg appears above the surface of the water and nine-tenths is submerged. The specific gravity of pine is about

.5; hence, about one-half of a pine log is submerged and one-half is above water.

The following examples show applications of the principles given in the preceding paragraphs:

EXAMPLE 1.—Water-tight canvas air bags are used for raising sunken ships. They are sunk when collapsed, attached to the ship by divers, and then filled with air by means of pumps from above. (*a*) If the capacity of a bag is 200 cubic feet, what is its buoyant effort? (*b*) How many bags will be required to lift 600 tons?

SOLUTION.—The weight of the bag and enclosed air may be neglected. (*a*) The buoyant effort of one bag is the weight of water displaced by the full bag, that is, $200 \times 62.5 = 12{,}500$ lb. Ans.

(*b*) To raise 600 tons, therefore, $\frac{600 \times 2{,}000}{12{,}500} = 96$ bags are required. Ans.

EXAMPLE 2.—A cast-iron cylinder is 14 inches long and 8 inches in diameter on the outside; it is closed at the ends and the metal is $\frac{1}{2}$ inch thick throughout. Will the cylinder float or sink in water?

SOLUTION.—The volume of the entire cylinder is $.7854 \times 8^2 \times 14 = 703.72$ cu. in. The hollow part has a length of 13 in. and a diameter of 7 in.; its volume is therefore $.7854 \times 7^2 \times 13 = 500.3$ cu. in. The volume of metal is therefore $703.72 - 500.3 = 203.42$ cu. in. Taking the weight of cast iron as 450 lb. per cu. ft., the weight of the cylinder is $\frac{203.42}{1{,}728} \times 450 = 52.973$ lb. If immersed, the cylinder displaces 703.72 cu. in. of water, which weighs $\frac{703.72}{1{,}728} \times 62.5 = 25.453$ lb. The buoyant effort being less than the weight, the cylinder will sink.

SPECIFIC GRAVITY

20. Specific Gravity of Solids.—The specific gravity of a body has been defined as the ratio between the weight of the body and the weight of an equal volume of water.

Archimedes's principle affords an easy and accurate method of finding the specific gravity of solids not easily soluble in water. The body is weighed first in air, then in water, suspended by a string from a scale pan, thus taking the place of the two cylinders shown in Fig. 17. *The difference between the two weights will be the weight of an equal volume of water. The ratio of the weight in air to the difference thus found will*

be the specific gravity. The abbreviation for specific gravity is Sp. Gr.

Let G = weight of solid in air;

G' = weight in water;

$G - G'$ = weight of a volume of water equal to volume of solid.

Then,
$$\text{Sp. Gr.} = \frac{G}{G - G'} \tag{1}$$

EXAMPLE 1.—A body in air weighs $36\frac{1}{4}$ ounces and in water 30 ounces; what is its specific gravity?

SOLUTION.—Substituting in formula **1**,

$$\text{Sp. Gr.} = \frac{36\frac{1}{4}}{36\frac{1}{4} - 30} = \frac{36\frac{1}{4}}{6\frac{1}{4}} = 5.8. \text{ Ans.}$$

If the body is lighter than water, a piece of iron or other substance sufficiently heavy to sink both must be attached to it. *Then, weigh both bodies in air and both in water. Weigh each separately in air, and weigh the heavier body in water. Subtract the weights of the bodies in air and in water, and the result will be the weight of a volume of the water equal to the volume of the two bodies. Find the difference of the weights of the heavy body in air and in water, and the result will be the weight of a volume of water equal to the volume of the heavy body. Subtract this last result from the former, and the result will be the weight of a volume of water equal to the volume of the light body. The weight of the light body in air divided by the weight of an equal volume of water is the specific gravity of the light body.*

Let G = weight of both bodies in air;

G' = weight of both bodies in water;

G_1 = weight of light body in air;

G_2 = weight of heavy body in air;

G_3 = weight of heavy body in water.

Then, the specific gravity of the light body is given by

$$\text{Sp. Gr.} = \frac{G_1}{(G - G') - (G_2 - G_3)} \tag{2}$$

EXAMPLE 2.—A piece of cork weighs 4.8 ounces in air; a piece of cast iron weighs 36 ounces in air and 31 ounces in water; the weight of the iron and cork together in water is 15.8 ounces. (*a*) What is the specific gravity of the cork? (*b*) Of the cast iron?

SOLUTION.—(*a*) Substituting in formula **2** the values given,

$$\text{Sp. Gr.} = \frac{4.8}{(40.8 - 15.8) - (36 - 31)} = \frac{4.8}{20} = .24$$

the specific gravity of the cork. Ans.

(*b*) By formula **1**, Sp. Gr. $= \frac{G}{G - G'} = \frac{36}{36 - 31} = 7.2$, the specific gravity of the iron. Ans.

21. Specific Gravity of Liquids.—To find the specific gravity of a liquid:

Weigh an empty flask; fill it with water, then weigh it again and find the difference between the two results; this difference will equal the weight of the water. Then weigh the flask filled with the liquid, and subtract the weight of the flask; the result is the weight of a volume of the liquid equal to the volume of the water. The weight of the liquid divided by the weight of the water is the specific gravity of the liquid.

Let G = weight of the flask and liquid;
G_1 = weight of the flask and water;
G_2 = weight of the flask.

Then, $$\text{Sp. Gr.} = \frac{G - G_2}{G_1 - G_2}$$

EXAMPLE.—If the weight of the flask is 8 ounces, the weight when filled with water is 33 ounces, and when filled with alcohol 28 ounces, what is the specific gravity of the alcohol?

SOLUTION.—Substituting in the formula,

$$\text{Sp. Gr.} = \frac{28 - 8}{33 - 8} = .8. \quad \text{Ans.}$$

22. Specific Gravity of Gases.—The specific gravity of a gas is found by dividing the weight of a given volume of the gas by the weight of an equal volume of air or hydrogen. Air is usually taken as the standard for gases, but hydrogen is sometimes used. Water is the standard for liquids and solids. The specific gravity of gases is usually taken at a temperature of 32° F. The determination of the weight of gases being a very difficult matter, the method of finding the specific gravity of gases will not be described.

23. Hydrometers.—Instruments called **hydrometers** are in general use for determining quickly and accurately the specific gravities of liquids and some forms of solids.

They are of two kinds: (1) hydrometers of constant weight, as **Beaume's**; (2) hydrometers of constant volume, as **Nicholson's.**

A hydrometer of constant weight is shown in Fig. 18. It consists of a glass tube near the bottom of which are two bulbs. The lower and smaller bulb is loaded with mercury or shot, so as to cause the instrument to remain in a vertical position when placed in the liquid. The upper bulb is filled with air, and its volume is such that the whole instrument is lighter than an equal volume of water.

FIG. 18

The point to which the hydrometer sinks when placed in water is usually marked, the tube being graduated above and below the mark in such a manner that the specific gravity of the liquid can be read directly. It is customary to have two instruments, one with the zero point near the top of the stem for use with liquids heavier than water, and the other with the zero point near the bulb for use with liquids lighter than water.

These instruments are more commonly used for determining the degree of concentration or dilution of certain liquids,

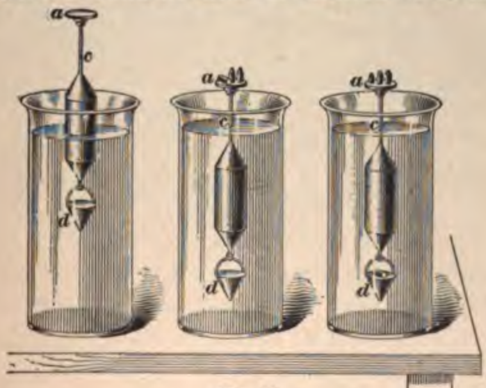


FIG. 19

as acids, alcohol, milk, solutions of sugar, etc., rather than their actual specific gravities. They are then known as *acidimeters*, *alcoholometers*, *lactometers*, *saccharimeters*, etc., according to the use to which they are put.

Nicholson's hydrometer is shown in Fig. 19. It consists of a hollow cylinder carrying at its lower end a basket d heavy enough to keep the apparatus upright when placed in water. At the top of the cylinder is a vertical rod, to which is attached a shallow pan a for holding weights, etc. The cylinder and its basket together must be so much lighter than water that a certain weight G must be placed in the pan in order to sink the apparatus to a given point c on the rod. The body whose specific gravity is to be found must weigh less than G. It is placed in the pan a, and enough weight G_1 is added to sink the point c to the water level. It is evident that the weight of the given body is $G - G_1$. The body is now removed from the pan a and placed in the basket d, an additional weight being added to sink the point c to the water level. Represent the weight now in the pan by G_2. The difference $G_2 - G_1$ is the weight of a volume of water equal to the volume of the body. Hence,

$$\text{Sp. Gr.} = \frac{G - G_1}{G_2 - G_1}$$

EXAMPLE.—The weight necessary to sink the hydrometer to the point c is 16 ounces; the weight necessary when the body is in the pan a is 7.3 ounces, and when the body is in the basket d, 10 ounces; what is the specific gravity of the body?

SOLUTION.—By the above formula, $\text{Sp. Gr.} = \frac{16 - 7.3}{10 - 7.3} = \frac{8.7}{2.7} = 3.222$. Ans.

24. Volume of Irregularly Shaped Bodies.—Archimedes's principle gives a very easy and accurate method of finding the volume of an irregularly shaped body. Thus, subtract its weight in water from its weight in air, and the difference will be the weight of an equal volume of water. Divide this weight by .03617, and the result will be the volume in cubic inches; or divide by 62.5, and the result will be the volume in cubic feet.

If the specific gravity of the body is known, its cubic contents can be found by dividing its weight by its specific gravity, and then dividing again by either .03617 or 62.5 to find the volume either in cubic inches or in cubic feet, as may be desired.

EXAMPLE.—A certain body has a specific gravity of 4.38 and weighs 76 pounds; how many cubic inches are there in the body?

SOLUTION.— $\frac{76}{4.38 \times .03617} = 479.72$ cu. in. Ans.

Since the weight of a cubic foot of water varies at different temperatures, and with the amount of impurities it contains, it is necessary to have some standard when getting the specific gravity. This standard is pure distilled water at its maximum density, which occurs at a temperature of 39.1° F. Owing to the difficulty of maintaining the low temperature, however, other temperatures, as 60° or 62° F., are frequently used in practice, while in some instances a temperature of 32° F. is used. At 39.1° F. pure water weighs 62.425 pounds per cubic foot; but for ordinary calculations it is customary to take it as weighing 1,000 ounces, or 62.5 pounds, per cubic foot.

EXAMPLES FOR PRACTICE

1. If a certain quantity of red lead weighs 5 pounds in air, and 4.441 pounds in water, what is its specific gravity? Ans. 8.94+

2. A piece of iron weighs 1 pound in air and .861 pound in water. A piece of wood weighing 1 pound in air is attached to the iron and together they weigh .2 pound in the water. What is the specific gravity (*a*) of the iron? (*b*) of the wood? Ans. $\begin{cases} (a)\ 7.194 \\ (b)\ .602 \end{cases}$

3. An empty flask weighed 13 ounces; when filled with water, it weighed 22 ounces; and when filled with sulphuric acid, 29.56 ounces. What was the specific gravity of the acid? Ans. 1.84

4. How many cubic feet of brick, having a specific gravity of 1.9, are required to weigh 260 pounds? Ans. 2.19 cu. ft., nearly

CAPILLARITY

25. Capillarity is the mutual attraction or repulsion between a liquid and a solid that causes the fluid in contact with the solid to rise above or sink below the level of the surrounding liquid. If a clean glass rod is placed vertically in water, the water will be drawn up around the rod, as shown at *a* in Fig. 20. If the same rod is placed in mercury, the liquid will be depressed instead of raised. On examination, it will be found that water wets the glass, while mercury does not. If the rod is greased and placed in water, the surface of the water will be depressed about the rod. If a

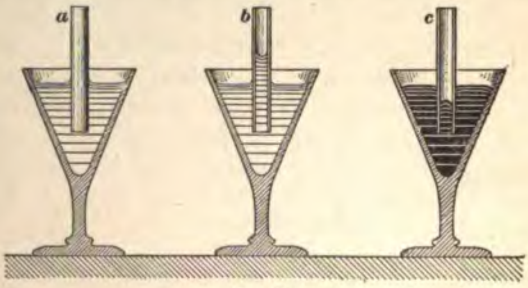


FIG. 20

clean lead or zinc strip is placed in mercury, the surface of the mercury will be raised about the strip. The greased rod when removed from the water will be dry, no water adhering to it, while the mercury will adhere to the lead or zinc strip, which will be wet.

In general, *all liquids that will wet the solids placed in them will be lifted, while those that do not wet them will be pushed down.*

These phenomena are due to a force known as capillarity, because they are best shown in very fine or hair-like tubes called *capillary tubes.* When the surface of the liquid rises at the surface of the rod or tube, the force causing the

phenomenon is called **capillary attraction,** and when the surface of the liquid is depressed the force is called **capillary repulsion.** At *b*, Fig. 20, is shown a glass tube with one end immersed in water, and at *c* is shown a glass tube immersed in mercury. The surface of the water in the tube *b* is concave, while the surface of the mercury in the tube *c* is convex.

The amount to which a liquid will ascend or be depressed, varies inversely as the diameter of the tube. Thus, water will rise twice as far in a tube $\frac{1}{32}$ inch in diameter as in one $\frac{1}{16}$ inch in diameter.

There are many illustrations of capillary action. It causes the oil to rise between the fibers of a lamp wick to the place of combustion. It enables cloth and sponges to take up moisture, and causes blotting paper to absorb ink. When paper is sized, however, so that its pores are filled, the ink dries by evaporation.

PNEUMATICS

PROPERTIES OF AIR AND GASES

ATMOSPHERIC PRESSURE

1. Pneumatics is the branch of mechanics that treats of the properties of gases.

2. Weight of Air.—As water is the most common of liquids, so air is the most common of gases. It was supposed by the ancients that air was imponderable—that is, that it weighed nothing—and it was not until about the year 1650 that it was proved that air really had weight. The ratio of the weight of a volume of air at 60° F., under atmospheric pressure, to that of an equal volume of water under the same conditions, is about 1 : 816; that is, under these conditions, air is only about $\frac{1}{816}$ as heavy as water. If a body is immersed in water and weighs less than the volume of water displaced, the body will rise and extend partly out of the water. The same is true to a certain extent of air. If a vessel made of light material is filled with a gas lighter than air and the total weight of vessel and gas together is less than the weight of the volume of air that they displace, the vessel will rise; the construction and operation of balloons are based on this principle.

3. Mercury Column Equivalent to Atmospheric Pressure.—Since air has weight, it is evident that the enormous quantity of air that constitutes the atmosphere must exert a considerable pressure on the earth. This is easily proved by taking a long glass tube, closed at one end, and filling it with mercury. If the finger is placed over the open

end, so as to keep the mercury from running out, and the tube is inverted and placed in a cup of mercury, as shown in Fig. 1, the mercury will fall, then rise, and after a few oscillations will come to rest at a height of 29.92, or roughly 30, inches above the top of the mercury in the cup. The height will always be the same under the same atmospheric conditions, allowance being made for the effects of capillary attraction.

FIG. 1

Now, if the atmosphere has weight, it must press on every square unit of the surface of the mercury in the cup with equal intensity, except on that part of the surface covered by the tube. According to Pascal's law, which is given in *Hydrostatics*, this pressure is transmitted equally in all directions. There being nothing in the tube, except the mercury, to counterbalance the upward pressure of the air, the mercury falls in the tube until it exerts a downward pressure on the upper surface of the mercury in the cup sufficiently great to counterbalance the upward pressure produced by the atmosphere. In order that there shall be equilibrium, the pressure of the air per unit of area on the upper surface of the mercury in the cup must be equal to the pressure exerted per unit of area by the mercury inside the tube at the level of the surface of the mercury in the cup.

Suppose that the area of the inside of the tube is 1 square inch; then, since mercury is 13.59 times as heavy as water, and water weighs .03613 pound per cubic inch, the weight of the mercurial column is $.03613 \times 13.59 \times 29.92 = 14.69$ pounds. More accurate determinations make the average value at sea level 14.696 pounds per square inch at a temperature of 32° F.

Since this weight, exerted on 1 square inch of the liquid in the cup, just produces equilibrium, it is plain that the pressure of the outside air is 14.696 pounds on every square inch of surface. In engineering practice, however, a value of 14.7 is generally used. Since a column of mercury 29.92 inches high corresponds to a pressure of 14.696 pounds per square inch, a height of 1 inch corresponds to a pressure of $\frac{14.696}{29.92}$ = .4911 pound per square inch. For ordinary calculations, this value is taken at .49 pound.

4. Vacuum.—The space between the upper end of the tube and the upper surface of the mercury inside the tube is absolutely devoid of pressure and contains no substance, solid, liquid, or gaseous. This total absence of pressure constitutes a perfect **vacuum.** If there were a gas of some kind in this space, no matter how small its quantity might be, it would expand, filling the space, and its pressure would cause the column of mercury to fall still lower and become shorter, according to the amount of gas present; a **partial vacuum** would then exist in this space. That is, there would be only a partial absence of pressure. If the mercury falls 1 inch, so that the column is only 29 inches high, it is said, in ordinary language, that there is a vacuum of 29 inches. If it falls 8 inches, it is said that there is a vacuum of 22 inches. If it falls 16 inehes, it is said that there is a vacuum of 14 inches, and so on. Suppose that the vacuum gauge of a condensing engine shows a vacuum of 26 inches; this indicates that there is enough air in the condenser to depress the mercury column 30 − 26 = 4 inches, and to produce a pressure of $\frac{4}{30} \times 14.7 = 1.96$ pounds per square inch inside the condenser. Written as a formula, this would be

$$p = \frac{14.7 \times (30 - r)}{30}$$

in which p = absolute pressure in condenser, in pounds per square inch;

r = reading of vacuum gauge, in inches of mercury.

In all cases where the mercury column is used to measure a vacuum, the height of the column, in inches, gives the number of inches of vacuum. Thus, if the column is 5 inches high, or the vacuum gauge reads 5 inches, the vacuum is 5 inches. That is, the difference between the pressure in the condenser and the pressure of the atmosphere outside the condenser is equivalent to 5 inches of mercury. The height of the column of mercury is therefore a measure of the difference of pressures existing inside and outside the tube.

FIG. 2

5. Water Column Equivalent to Atmospheric Pressure.—If the tube had been filled with water instead of mercury, the height of the column of water necessary to balance the pressure of the atmosphere would have been $30 \times 13.59 = 407.7$ inches $= 33.975$ feet, generally taken in practice as 34 feet. This means that if a tube is filled with water, inverted, and placed in a dish of water in the same way as in the experiment made with the mercury, the resulting height of the column of water will be about 34 feet.

BAROMETERS

6. The **barometer** is an instrument used for measuring the pressure of the atmosphere. There are two kinds in general use—the *mercurial* and the *aneroid*. The **mercurial barometer** is shown in Fig. 2. The principle is the same as in the case of the inverted tube shown in Fig. 1. The tube and cup at the bottom are protected by a brass or iron casing. At the top of the tube is a graduated scale that can be read to $\frac{1}{1000}$ inch by means of a vernier. Attached to the casing is an accurate thermometer for determining the temperature of the outside air at the time the barometric observation is taken. This is necessary, since mercury expands when

the temperature is increased, and contracts when the temperature falls; for this reason, a standard temperature is assumed and all barometer readings are reduced to this temperature. This standard temperature is usually taken at 32° F., at which temperature the height of the column of mercury is about 30 inches. Another correction is made for the altitude of the place above sea level, and a third correction for the effects of capillary attraction.

FIG. 3

7. An **aneroid barometer** is shown in Fig. 3. This instrument is made in various sizes, from the size of a large watch up to one with an 8- or 10-inch face. The barometer consists of a cylindrical box of metal, with a top of thin, elastic, corrugated metal. The air is removed from the box. When the atmospheric pressure increases, the top is pressed inwards; and when it is diminished the top is pressed outwards by its own elasticity aided by a spring beneath.

These movements of the cover are transmitted and multiplied by a combination of delicate levers that act on an index hand and cause it to move either to the right or to the left over a graduated scale. These barometers are self-correcting; that is, compensated for variations in temperature. They are portable, occupy but a small space, and are so delicate that they are said to show a difference in the atmospheric pressure when transferred from a table to the floor. They must be handled with care, as they are easily injured. The mercurial barometer is the standard.

It will be observed that in Fig. 3 the zero of the outer scale does not coincide with the 30 mark on the inner scale, as would seem consistent. The reason for placing the zero opposite the 31 division is thus explained. The average reading at sea level, or zero, is 29.92 inches. If the zero were set at 30, therefore, the reading would be on the scale part of the time, and off the scale the remainder of the time, according to the variation of pressure while working at sea level. Consequently, the zero is placed at the highest reading obtained at sea level, which is about 31 inches.

8. With air as with water, the lower the location the greater the pressure, and the higher the location the less the pressure. At the level of the sea, the height of the mercurial column is about 30 inches; at 5,000 feet above the sea, it is 24.7 inches; at 10,000 feet above the sea, it is 20.5 inches; at 15,000 feet above the sea, it is 16.9 inches; at 3 miles, it is 16.4 inches; and at 6 miles above the sea level it is 8.9 inches.

The heaviness also varies with the altitude; that is, a cubic foot of air at an elevation of 5,000 feet above sea level will not weigh as much as a cubic foot at sea level. This is proved conclusively by the fact that at a height of 3½ miles the mercurial column measures but 15 inches, indicating that half the weight of the entire atmosphere is below that. It is known that the height of the earth's atmosphere is at least 50 miles; hence, the air just before reaching the limit must be in an exceedingly rarefied state. It is by means

of barometers that great heights are measured. The aneroid barometer has the heights marked on the dial, so that they can be read directly. With the mercurial barometer, the heights must be calculated from the reading.

9. Atmospheric pressure is everywhere present and presses on all objects in all directions with equal intensity. If a book is laid on the table, the air presses on it in every direction with an average force of 14.7 pounds per square inch. It would seem as if it would take considerable force to raise a book from the table, since, if the size of the book were 8 inches by 5 inches, the pressure on it is $8 \times 5 \times 14.7 = 588$ pounds; but there is an equal pressure beneath the book to counteract the pressure on the top. It would now seem as if it would require a great force to open the book, since there are two pressures of 588 pounds each, acting in opposite directions, and tending to crush the book. This would be so but for the fact that there are layers of air between the leaves acting upwards and downwards with a pressure of 14.7 pounds per square inch.

If two metal plates are made as smooth and flat as it is possible to get them, and the face of one is laid on the face of the other, so that the air is almost entirely excluded from between them, it will take an immense force, compared with the weight of the plates, to separate them. This is because the full pressure of 14.7 pounds per square inch is then exerted on each of the plates with no counteracting equal pressure between them.

If a piece of flat glass be laid on a flat surface that has been previously moistened with water, it will require considerable force to separate them; this is because the water helps to fill up the pores in the flat surface and glass, and thus creates a partial vacuum between the glass and the surface, thereby reducing the counter pressure beneath the glass.

PROPERTIES OF GASES

10. Pressure of Gases.—According to modern and now generally accepted theories, a gas consists of molecules that are relatively far apart and are moving incessantly. The molecules in their motion frequently strike each other, and those near the walls of the vessel containing the gas frequently strike the walls and rebound. It is this continual striking of molecules against the containing walls that gives rise to what we call **pressure.**

Suppose that the cylinder, Fig. 4, contains any gas, as air. There is a definite number of molecules in the enclosed space, all moving to and fro. The number of molecules striking any portion of the containing wall per second remains on an average the same and produces a constant pressure that is the same at all points of the enclosing walls. If the piston be pushed into the dotted position, the molecules are crowded closer together and, provided that they move with the same average velocity as before, more of them must strike any given part of the enclosing walls, say 1 square inch, in a given interval of time than in the first case. This means that the pressure is greater in the second case than in the first.

FIG. 4

Conversely, if the piston be moved to the right so as to increase the space filled by the molecules, the molecules will be farther apart along the enclosing walls and each square inch of wall will receive fewer impacts than before; thus the pressure per square inch will be less. In general, therefore, with a given number of molecules, or, what is the same thing, with a given weight of gas, the pressure is greater the smaller the space into which the gas is crowded.

11. Expansiveness of Gases.—No matter how large the enclosing vessel may be, the gas will fill it. Thus, if the cylinder in Fig. 4 is considered indefinitely long and the piston is moved a great distance to the right, the molecules

will still move in all parts of the enlarged space. They will, of course, be farther apart and will move greater distances before colliding with each other; and any part of the bounding wall will be struck less frequently.

If a bladder or a football, partly filled with air, is placed under a glass jar, called a **receiver**, and the air is then exhausted from the receiver, the bladder or football will immediately expand, as shown in Fig. 5. The pressure being removed from the outside of the bladder by exhausting the air from the receiver, the striking against the inner walls of the bladder by the molecules of contained air causes the bladder to distend.

FIG. 5

12. Measurement of Pressure.—There are two ways of measuring the pressure of a gas; by means of an instrument called a **manometer**, and by means of a **gauge**. The manometer generally used is practically the same as a mercurial barometer, except that the tube is much longer, so that pressures equal to several atmospheres may be measured; and it is enlarged and bent into a **U** shape at the lower end. Both lower and upper ends are open, the lower end being connected to the vessel containing the gas whose pressure it is desired to measure.

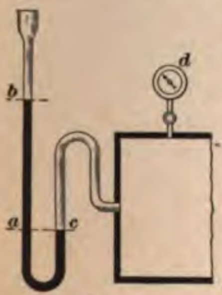


FIG. 6

The pressure is measured by the change of level of the mercury in the tube. In Fig. 6, suppose the pressure of the gas is such as to depress the level of the mercury to *c*, while in the other branch of the tube it rises to *b*. Now, the points *a* and *c* being at the same level, the pressure at those points must be equal. The pressure at *a* is due partly to

the weight of the column of mercury $a b$ above a and partly to the pressure of the atmosphere on the top of the column at b. In Art. **3**, it was shown that 1 inch of mercury corresponds to .49 pound per square inch; hence, if h denotes the difference in level, in inches, between the points b and a, .49 h is the pressure per square inch due to the weight of the column $a b$. Add to this the atmospheric pressure, 14.7 pounds per square inch, and the total pressure at a, and therefore at c, is $p = .49\ h + 14.7$. The pressure thus obtained is the pressure above vacuum, or the **absolute pressure.**

The ordinary pressure gauge d, Fig. 6, measures not the absolute pressure but the *excess* of the pressure of the gas over atmospheric pressure. The pressure thus measured is called the **gauge pressure.** Evidently, the column $a b$ measures the gauge pressure, for since $p = .49\ h + 14.7$, it follows that $.49\ h = p - 14.7$, which is the excess of pressure of the gas over atmospheric pressure.

In speaking of the pressure of a gas, the gauge pressure is usually meant; thus, to say the pressure of steam in a boiler is 70 pounds per square inch, means that that is the pressure shown by the gauge and is the excess of the steam pressure over the pressure of the atmosphere.

To obtain the absolute pressure when the gauge pressure is given, the atmospheric pressure is added. Thus, if the gauge pressure is 70 pounds per square inch, the absolute pressure is $70 + 14.7 = 84.7$ pounds per square inch. Conversely, the atmospheric pressure is subtracted from the absolute pressure to obtain the gauge pressure.

13. Units of Pressure.—Pressures are ordinarily expressed in *pounds per square inch*. Thus, in speaking of the pressure of steam in a boiler or steam cylinder, it is said to be 60, 80, or 100 pounds per square inch, as the case may be. Pressures may also be expressed in *pounds per square foot*. To reduce pounds per square inch to pounds per square foot, multiply by 144; thus, atmospheric pressure, which is 14.7 pounds per square inch, is $14.7 \times 144 = 2{,}116.8$

pounds per square foot. Conversely, pressures per square foot are divided by 144 to get pressures per square inch.

Small pressures, such as are produced by fans, are often measured in *ounces per square inch*, and sometimes in inches of water. It has been shown that a head of 1 foot of water gives a pressure of .434 pound per square inch, and hence 1 inch of water is $\frac{1}{12}$ of .434 = .0362 pound per square inch = 5.2 pounds per square foot.

Another unit of pressure is the *inch of mercury*, which equals .49 pound per square inch, as explained in Art. **3.** The relations between these five units of pressure are shown in Table I. The pressures given in any horizontal line are

TABLE I

RELATIVE UNIT PRESSURES

Pounds per Square Foot	Pounds per Square Inch	Ounces per Square Inch	Inches of Water	Inches of Mercury
1	$\frac{1}{144}$	$\frac{1}{9}$	.192	.0142
144	1	16	27.7	2.04
9	$\frac{1}{16}$	1	1.73	.13
5.2	.0362	.58	1	.074
70.56	.49	7.84	13.6	1
2,116.8	14.7	235.2	408 = 34 ft.	30

the same, using the units at the tops of the columns. Thus, 1 inch of water equals 5.2 pounds per square foot, equals .0362 pound per square inch, equals .58 ounce per square inch, etc. The last line gives atmospheric pressure expressed in the various units. These values deviate slightly in some cases from the theoretically exact values, but they are the values generally used in practical problems.

EXAMPLE.—The pressure of the forced draft of a marine boiler is 4.3 inches of water; what is the pressure expressed in the other units?

SOLUTION.— 1 in. of water = 5.2 lb. per sq. ft.; hence,

$p = 4.3 \times 5.2 = 22.36$ lb. per sq. ft.
$= 4.3 \times .0362 = .1557$ lb. per sq. in.
$= 4.3 \times .58 = 2.494$ oz. per sq. in.
$= 4.3 \times .074 = .3182$ in. of mercury. Ans.

14. Temperature.—With solids and liquids the **temperature**, or *hotness*, of a body has little or no influence on its behavior, and usually need not be taken into account. In the case of gases, however, the temperature exerts a marked influence and must always be considered.

Consider the cylinder, Fig. 4, to be filled with gas at some definite temperature as shown by a thermometer. Now, keeping the piston in the same position, let the gas be heated, say by a flame applied to the cylinder walls. The gas will grow hotter, that is, the temperature indicated by the thermometer will rise; and at the same time it will be found that the pressure of the gas, as shown by a gauge, will also rise. Now, since the volume of the gas has not changed, there will be the same number of molecules per cubic inch as before. But since the pressure has risen, the number of impacts in a second must have increased, which means that the molecules are moving at a greater speed than before.

According to the modern theory of heat, the temperature depends directly on the speed with which the molecules of a body are moving. When the temperature of a body rises, its molecules move faster; when the temperature falls, the molecules move more slowly. This subject will be treated more fully in *Heat*, Part 1.

15. Absolute Temperature.—Suppose that the gas in the cylinder shown in Fig. 4 is at the temperature of melting ice, 32° F., and has some definite pressure denoted by p. If the gas is heated, so that its temperature rises 1°, that is, to 33° F., the pressure will increase, provided that the piston is fixed so that the gas cannot change in volume. It has been found, by experiment, that the change of pressure is $\frac{1}{492}$ of the pressure at 32°; therefore the pressure at 33° is $p + \frac{1}{492}p = p\left(1 + \frac{1}{492}\right)$. If the temperature rises another degree, that is, to 34° F., the pressure increases by the same amount, $\frac{1}{492}p$, and at 34° the pressure is therefore

Suppose that the pressure at 32° is 100 pounds per square inch; then the rise of pressure for a rise in temperature of 1° F. is $100 \times \frac{1}{492} = .20325$ pound per square inch. If the temperature of the gas is raised to 212°, the boiling point of water, the rise in temperature is $212° - 32° = 180°$, the change in pressure is $.20325 \times 180 = 36.585$ pounds per square inch, and the new pressure is $100 + 36.585 = 136.58$ pounds per square inch.

The change of pressure during a change of temperature may be represented graphically, as shown in Fig. 7. Through some point a, a vertical line is drawn and on it are marked points b, c, d, etc., corresponding to the temperatures shown. The vertical line thus corresponds to a thermometer scale. From the point b, which corresponds to 32°, the horizontal distance be is laid off to represent a pressure of 100 pounds per square inch, and from d, which represents a temperature of 212°, df is laid off, representing to the same scale the new pressure of 136.58 pounds per square inch.

FIG. 7

Let f and e be joined by a straight line; then the horizontal distance between bd and ef at any point gives the pressure for the corresponding temperature. The point c, for example, corresponds to 100° and cg represents the pressure at 100° to the same scale that be represents 100 pounds per square inch.

If the temperature is lowered below 32° the pressure decreases at the same rate; that is, for each degree it decreases $\frac{1}{492}$ of the pressure at 32°. Hence, if the line fe is prolonged, the horizontal lines below be give the pressures for temperatures below 32°. Thus, ah represents the pressure at 0° and kl that at $-100°$, that is, 100° below 0°.

As the temperature lowers, the horizontal distances between the lines grow smaller, and at m, where the lines intersect, the

distance becomes zero. This means that if in some way the temperature could be lowered to that represented by the point *m*, the gas would exert no pressure on the enclosing walls. The molecules would be motionless and would no longer strike the walls, and the gas would be entirely without heat. The temperature corresponding to *m* is called the **absolute zero of temperature,** and temperatures reckoned from this zero as a starting point are called **absolute temperatures.**

Now let the position of the point *m* be determined. Starting at 32°, the decrease of pressure per degree is $\frac{1}{492}$ of the pressure at 32°. For a cooling of 10°, the decrease is $\frac{10}{492}$; for a cooling of 100°, $\frac{100}{492}$; and so on. Let the temperature be lowered 492° below 32°; then the decrease in pressure is $\frac{492}{492}$ of the pressure at 32°, or the whole of the original pressure. Hence, the gas exerts no pressure at 492° below 32° F., and this temperature is the absolute zero indicated by the point *m*.

Since from *b* to *m*, Fig. 7, is 492° and from *b* to *a* is 32°, *m* must be 492° − 32° = 460° below 0°. Hence, when the Fahrenheit scale is used, the absolute zero is 460° below the ordinary zero, and 460 must be added to the ordinary temperature to obtain the corresponding absolute temperature.

Let t = ordinary temperature as given by Fahrenheit thermometer;

T = corresponding absolute temperature.

Then, $$T = t + 460°$$

and $$t = T - 460°$$

The temperature at which water boils is 212°. The corresponding absolute temperature is 212° + 460° = 672°. The ordinary temperature corresponding to an absolute temperature of 900° is 900° − 460° = 440°. When the word temperature is used alone, the ordinary temperature is meant.

16. Gases and Vapors.—When liquids are heated to sufficiently high temperatures, they are converted into gases. When water, for example, is heated, it boils, giving off steam. If the heating is continued long enough, all the water will be

evaporated. Steam is simply a gas formed by boiling water. The water, however, does not change immediately into a perfect gas. It first takes the form of **vapor,** in which state it is readily condensed. As the heating is continued, it takes a more stable gaseous form, and eventually becomes a perfect gas.

LAWS RELATING TO CHANGE OF STATE

17. The **state** of a gas is defined by three things: pressure, volume, temperature. Suppose a given quantity of gas has a definite pressure, volume, and temperature. If the volume is changed either by compressing the gas or by permitting it to expand, either the pressure or the temperature or both will change at the same time. The change from the original to the new pressure, volume, and temperature is called a *change of state.* If p_1, v_1, and t_1 denote the original, and p_2, v_2, and t_2 the final pressure, volume, and temperature, respectively, the gas is said to change from the state p_1, v_1, t_1 to the state p_2, v_2, t_2. The laws governing these changes of state are here given and explained.

BOYLE'S LAW

18. Pressure and Volume of a Gas.—*If the temperature of a given quantity of gas remains the same and the volume is changed, the pressure varies inversely as the volume.*

This is known as **Boyle's law.** If the piston, Fig. 4, be moved to the left until the gas occupies one-half of its original volume, double the number of molecules will strike a square inch of the wall in 1 second, provided that they have the same speed as at first; that is, provided that the gas remains at the same temperature; but this means that the new pressure is double the original pressure. If the piston is moved still farther until the new volume is one-third of the original volume, the new pressure will be three times the original pressure. If, on the other hand, the piston is moved to the right until the new volume is three times the original volume, the new pressure is one-third of the original

pressure; and so on. This relation may also be expressed as follows: *The volume of a given quantity of gas at constant temperature varies inversely as the pressure.*

Suppose 3 cubic feet of air to be under a pressure of 60 pounds per square inch in a cylinder fitted with a movable piston; then, the product of the volume and pressure is $3 \times 60 = 180$. Let the pressure be decreased to 30 pounds per square inch; then the volume will be 6 cubic feet, and $30 \times 6 = 180$, as before. Let the pressure be decreased to $7\frac{1}{2}$ pounds per square inch, the volume will then be increased to $60 \div 7\frac{1}{2} = 8$ times the original, or $8 \times 3 = 24$ cubic feet, and $24 \times 7\frac{1}{2} = 180$, as in the two preceding cases. It will now be noticed that if a gas is enclosed within a confined space and allowed to expand without change of temperature, *the product of the pressure and the corresponding volume for one position of the piston is the same as for any other position of the piston.* If the piston were to compress the air, the same result would be obtained.

Let p_0 and v_0 be the original pressure and volume of gas, and p_1 and v_1 any other pressure and corresponding volume. Then, if the temperature does not change,

$$p_0 v_0 = p_1 v_1 = \text{a constant quantity} \qquad (1)$$

In general, if p_1, v_1 and p_2, v_2 denote the pressures and volumes for any two states,

$$p_1 v_1 = p_2 v_2 \qquad (2)$$

19. Graphic Representation of Boyle's Law.—The change of state of a gas according to Boyle's law may be represented graphically, as shown in Fig. 8. On cross-section paper take two section lines OP and OV intersecting at O; let the spaces along OV represent volumes, and let those along OP represent pressures. In the figure, one space on OV represents 1 cubic foot, and on OP a pressure of 5 pounds per square inch. Where each horizontal line cuts OP is marked the pressure, as *5*, *10*, *15*, etc., and on the intersections of the vertical lines with OV are marked the proper volumes represented by the lines, as *2*, *4*, *6*, etc.

Suppose, now, that the gas in its original state has a pressure of 50 pounds per square inch and a volume of 2 cubic feet. This state is represented by the point *A*, opposite *50*, on *O P*, and above *2* on *O V*. The product of the pressure and volume is $50 \times 2 = 100$, and this is the constant quantity of formula **1** of Art. **18.** As the volume increases to 3, 4, 5, etc. cubic feet, the corresponding pressures are found as follows: Let p_o and v_o be the initial pressure and volume, and p and v the pressure and volume at any other point. Then $p v = p_o v_o = 50 \times 2 = 100$.

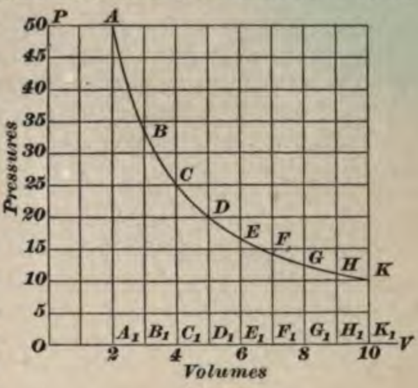


FIG. 8

For $v = 3$, $p \times 3 = 100$, or $p = \frac{100}{3} = 33\frac{1}{3}$ lb. per sq. in.
For $v = 4$, $p \times 4 = 100$, or $p = \frac{100}{4} = 25$ lb. per sq. in.
For $v = 5$, $p \times 5 = 100$, or $p = \frac{100}{5} = 20$ lb. per sq. in.
For $v = 6$, $p \times 6 = 100$, or $p = \frac{100}{6} = 16\frac{2}{3}$ lb. per sq. in.
For $v = 7$, $p \times 7 = 100$, or $p = \frac{100}{7} = 14\frac{2}{7}$ lb. per sq. in.
For $v = 8$, $p \times 8 = 100$, or $p = \frac{100}{8} = 12\frac{1}{2}$ lb. per sq. in.
For $v = 9$, $p \times 9 = 100$, or $p = \frac{100}{9} = 11\frac{1}{9}$ lb. per sq. in.
For $v = 10$, $p \times 10 = 100$, or $p = \frac{100}{10} = 10$ lb. per sq. in.

These different states are represented by the points *B*, *C*, *D*, etc.; thus, for $v = 5$ and $p = 20$, find the intersection of the section line through *20* on *O P* and that through *5* on *O V*; this is the point *D*. A curve drawn through these points, as shown, will represent the pressures and volumes at any positions.

In problems that involve Boyle's law and in which formula **1** of Art. **18** is used, the pressures must be absolute, not gauge, pressures. Throughout the remainder of this Section, all pressures will be understood as absolute pressures unless the contrary is distinctly stated.

20. The following examples illustrate the use of Boyle's law, as expressed in formula **1** of Art. **18.**

EXAMPLE 1.—If 10 cubic feet of air at atmospheric pressure, 14.7 pounds per square inch, is compressed at constant temperature until the gauge pressure reaches 36 pounds per square inch, what is the new volume?

SOLUTION.—The original pressure p_0 is 14.7, the volume v_0 is 10, and the final absolute pressure p is $36 + 14.7 = 50.7$. From the formula $p\,v = p_0\,v_0$,

$$v = \frac{p_0\,v_0}{p} = \frac{14.7 \times 10}{50.7} = 2.9 \text{ cu. ft., nearly. Ans.}$$

EXAMPLE 2.—An engine uses compressed air. When the flow of air to the cylinder is stopped, the volume of air in the cylinder is .84 cubic foot and the pressure is 60 pounds gauge; when the piston reaches the end of its stroke the air has expanded to a volume of 1.35 cubic feet. If the temperature remains constant, what is the gauge pressure of the air at the end of the expansion?

SOLUTION.—The initial absolute pressure p_0 is $60 + 14.7 = 74.7$, $v_0 = .84$, and $v_1 = 1.35$. From formula **1** of Art. **18**, $p_0\,v_0 = p_1\,v_1$, or

$$p_1 = \frac{p_0\,v_0}{v_1} = \frac{74.7 \times .84}{1.35} = 46.48 \text{ lb. per sq. in.}$$

This is the absolute pressure; hence the gauge pressure is $46.48 - 14.7 = 31.78$ lb. per sq. in. Ans.

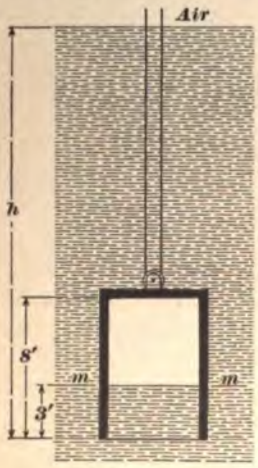


FIG. 9

EXAMPLE 3.—A cylindrical diving-bell 8 feet high, shown in Fig. 9, is lowered into sea-water until the water rises in the bell to a height of 3 feet above the bottom edge of the bell; what is the depth of the bottom of the bell below the surface of the water?

SOLUTION.—The head of water above the level $m\,m$ of the water in the bell is $(h - 3)$ ft. The weight of a column of sea-water 1 ft. high and 1 sq. in. in area is .445; hence the pressure at $m\,m$ per square inch is $.445\,(h - 3) + 14.7$, the 14.7 being the atmospheric pressure on the surface, which is transmitted without loss. Now, the original pressure of air in the bell was 14.7 lb. per sq. in., and denoting by A the area in square feet of the horizontal circular section of the bell, the original volume was $8\,A$ cu. ft. The air is now compressed so that it occupies only $(8 - 3)\,A$ cu. ft.

$= 5\,A$ cu. ft. From formula **1** of Art. **18**, the pressure is

$$p_1 = \frac{p_0 v_0}{v_1} = \frac{14.7 \times 8\,A}{5\,A} = 23.52 \text{ lb. per sq. in.}$$

Since the pressure of the air on the surface $m\,m$ inside the bell must just balance the upward pressure of the water, $.445(h - 3) + 14.7 = 23.52$, and

$$h = \frac{23.52 - 14.7}{.445} + 3 = 22.82 \text{ ft. Ans.}$$

GAY-LUSSAC'S LAW

21. Pressure and Temperature of Gas.—The **law of Gay-Lussac**, or of **Charles**, is essentially that stated in Art. **15** and illustrated in Fig. 7. *If the volume of a gas remains constant, the increase of pressure per degree rise in temperature is* $\frac{1}{492}$ *of the pressure at the temperature of melting ice, that is, 32° F.*

Let p_0 = pressure at 32° F.;
p = pressure at some other temperature $t°$ F.;
$T = t + 460$, absolute temperature corresponding to temperature t;
$32 + 460 = 492$, absolute temperature of melting ice.

In raising the temperature from 32° to $t°$, the change is $(t - 32)$ degrees, and since for each degree the increase of pressure is $\frac{1}{492}$ of p_0, the increase for $(t - 32)$ degrees is $\frac{t-32}{492}$ of p_0; hence, $p = p_0 + \frac{t-32}{492}p_0 = p_0\left(1 + \frac{t-32}{492}\right)$.

Now, $1 + \frac{t-32}{492} = \frac{492 + t - 32}{492} = \frac{t + 460}{492} = \frac{T}{T_0}$, and $p = p_0\frac{T}{T_0}$, or,

$$\frac{p}{p_0} = \frac{T}{T_0} \qquad (1)$$

Let p_1 and T_1 equal pressure and absolute temperature for one state of a gas and p_2 and T_2 equal pressure and absolute temperature for a second state of the same gas; from the formula just given, $\frac{p_1}{p_0} = \frac{T_1}{T_0}$, also $\frac{p_2}{p_0} = \frac{T_2}{T_0}$. Dividing one equation by the other, member by member,

$$\frac{p_1}{p_2} = \frac{T_1}{T_2} \qquad (2)$$

The following statement of Gay-Lussac's law follows from formulas **1** and **2**: *The volume remaining the same, the pressure of a gas varies directly as the absolute temperature.*

EXAMPLE 1.—A certain weight of air is heated from 60° to 144° at constant volume; if the initial absolute pressure was 20 pounds per square inch, what is the final pressure?

SOLUTION.— $p_1 = 20$, $T_1 = 60 + 460 = 520$, and $T_2 = 144 + 460 = 604$; using formula **2**, $\frac{20}{P_2} = \frac{520}{604}$, or

$$P_2 = \frac{20 \times 604}{520} = 23.23 \text{ lb. per sq. in., absolute. Ans.}$$

EXAMPLE 2.—If 10 cubic feet of air has a pressure of 30 pounds per square inch, absolute, at a temperature of 70° F., to what temperature must it be raised in order that the final pressure may be 50 pounds per square inch, absolute, the volume being unchanged?

SOLUTION.—Here $p_1 = 30$, $p_2 = 50$, and $T_1 = 70 + 460 = 530$. From formula **2**, $\frac{30}{50} = \frac{530}{T_2}$, whence

$$T_2 = \frac{530 \times 50}{30} = 883\tfrac{1}{3}°. \quad t_2 = T_2 - 460° = 883\tfrac{1}{3}° - 460° = 423\tfrac{1}{3}° \text{ F. Ans.}$$

COMBINATION OF BOYLE'S AND GAY-LUSSAC'S LAWS

22. Pressure, Volume, and Temperature of a Gas. Consider a certain weight of gas contained in a vessel, and in its first state a, Fig. 10, let the pressure, volume, and absolute temperature be p_1, v_1, and T_1, respectively. Now, keeping the temperature at T_1, let the pressure and volume change to p' and v_2, so that in the state b it is p', v_2, and T_1. Finally, let the state change from b to c during which the volume v_2 remains the same, but p' changes to p_2 and T_1 to T_2. Comparing state c with state a, it is seen that pressure, volume, and temperature have changed.

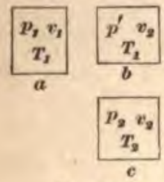


FIG. 10

There is a relation between the p_1, v_1, and T_1 of state a and the p_2, v_2, and T_2 of state c. In passing from state a to state b, the temperature remains constant and the change follows Boyle's law; hence, from formula **2**, Art. **18**,

$$p_1 v_1 = p' v_2$$

In passing from state b to state c, the volume is constant and the change is made according to Gay-Lussac's law; hence,

$$\frac{p'}{p_2} = \frac{T_1}{T_2}$$

From the first equation, $p' = \frac{p_1 v_1}{v_2}$; and from the second, $p' = p_2 \frac{T_1}{T_2}$. Placing these two values of p' equal to each other,

$$\frac{p_1 v_1}{v_2} = p_2 \frac{T_1}{T_2}$$

or,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

This formula is very important. It shows that with a given weight of a gas, the quotient obtained by dividing the product of the pressure and volume by the absolute temperature is the same for any state of the gas.

EXAMPLE.—A certain quantity of air has a volume of 40 cubic feet, a pressure of 30 pounds per square inch, absolute, and a temperature of 80° F.; the air expands until its volume is 56 cubic feet and the temperature falls to 40° F. What is the absolute pressure at the end of expansion?

SOLUTION.—In the first state, $p_1 = 30$, $v_1 = 40$, and $T_1 = 80 + 460 = 540$. In the second state, p_2 is unknown, $v_2 = 56$, and $T_2 = 40 + 460 = 500$. Using the formula given above,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}, \quad \frac{30 \times 40}{540} = \frac{p_2 \times 56}{500}$$

whence, $p_2 = \frac{30 \times 40 \times 500}{540 \times 56} = 19.84$ lb. per sq. in. Ans.

23. Expansion at Constant Pressure.—Let a gas change its state in such a way that the pressure remains the same; then, in the formula derived in Art. **22**, $p_1 = p_2$ and therefore

$$\frac{v_1}{T_1} = \frac{v_2}{T_2}, \text{ or } \frac{v_1}{v_2} = \frac{T_1}{T_2}$$

That is, *the pressure remaining the same, the volume of a given weight of gas varies directly as the absolute temperature.*

EXAMPLE.—A quantity of gas having a volume of 6.4 cubic feet is heated at constant pressure from 62° F. to 315° F.; what is the volume after heating?

SOLUTION.— $v_1 = 6.4$, v_2 is unknown, $T_1 = 62 + 460 = 522$, and $T_2 = 315 + 460 = 775$. From the above formula,

$$\frac{6.4}{v_2} = \frac{522}{775}, \text{ or } v_2 = \frac{6.4 \times 775}{522} = 9.5 \text{ cu. ft. Ans.}$$

24. General Equations.—Consider a unit weight of gas, say 1 pound, and let v denote its volume in any given state. Then, by the formula in Art. 22,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3} = \frac{p v}{T}$$

where p_1, v_1, T_1, and p_2, v_2, T_2, etc. denote different states. The quantity $\frac{p v}{T}$ is therefore a constant; that is, it remains the same for all states of the pound of gas. Let R denote this constant quantity; then, $\frac{p v}{T} = R$, or $p v = R T$.

The value of R varies for different gases. For air, it may be determined if the pressure, volume, and temperature are known at some standard state. Let a pound of air be taken at atmospheric pressure and at the temperature of melting ice; the volume of 1 pound of air under these conditions is 12.387 cubic feet, the pressure is 14.696 pounds per square inch, and the absolute temperature is $460° + 32° = 492°$. Substituting these values in the formula above,

$$14.696 \times 12.387 = R \times 492, \text{ or } R = \frac{14.696 \times 12.387}{492} = .37$$

The exact values are used here because this value for R will be used in future problems. With this value of R, p in this formula must be expressed in pounds per square inch, and v in cubic feet.

EXAMPLE 1.—What is the volume of 1 pound of air at a pressure of 63 pounds per square inch absolute and a temperature of 80° F.?

SOLUTION.—The absolute temperature is $460 + 80 = 540$; from the formula $p v = R T$,

$$v = \frac{R T}{p} = \frac{.37 \times 540}{63} = 3.17 \text{ cu. ft. Ans.}$$

EXAMPLE 2.—At what temperature will 1 pound of air occupy a volume of 16 cubic feet at a pressure of 22 pounds per square inch absolute?

SOLUTION.—Using the formula given above, $22 \times 16 = .37\ T$, whence $T = \frac{22 \times 16}{.37} = 951.35°$. The ordinary temperature is therefore $951.35° - 460° = 491.35°$. Ans.

25. If the weight of gas considered is not precisely 1 pound, the formula in Art. **24** is modified as follows:

Let G = weight of gas, in pounds;
v = volume of the G pounds, in cubic feet.

Then the volume of 1 pound is $\frac{v}{G}$, and this value takes the place of v in the formula in Art. **24**. Thus,

$$p\frac{v}{G} = R\,T, \text{ and } p\,v = G\,R\,T$$

EXAMPLE 1.—In an air compressor is 2.7 cubic feet of air at a pressure of 54 pounds per square inch gauge and a temperature of 210° F.; what is the weight of the air?

SOLUTION.—The absolute pressure is $54 + 14.7 = 68.7$, and the absolute temperature is $460° + 210° = 670°$. Substituting in the formula $p\,v = G\,R\,T$,

$$68.7 \times 2.7 = G \times .37 \times 670, \text{ or } G = \frac{68.7 \times 2.7}{.37 \times 670} = .748 \text{ lb. Ans.}$$

EXAMPLE 2.—A hot-air balloon has a capacity of 200 cubic feet; the temperature of the air inside is 280° F., and of that outside is 70° F. Find the weight of air in the balloon, the weight of air at 70° displaced by the balloon, and the lifting power of the balloon.

SOLUTION.—Both inside and outside the balloon the pressure is that of the atmosphere, 14.7 lb. From the formula, the weight of the hot air is

$$G = \frac{p\,v}{R\,T} = \frac{14.7 \times 200}{.37 \times (460 + 280)} = 10.738 \text{ pounds}$$

and the weight of the same volume of air at 70° is

$$G = \frac{14.7 \times 200}{.37 \times (460 + 70)} = 14.992 \text{ pounds}$$

The lifting power is the difference between these weights, or $14.992 - 10.738 = 4.254$ lb. Ans.

26. Heaviness.—The **heaviness** of a gas, that is, the weight of a cubic foot of it, is the reciprocal of the volume of 1 pound. For example, if 1 pound of air has a volume of 13 cubic feet, then the weight of 1 cubic foot is evidently

$\frac{1}{13}$ pound. Let H denote the heaviness, and v the volume of 1 pound, then

$$H = \frac{1}{v}, \text{ or } v = \frac{1}{H}$$

If this expression for v be substituted for the volume in the formula in Art. **22**, the resulting equation is,

$$\frac{p_1 \frac{1}{H_1}}{T_1} = \frac{p_2 \frac{1}{H_2}}{T_2}, \text{ or } \frac{p_1}{T_1 H_1} = \frac{p_2}{T_2 H_2}$$

This may be written

$$\frac{H_1}{H_2} = \frac{p_1 T_2}{p_2 T_1} \qquad \textbf{(1)}$$

Let the temperature be constant; then, in formula **1**, $T_2 = T_1$ and

$$\frac{H_1}{H_2} = \frac{p_1}{p_2} \qquad \textbf{(2)}$$

Let the pressure remain constant; then, in formula **1**, $p_1 = p_2$ and

$$\frac{H_1}{H_2} = \frac{T_2}{T_1} \qquad \textbf{(3)}$$

Formulas **2** and **3** may be expressed, in words, as follows: *The heaviness of a given weight of gas varies directly as the absolute pressure when the temperature remains constant, and inversely as the absolute temperature when the pressure remains constant.*

Since $\frac{1}{v}$ expresses the heaviness when the weight of the gas is 1 pound, it is apparent that $\frac{G}{v}$ expresses the heaviness when the weight is greater or less than 1 pound, G being the weight of gas in pounds, and v the corresponding volume in cubic feet. Thus, for any states, $H_1 = \frac{G}{v_1}$, $H_2 = \frac{G}{v_2}$, etc. Dividing, $H_1 \div H_2 = \frac{G}{v_1} \div \frac{G}{v_2}$.

That is,

$$\left.\begin{aligned} \frac{H_1}{H_2} &= \frac{v_2}{v_1} \\ \text{and} \quad v_1 H_1 &= v_2 H_2 \end{aligned}\right\} \qquad (4)$$

For any change of state, therefore, *the heaviness of a given quantity of gas is inversely as the volume.*

EXAMPLE 1.—The weight of 1 cubic foot of air at a temperature of 60° F. and under a pressure of one atmosphere, or 14.7 pounds per square inch, is .0764 pound; what will be the weight per cubic foot if the volume is compressed until the pressure is 73.5 pounds per square inch, or five atmospheres, the temperature still being 60° F.?

SOLUTION.—Applying formula **2**, $\frac{H_1}{H_2} = \frac{p_1}{p_2}, \frac{.0764}{H_2} = \frac{14.7}{73.5}$,

and, $H_2 = .0764 \times 5 = .382$ lb. per cu. ft. Ans.

EXAMPLE 2.—If 6.75 cubic feet of air, at a temperature of 60° F., and a pressure of one atmosphere is compressed to 2.25 cubic feet, the temperature remaining the same, what is the weight of 1 cubic foot of the compressed air?

SOLUTION.—From formula **4**, $v_1 H_1 = v_2 H_2$, $6.75 \times .0764 = 2.25 \times H_2$;

then, $H_2 = \frac{6.75 \times .0764}{2.25} = .2292$ lb. Ans.

27. Homogeneous Formulas.—The formulas in the preceding articles, except those derived in Arts. **24** and **25**, are **homogeneous**; that is, they hold good whatever units are used for pressures and volumes. For volumes, cubic feet, cubic inches, or cubic meters may be used; for pressures, pounds per square inch, pounds per square foot, inches of water, or inches of mercury. However, the same units must be used for both pressures and for both volumes; thus, if p_1 is expressed in inches of mercury, p_2 must also be expressed in inches of mercury; and if v_1 is expressed in cubic inches, v_2 must also be expressed in cubic inches.

The formulas derived in Arts. **24** and **25** are not homogeneous. Using .37 as the value of R, the formulas hold good only when p is expressed in pounds per square inch, v in cubic feet, and G in pounds. If other units are used, R must have a different value. Furthermore, while the general statements of these formulas are true for any gas, the character of the constant R is such that, when determined, it limits the application of the formulas not only to certain units, but also to a particular gas, while the other formulas hold for all gases.

EXAMPLE.—A quantity of hydrogen occupies a volume of 60 cubic inches at a temperature of 70° when the barometer stands at 30 inches; what volume will be occupied when the temperature is 44° and the barometer stands at 29.4 inches?

SOLUTION.—The volumes are expressed in cubic inches and the pressures in inches of mercury. Using the formula in Art. **22**, $\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$, and substituting,

$$\frac{30 \times 60}{460 + 70} = \frac{29.4 \times v_2}{460 + 44}; \text{ hence, } v_2 = \frac{30 \times 60 \times 504}{530 \times 29.4} = 58.22 \text{ cubic inches.}$$

Ans.

EXAMPLES FOR PRACTICE

1. A vessel contains 25 cubic feet of gas at a pressure of 18 pounds per square inch, absolute; if 125 additional cubic feet of gas having the same pressure are forced into the vessel, what will be the resulting pressure? Ans. 108 lb. per sq. in.

2. A pound of air has a temperature of 126°, and a pressure of one atmosphere; what volume does it occupy? Ans. 14.75 cu. ft.

3. The volume of steam in the cylinder of a steam engine at cut-off is 1.35 cubic feet, and the absolute pressure is 85 pounds per square inch; if the absolute pressure at the end of the stroke is 25 pounds per square inch, what is the new volume, assuming that the expanding steam follows Boyle's law? Ans. 4.59 cu. ft.

MIXTURES OF GASES

28. Diffusion of Gases.—If two liquids that do not act chemically on each other are mixed together and allowed to stand, it will be found that after a time the liquids have separated and that the heavier has fallen to the bottom. If two equal vessels, containing gases of different heaviness, are put in communication with each other, the gases will be found to have mixed in equal proportions after a short time. If one vessel is higher than the other, and the heavier gas is in the lower vessel, the result will be the same. The greater the difference in heaviness of the two gases, the quicker they will mix. It is assumed that no chemical action takes place between the two gases. When the two gases have the same temperature and pressure, the pressure of the mixture will be the same; this is evident, since the total

volume has not been changed, and unless the volume or temperature changes, the pressure cannot change. This property of the mixing of gases is a very valuable one; for if gases acted like liquids, carbonic-acid gas, the result of combustion, which is $1\frac{1}{2}$ times as heavy as air, would remain next to the earth instead of dispersing into the atmosphere, and, in consequence, no animal life could exist.

29. Mixture of Equal Volumes of Gases Having Unequal Pressures.—*If two gases having equal volumes and temperatures, but different pressures, are mixed in a vessel whose volume is equal to one of the equal volumes of the gas, the pressure of the mixture will be equal to the sum of the two pressures, provided the temperature remains the same as before.*

EXAMPLE.—Each of two vessels contains 3 cubic feet of gas, subjected to pressures of 40 pounds and 25 pounds per square inch, respectively, and at a temperature of 60° F. The vessels are placed in communication with each other, and all the gas is compressed into one vessel. If the temperature of the mixture is also 60°, what is the pressure?

SOLUTION.—According to the rule just given, the pressure will be $40 + 25 = 65$ lb. per sq. in. This may be proved by applications of Boyle's law; thus, compress the gas whose pressure is 25 lb. per sq. in. until its pressure is 40 lb.; its volume may be found thus: $p\,v = p_1\,v_1$, or $25 \times 3 = 40 \times v$; whence, $v = 1.875$ cu. ft. Let communication be established between the two vessels; the pressure will evidently be 40 lb. and the total volume $3 + 1.875 = 4.875$ cu. ft. If this is compressed until the volume is 3 cu. ft., the temperature remaining at 60° throughout the whole operation, the final pressure may be found by the formula, $p\,v = p_1\,v_1$. Thus, $40 \times 4.875 = p_1 \times 3$, and $p_1 = \frac{40 \times 4.875}{3}$ $= 65$ lb. per sq. in., as before.

30. Mixture of Two Gases Having Unequal Volumes and Pressures.—Let v_1 and p_1 be the volume and pressure, respectively, of one of the gases; v_2 and p_2 be the volume and pressure, respectively, of the other gas; and V and P be the volume and pressure, respectively, of the mixture. Then, if the temperature remains the same,

$$P\,V = p_1\,v_1 + p_2\,v_2$$

That is, *if the temperature is constant, the product of the pressure and volume after mixing is equal to the sum of the*

products obtained by multiplying each volume by its corresponding pressure previous to mixing.

EXAMPLE.—Two gases of the same temperature, having volumes of 7 cubic feet and $4\frac{1}{2}$ cubic feet, and whose pressures are 27 pounds and 18 pounds per square inch, respectively, are mixed together in a vessel whose volume is 10 cubic feet. The temperature of the two gases and of the mixture being 60° F., what is the resulting pressure?

SOLUTION.—Applying the formula $PV = p_1 v_1 + p_2 v_2$,

$$P \times 10 = 27 \times 7 + 4\tfrac{1}{2} \times 18$$

$$P = \frac{189 + 81}{10} = 27 \text{ lb. per sq. in.} \quad \text{Ans.}$$

31. Mixture of Two Bodies of Air Having Unequal Pressures, Volumes, and Temperatures.—If a body of air having a temperature t_1, a pressure p_1, and a volume v_1 is mixed with another volume of air having a temperature t_2, a pressure p_2, and a volume v_2, to form a volume V, having a pressure P and a temperature t, then, either the new temperature t, the new volume V, or the new pressure P may be found, if the other quantities are known, by the following formula, in which T_1, T_2, and T are the absolute temperatures corresponding to t_1, t_2, and t;

$$\frac{PV}{T} = \frac{p_1 v_1}{T_1} + \frac{p_2 v_2}{T_2}$$

EXAMPLE.—Five cubic feet of air having a pressure of 30 pounds per square inch, absolute, and a temperature of 80° F., is to be compressed together with 11 cubic feet of air having a pressure of 21 pounds per square inch, absolute, and a temperature of 45° F., in a vessel whose cubical capacity is 8 cubic feet; if the resulting pressure is 45 pounds per square inch, what is the temperature of the mixture?

SOLUTION.—Substituting in the formula,

$$\frac{45 \times 8}{T} = \left(\frac{30 \times 5}{540} + \frac{21 \times 11}{505}\right), \text{ or } \frac{360}{T} = .7352. \quad \text{Hence, } T = \frac{360}{.7352}$$

$$= 489.66°, \text{ nearly, and } t = 29.66°. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Two vessels contain air at pressures of 60 and 83 pounds per square inch, absolute; the volume of each vessel is 8.47 cubic feet. If all of the air in both vessels is removed to another vessel, and the new

pressure is 100 pounds per square inch, absolute, what is the volume of the vessel, the temperature being the same throughout?

Ans. 12.11 cu. ft.

2. A vessel contains 11.83 cubic feet of air at a pressure of 33.3 pounds per square inch, absolute; it is desired to increase the pressure to 40 pounds per square inch, absolute, by supplying air from a second vessel which contains 19.6 cubic feet of air at a pressure of 60 pounds per square inch, absolute. What will be the pressure in the second vessel after the pressure in the first has been raised to 40 pounds per square inch?

Ans. 55.96 lb. per sq. in.

3. If 4.8 cubic feet of air having a pressure of 52 pounds per square inch, absolute, and a temperature of 170° is mixed with 13 cubic feet having a pressure of 78 pounds per square inch, absolute, and a temperature of 265°, what must be the volume of the vessel containing the mixture in order that the pressure of the mixture may be 30 pounds per square inch, absolute, and the temperature 80°?

Ans. 32.31 cu. ft.

PNEUMATIC MACHINES AND DEVICES

THE AIR PUMP

32. Action of the Air Pump.—The **air pump** is an instrument for removing air from an enclosed space. A sec-

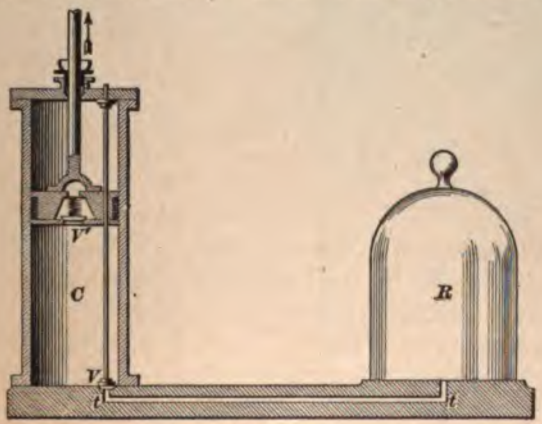


FIG. 11

tion of the principal parts is shown in Fig. 11, and the complete instrument in Fig. 12. The closed vessel R is called

the **receiver**, and the space that it encloses is that from which it is desired to remove the air. The receiver is usually made of glass, and the edges are ground so as to make a perfectly air-tight joint with the plate on which it rests. When made in the form shown, it is called a **bell-jar receiver.** The receiver rests on a horizontal plate in the center of which is an opening that communicates with the pump cylinder *C* by means of the air passage *t*. The pump piston fits the cylinder accurately, and has a valve *V'* opening upwards. At the junction of the air passage with the cylinder is another valve *V*, also opening upwards. When the piston is raised, the valve *V'* closes; and, since no air can get into the cylinder from above, the piston leaves a partial vacuum behind it. The pressure on top of *V* being now partially removed, the pressure of the air in the receiver *R* causes *V* to rise; the air in the receiver then expands and occupies the space behind the piston and the space in the passage *t* and the receiver *R*. The piston is now pushed down, the valve *V* closes, the valve *V'* opens, and the air in *C* escapes. The lower valve *V* is sometimes supported, as shown in Fig. 11, by a metal rod passing through the piston and fitting it tightly. When the piston is raised or lowered, this rod moves with it. A button near the upper end of the rod confines its motion to very narrow limits, the piston sliding on the rod during the greater part of the motion.

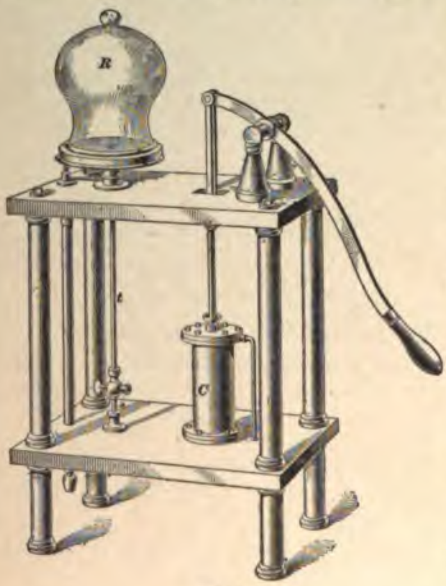


FIG. 12

33. Degrees and Limits of Exhaustion.—Suppose that the volume of R and t together is four times that of C, and that there are, say, 200 grains of air in R and t, and 50 grains in C, when the piston is at the top of the cylinder.

FIG. 13

At the end of the first stroke, when the piston is again at the top, 50 grains of air in the cylinder C will have been removed, and the 200 grains in R and t will occupy the spaces R, t, and C. The ratio between the sum of the spaces R and t and the total space $R + t + C$ is $\frac{4}{5}$; hence, $200 + \frac{4}{5} = 160$ grains = the weight of air in R and t after the first stroke. After the second stroke, the weight of the air in R and t will be $200 \times \frac{4}{5} \times \frac{4}{5} = 200 \times (\frac{4}{5})^2 = 200 \times \frac{16}{25} = 128$ grains. At the end of the third stroke, the weight will be $[200 \times (\frac{4}{5})^2] \times \frac{4}{5} = 200 \times (\frac{4}{5})^3 = 200 \times \frac{64}{125} = 102.4$ grains. At the end of n strokes, the weight will be $200 \times (\frac{4}{5})^n$. It is evident that *it is impossible to remove all the air that is contained in R and t by this method.* It requires an exceedingly good air pump to reduce the pressure of the air in R to $\frac{1}{50}$ inch of mercury. When the air has become so rarefied as this, the valve V' will not lift, and no more air can be exhausted.

34. Sprengel's Air Pump.—In Fig. 13 is shown a glass tube longer than 30 inches, open at both ends, and connected by means of rubber tubing with a funnel *A* filled with mercury and supported by a stand. Mercury is allowed to fall into this tube at a rate regulated by a clamp at *c*. The lower end of the tube *cd* fits in the flask *B*, which has a spout at the side a little higher than the lower end of *cd*; the upper part has a branch at *x* to which a receiver *R* can be tightly fixed. When the clamp at *c* is opened, the first portions of the mercury that run out close the tube and prevent air from entering from below. These drops of mercury act like little pistons, carrying the air in front of them and forcing it out through the bottom of the tube. The air in *R* expands to fill the tube every time that a drop of mercury falls, thus creating a partial vacuum in *R*, which becomes more nearly complete as the process goes on. The escaping mercury falls into the dish *H*, from which it can be poured back into the funnel from time to time. As the exhaustion from *R* goes on, the mercury rises in the tube *cd* until, when the exhaustion is complete, it forms a continuous column about 30 inches high.

This instrument necessarily requires a great deal of time for its operation, but the results are very complete. The pressure in *R* has been reduced to $\frac{1}{430000}$ inch of mercury. By the use of chemicals in addition to the above, a vacuum of $\frac{1}{650000}$ inch of mercury has been obtained.

APPARATUS SHOWING WEIGHT AND PRESSURE OF THE ATMOSPHERE

35. Magdeburg Hemispheres.—By means of the two hemispheres shown in Fig. 14, it can be proved that the atmosphere presses on a body equally in all directions. Such hemispheres were invented by Otto Von Guericke, of Magdeburg, Germany, and are called the **Magdeburg hemispheres.** One of the hemispheres is provided with a stop-cock, by which it can be attached to an air pump. The rims fit accurately and are well greased, so as to make an air-tight

joint. As long as the hemispheres contain air, they can be separated with ease; but when the air in the interior is pumped out by means of an air pump, they can be separated only with great difficulty. The force required to separate them will be equal to the area of the largest circle of the hemisphere, which is the projected area, in square inches, multiplied by 14.7 pounds. This force will be the same in whatever position the hemisphere may be held, thus proving that the pressure of air on it is the same in all directions.

36. The Weight Lifter.—The pressure of the atmosphere is very clearly shown by means of an apparatus like that illustrated in Fig. 15. Here, a cylinder fitted with a piston is held in suspension by a chain. At the top of the cylinder is a plug *a* that can be taken out. This plug is removed and the piston pushed up, the force necessary being equal to the weight of the piston and rod *b*, until it touches the cylinder head. The plug is then screwed in, and assuming that no air is left in the cylinder, the piston will remain at the top until a weight has been hung on the rod, equal to the area of the piston multiplied by 14.7 pounds, less the weight of the piston and rod. If a force just great enough to move the piston downwards were applied to the rod, the pressure of the air would raise any weight less than this to the top of the cylinder.

FIG. 14

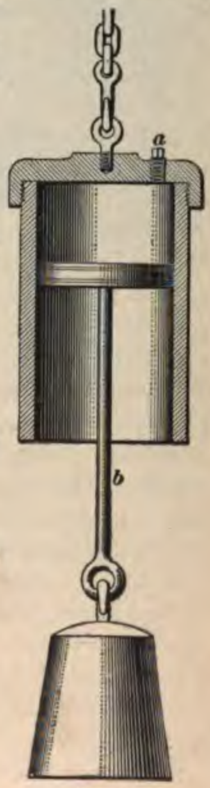


FIG. 15

Suppose the weight to be removed, and the piston to be supported, say at the middle of the length of the cylinder. Let the plug be removed and air admitted above the piston; then, with the piston in the middle of the cylinder, screw the plug back into its place; if the piston be forced upwards, the farther up it goes the greater will be the force necessary to push it, on account of the compression of the air. If the piston is of large diameter, it will also require a great force to pull it out of the cylinder. For example, let the diameter of the piston be 20 inches, the length of the cylinder 36 inches plus the thickness of the piston, and the weight of the piston and rod 100 pounds. If the piston is at the middle of the cylinder, there will be 18 inches of space above it and 18 inches of space below it. The area of the piston is $20^2 \times .7854 = 314.16$ square inches, and with the atmospheric pressure on it, there will be $314.16 \times 14.7 = 4,618$ pounds, nearly, pressing on each side of the piston. In order to hold the piston central, an upward force of 100 pounds must be exerted to balance the weight of the piston and rod. In order to move the piston upwards 9 inches, reducing the volume one-half and doubling the pressure, the upward pressure on it must be twice the atmospheric pressure, plus 100 pounds, provided that the temperature remains constant. The total upward force is therefore $2 \times 4,618 + 100 = 9,336$ pounds. The force in excess of that of the atmosphere necessary to cause the piston to move upwards 9 inches will then be $9,336 - 4,618 = 4,718$ pounds.

Now suppose the piston to be moved downwards until it is just at the point of being pulled out of the cylinder. The volume above it will then be twice as great as before, and the pressure one-half as great, or $4,618 \div 2 = 2,309$ pounds. The total upward pressure will be the pressure of the atmosphere, or 4,618 pounds. The force necessary to pull the piston downwards to this point will be the difference between the total upward pressure and the downward pressure due to the pressure inside the cylinder and the weight of the piston and rod. This gives the force necessary to pull the piston down as $4,618 - (2,309 + 100) = 2,209$ pounds.

37. The Baroscope.—The buoyant effect of air is very clearly shown by means of an instrument called the **baroscope,** shown in Fig. 16. It consists of a scale beam, from one extremity of which is suspended a small weight, and from the other a hollow copper sphere. In air, they exactly balance each other; but when placed under the receiver of an air pump and the air exhausted, the sphere sinks, showing that it is really heavier than the small weight. Before the air is exhausted, each body is buoyed up by the weight of the air it displaces, and since the sphere displaces the more air, it loses more weight by reason of this displacement than the small weight. Suppose that the volume of the sphere exceeds that of the weight by 10 cubic inches; the weight of this volume of air is 3.1 grains. If this weight is added to the small weight, it will overbalance the sphere in air, but will exactly balance it in a vacuum.

Fig. 16

Fig. 17

38. The Cartesian Diver.—The device shown in Fig. 17 illustrates the elasticity of air and the transference of pressure in all directions in water. It is called the **Cartesian diver** and consists of a glass jar nearly filled with water, having a rubber bulb at the top filled with air. The image in the jar is made of glass and is hollow, the weight being less than that of an equal volume of water, so that it will float at the top of the jar. The tail

of the image has a hole in it, the water being prevented from getting inside of the image by the pressure of the air within it. If the bulb is squeezed, the air in it will be forced out, creating a pressure on the water which, being transferred in all directions, causes the water to flow into the tail of the image, compressing the air inside and thus causing it to fall to the bottom of the jar. When the pressure on the bulb is released, the air flows back into the bulb, the pressure on the water is removed, and the air within the image expands; the image again becoming lighter than water, rises to the top of the jar.

FIG. 18

39. Hero's Fountain. **Hero's fountain** derives its name from its inventor, Hero, who lived at Alexandria about 120 B. C. It depends for its operation on the elastic properties of air. It is shown in Fig. 18 and consists of a brass dish *A* and two glass globes *B* and *C*. The dish communicates with the lower part of the globe *C* by a long tube *D*, and another tube *E* connects the two globes. A third tube passes through the dish *A* to the lower part of the globe *B*. This last tube being taken out, the globe *B* is partially filled with water; the tube is then replaced and water is poured into the dish. The water flows downwards through the tube *D* into the lower globe, and expels the air, which is forced into the upper globe. The air thus compressed acts on the water and makes it shoot out through the shortest tube in the form of a jet, as represented

in the figure. Were it not for the resistance of the atmosphere and friction, the water would rise to a height above the water in the dish equal to the difference of the level of the water in the two globes.

THE SIPHON

40. The Principle of the Siphon.—The action of the **siphon** illustrates the effect of atmospheric pressure. The siphon is simply a bent tube with branches of unequal length open at both ends, and is used to convey a liquid from a higher point to a lower one, over an intermediate point higher than either. In Fig. 19, *a* and *b* are two vessels, *b* being lower than *a*, and *a c b* is the bent tube or siphon. Suppose this tube to be filled with water and placed in the vessels, as shown, with the short branch *a c* in the vessel *a*. The water will flow from the vessel *a* into the vessel *b*, so long as the level of the water in *b* is below the level of the water in *a*, and the level of the water in *a* is above the lower end of the tube *a c*. The atmospheric pressure on the surfaces of *a* and *b* tends to force the water up the tubes *a c* and *b c*. When the siphon is filled with water, each of these pressures is counteracted in part by the pressure of the water in that branch of the siphon which is immersed in the water on which the pressure is exerted. The atmospheric pressure opposed to the weight of the longer column of water will, therefore, be resisted with greater force than that opposed to the weight of the shorter column; consequently, the pressure exerted on the shorter column will be greater than that on the longer column, and this excess pressure will produce motion.

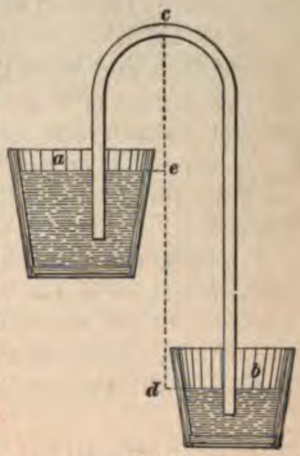


Fig. 19

Let $h_1 = dc =$ vertical distance, in feet, between surface of water in b and highest point of center line of tube;

$h_2 = ec =$ distance, in feet, between surface of water in a and highest point of center line of tube.

The downward pressure at the level ae due to the head ce is $.434\,h_2$ pounds per square inch, while the upward pressure due to the atmospheric pressure on the surface a is 14.7 pounds per square inch; then, the net upward pressure per square inch is $(14.7 - .434\,h_2)$ pounds. Similarly, at the level $b\,d$ the net upward pressure per square inch is $(14.7 - .434\,h_1)$ pounds. The water column $a\,c\,b$ is urged from a toward b by a force of $(14.7 - .434\,h_2)$ pounds per square inch and in the opposite direction by a force of $(14.7 - .434\,h_1)$ pounds per square inch. The difference between the forces is $(14.7 - .434\,h_2) - (14.7 - .434\,h_1) = .434(h_1 - h_2)$ pounds per square inch and since the upward force at a is the greater, the water moves from a toward b, that is, upwards in the shorter column and downwards in the longer. The net head $(h_1 - h_2)$ producing the flow is $d\,e$, the difference between the level of the surface of the water in the two vessels.

It will be noticed that the short column must not be higher than 34 feet for water, or the siphon will not work, since the pressure of the atmosphere will not support a column of water that is higher than 34 feet; 28 feet is about the greatest height at which a siphon will work well.

41. Intermittent Springs.—Sometimes a spring is observed to flow for a time and then cease; then, after an interval to flow again for a time. The generally accepted explanation of this is that there is an underground reservoir fed with water through fissures in the earth, as shown in Fig. 20. The outlet for the water is shaped like a siphon, as shown. When the water in the reservoir reaches the same point as the highest point of the outlet, it flows out until the level of the water in the reservoir falls below the mouth of

the siphon, the water flowing out of the reservoir faster than it is supplied to it. This flow then ceases until the water in

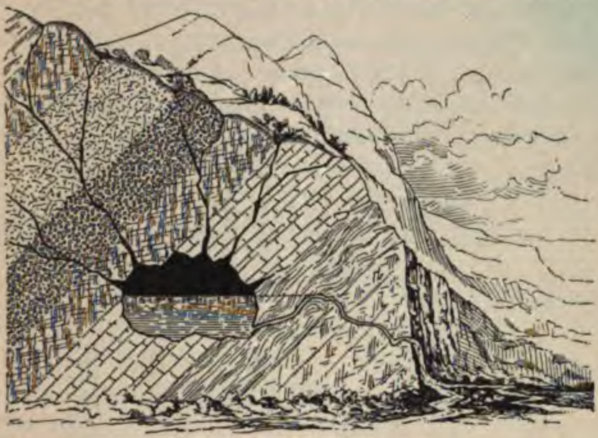


FIG. 20

the reservoir has again reached the level of the highest point of the siphon.

AIR COMPRESSORS

42. For many purposes, compressed air is preferable to steam or any other gas for use as a motive power. In such cases, **air compressors** are used to compress the air. These are made in many forms, but the most common one consists of a cylinder, called the *air cylinder*, placed in front of the crosshead of a steam engine, so that the piston of the air cylinder can be driven by attaching its piston rod to the crosshead, in a manner similar to a direct-acting steam pump. A cross-section of the air cylinder of a compressor of this kind is shown in Fig. 21, in which a is the piston and b is the piston rod, driven by the crosshead of a steam engine not shown in the figure. Both ends of the lower half of the cylinder are fitted with inlet valves d and d', which allow the air to enter the cylinder, and both ends of the upper half are fitted with discharge valves f and f', which allow the air to escape from the cylinder after it has been compressed to the required pressure.

Suppose the piston *a* to be moving in the direction of the arrow; then the inlet valves *d* in the left-hand end of the cylinder from which the piston is moving will be forced inwards by the pressure of the atmosphere, which overcomes the resistance of the light spring *c*, thus allowing the air to flow in and fill the cylinder. On the other side of the piston, the air is being compressed, and, consequently, it acts with

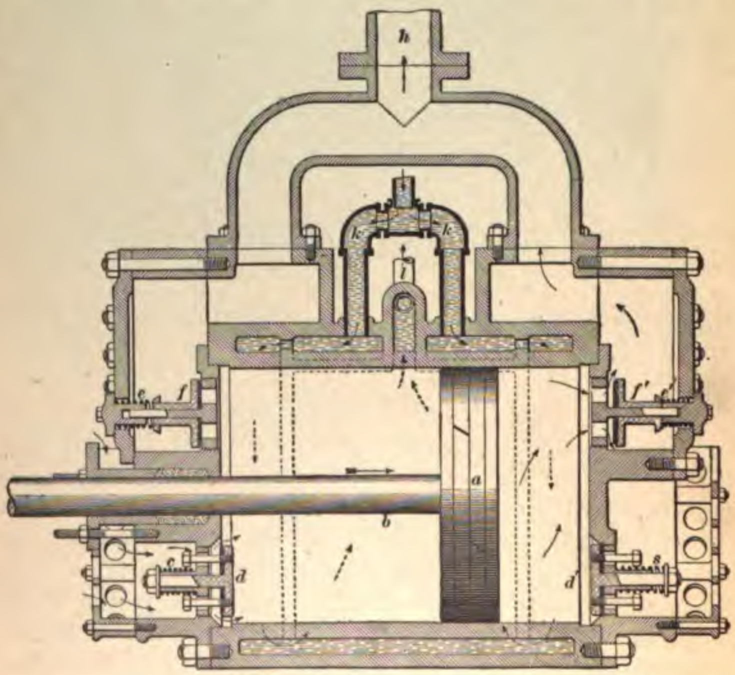


FIG. 21

the springs *s* to force the inlet valves *d'* in the right-hand end of the cylinder to their seats. In the right-hand end of the cylinder, the discharge valves *f'* are opened when the pressure of the air in the cylinder is great enough to overcome the resistance of the light springs *e'* and the pressure of the air in the passages leading to the discharge pipe *h*, and the discharge valves *f* are pressed against their seats by the springs *e* and the pressure of the air in the passages.

Suppose that it is desired to compress the air to 59 pounds per square inch, and it is necessary to find at what point of the stroke the discharge valves will open. Now, 59 pounds per square inch is equal to a pressure of four atmospheres, that is, four times the pressure of the atmosphere, very nearly; hence, when the pressure in the cylinder becomes great enough to force air out through the discharge valves, the volume must be one-quarter of the volume at atmospheric pressure, or, the valves will open when the piston has traveled three-quarters of its stroke, provided that the air is compressed at constant temperature. The air, after being discharged from the cylinder, passes out through the delivery pipe *h*, and is then conveyed to its destination.

When air or any other gas is compressed its temperature rises. For high pressures, this rise of temperature becomes a serious consideration, for two reasons: (1) When the air is discharged at a high temperature, the pressure falls considerably when it is cooled down to its normal temperature, and this represents a serious loss in the economical working of the machine. (2) The alternate heating and cooling of the compressor cylinder by the hot and cold air is very destructive to it, and increases the wear to a great extent. To prevent excessive heating, cooling devices are resorted to, the most common one being the so-called **water-jacket.** The cylinder walls are hollow, as shown in Fig. 21; the cold water enters this hollow space in the cylinder wall through the pipes *k*, *k*, flows around the cylinder, and finally passes out through the discharge pipe *l*. Much of the heat generated by the compression of the air passes through the cylinder walls and is absorbed and carried off by the water; thus the temperature of the air does not rise to the point at which it will be seriously detrimental.

HYDRAULICS

(PART 1)

FLOW OF WATER

FUNDAMENTAL PRINCIPLES

1. Flow in a Pipe of Uniform Diameter.—When water flows through a pipe of uniform cross-section, the quantity passing any section in a given interval of time depends on the area of the cross-section and the velocity with which the water moves. It is usually assumed that all the particles of water have the same velocity; but if they have different velocities the mean or average may be taken as the velocity of the flow.

Let A = area of cross-section of pipe, in square feet;
v = mean velocity of flow, in feet per second;
Q = quantity, in cubic feet, flowing past any section in 1 second.

Evidently, the quantity Q is equal to the volume of a column whose base is the area A and whose height is equal to v; hence,

$$Q = A\,v \qquad (1)$$

or

$$v = \frac{Q}{A} \qquad (2)$$

EXAMPLE.—With a velocity of 8 feet per second, what quantity of water will be discharged by a 3-inch pipe: (*a*) in 1 second? (*b*) in 1 hour?

SOLUTION.—(*a*) The area A is $\frac{.7854 \times 3^2}{144}$ sq. ft.; hence,

$$Q = A\,v = \frac{.7854 \times 3^2}{144} \times 8 = .3927 \text{ cu. ft. per sec. Ans.}$$

(*b*) The flow per hour is

$$.3927 \times 60 \times 60 = 1{,}413.7 \text{ cu. ft. Ans.}$$

2. Flow in a Pipe of Varying Diameter.—Since water is practically incompressible, the quantity flowing past any section must be the same as the quantity flowing past any other section in the same interval of time, provided that the pipe is full of water. Hence, if the cross-section of the pipe varies, the velocity also must vary. Where the cross-section is least, the velocity is greatest; and where the cross-section is greatest, the velocity is least. If, therefore, A_1, A_2, A_3 denote the areas of successive cross-sections and v_1, v_2, v_3 denote the corresponding velocities, then $Q = A_1 v_1$; $Q = A_2 v_2$, and $Q = A_3 v_3$; hence,

$$A_1 v_1 = A_2 v_2 = A_3 v_3$$

Assuming that the pipe always remains full, it appears that the velocities at various sections of the pipe vary inversely as the areas of those sections.

EXAMPLE.—The velocity through a part of a pipe that is 3 inches in diameter is 5 feet per second. (*a*) What is the velocity through an enlarged part of the pipe whose diameter is 4 inches? (*b*) What is the velocity through a valve that has an opening of $3\frac{1}{2}$ square inches?

SOLUTION.—(*a*) Since this formula applies when the areas are expressed in square inches, $A_1 = .7854 \times 3^2$, $A_2 = .7854 \times 4^2$, $v_1 = 5$. Substituting these values in the formula,

$$v_2 \times .7854 \times 4^2 = 5 \times .7854 \times 3^2$$

As the common factor .7854 cancels,

$$v_2 = 5 \times \frac{3^2}{4^2} = 2.8125 \text{ ft. per sec.} \quad \text{Ans.}$$

(*b*)
$$v_3 \times 3.5 = 5 \times .7854 \times 3^2$$

or
$$v_3 = \frac{5 \times .7854 \times 9}{3.5} = 10.098 \text{ ft. per sec.} \quad \text{Ans.}$$

3. Internal Pressure of Flowing Water.—The reservoir *a*, Fig. 1, is connected with a second reservoir *b* by a bent pipe of varying cross-section. In the first place, suppose the lower end of the pipe to be closed so that there is no flow. The water in *a* and in the pipe being at rest, the pressure at any section of the pipe is determined by the principles of hydrostatics. Thus, at the section s_1 the level *c d* of the water in *a* is at a distance h_1 above the center of the section, and the gauge pressure of the water at that point is therefore .434 h_1 pounds per square inch, when h_1 is

expressed in feet. For example, if a section of the pipe is 10 feet below the level $c\,d$, the gauge pressure at that section is $.434 \times 10 = 4.34$ pounds per square inch. To obtain the absolute pressure, the pressure of the atmosphere, 14.7 pounds per square inch, must be added to the gauge pressure.

Let a small tube t_1, open at both ends, be screwed into the pipe at s_1; when no water flows from the reservoir a to the reservoir b the water will rise in this tube to the level $c\,d$, and the height h_1 of the water in the tube therefore measures

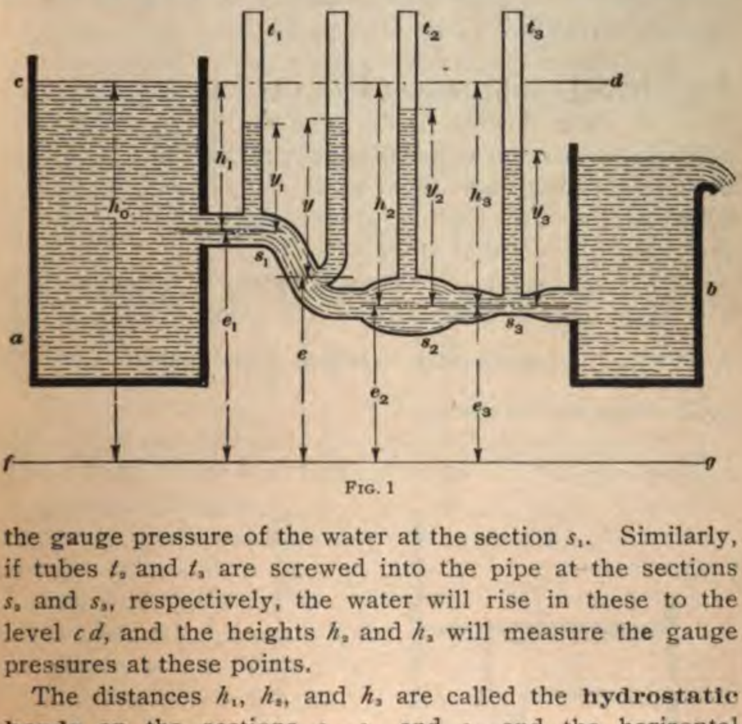


FIG. 1

the gauge pressure of the water at the section s_1. Similarly, if tubes t_2 and t_3 are screwed into the pipe at the sections s_2 and s_3, respectively, the water will rise in these to the level $c\,d$, and the heights h_2 and h_3 will measure the gauge pressures at these points.

The distances h_1, h_2, and h_3 are called the **hydrostatic heads** on the sections s_1, s_2, and s_3, and the horizontal line $c\,d$, to which the water in the tubes will rise when no water flows, is called the **hydrostatic level.**

When the water is flowing from the vessel a to the vessel b, it will be found that the water in the tube t_1 has a height y_1,

which is less than h_1; and likewise, in the tubes t_2 and t_3, the height y_2 is less than h_2 and y_3 is less than h_3. The decrease in height is found to be greatest in the tube t_3 inserted at s_3, the smallest section, and is least in the tube t_2 inserted at s_2, the largest section.

The heights y_1, y_2, and y_3 measure the gauge pressures at the sections s_1, s_2, and s_3, respectively, and are called the **pressure heads** on those sections.

In the case of water flowing in a pipe, *the pressure head at any section is less than the hydrostatic head, and the difference between the two is greater the smaller the section or the greater the velocity of flow.*

4. Energy of a Mass of Flowing Water.—Consider a mass of water flowing in the pipe of Fig. 1. The mass possesses a certain amount of energy, that is, it is capable of doing a definite amount of work, and this energy is composed of the three factors, kinetic energy, energy due to pressure, and potential energy.

1. *Kinetic Energy.*—If the mass weighs G pounds and has a velocity of v feet per second, the amount of the kinetic energy is $\frac{G v^2}{2g}$ foot-pounds, in which g is equal to 32.16, the acceleration due to gravity.

2. *Energy Due to Pressure.*—The mass of water possesses some energy because of its internal fluid pressure. To estimate the amount of this energy, proceed as follows: Suppose the water to enter a cylinder a, Fig. 2, from a reservoir, b, and to push against a piston on the other side of which is atmospheric pressure. Let p denote the absolute pressure, in pounds per square inch, of the water against

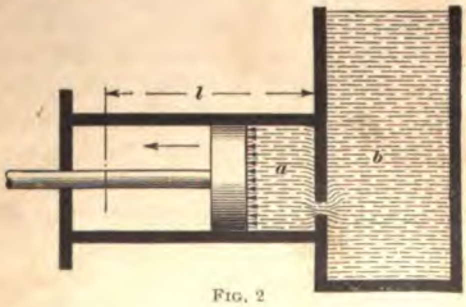


FIG. 2

the piston. When the piston has moved to the end of its stroke, let the water in the cylinder be shut off from the reservoir and opened to the atmosphere; the pressure of the water in the cylinder will then be that of the atmosphere—14.7 pounds per square inch. If A denotes the area of the piston, in square inches, the total net pressure urging the piston to the left, during the stroke, is $A(p - 14.7)$; hence, the work done is $A\,l\,(p - 14.7)$ foot-pounds, when l denotes the length of the stroke of the piston, in feet. The motion is assumed to take place very slowly, so that there is practically no change in the kinetic energy of the water; and as the water remains at the same level, there is no change of potential energy; hence, the work done is equal to the energy given up by the cylinder of water when its pressure is decreased to that of the atmosphere.

Now $p - 14.7$ is the gauge pressure of the water, and if y denotes the **pressure head,** in feet, corresponding to the gauge pressure, $p - 14.7 = .434\,y$. Further, since $\frac{A}{144}$ is the area of cylinder, in square feet, $\frac{A\,l}{144}$ is the volume of cylinder, in cubic feet, and $\frac{A\,l}{144} \times 62.5 = .434\,A\,l$ is the weight of water in the cylinder. Let G denote this weight; then $.434\,A\,l = G$, or $A\,l = \frac{G}{.434}$. Now, substituting the values thus found for the terms $p - 14.7$ and $A\,l$, in the expression $A\,l\,(p - 14.7)$, the energy due to pressure is

$$\frac{G}{.434} \times .434\,y = G\,y \text{ foot-pounds}$$

The pressure energy of a weight G of water is therefore equal to the product of the weight and the pressure head.

3. *Potential Energy.*—If the water is elevated a distance e feet above some assumed datum line fg, Fig. 1, it has a potential energy due to its position. A *datum line* is a reference line from which measurements are taken, and is usually assumed at the most convenient point for taking such measurements. The amount of this potential energy is the

work that would be required to raise the weight G through a height e feet, or Ge foot-pounds.

The *total energy* of the water is the sum of these three energies; denoting it by E,

$$E = \frac{Gv^2}{2g} + Gy + Ge = G\left(\frac{v^2}{2g} + y + e\right)$$

5. Equation of Energy.—If the flow through the pipe shown in Fig. 1 is assumed to be frictionless, the energy of a mass of water passing any section must be the same as the energy in passing any other section. The energy of 1 pound of water at the section s_1, with the velocity v_1 is

$$E_1 = \frac{v_1^2}{2g} + y_1 + e_1 \qquad (1)$$

For the section s_2, where the velocity is v_2, the energy is

$$E_2 = \frac{v_2^2}{2g} + y_2 + e_2 \qquad (2)$$

For the section s_3, it is

$$E_3 = \frac{v_3^2}{2g} + y_3 + e_3 \qquad (3)$$

Now, $E_1 = E_2 = E_3$, since no energy is lost in doing work against friction; hence,

$$\frac{v_1^2}{2g} + y_1 + e_1 = \frac{v_2^2}{2g} + y_2 + e_2 = \frac{v_3^2}{2g} + y_3 + e_3 \qquad (4)$$

It will be observed that y and e are distances or heights. To the height e of the water above the assumed datum line, the name **potential head** is given. It is apparent that the quantity $\frac{v^2}{2g}$ may also represent a height; in fact, it is the height that a body will rise when projected upwards with an initial velocity v. To this quantity, therefore, the name **velocity head** is given.

According to formula **4**, in frictionless steady flow, the sum of the velocity head, pressure head, and potential head is the same at all sections of the pipe. This statement is known as **Bernoulli's law**.

6. The Velocity Head.—Let h_0, Fig. 1, denote the height of the water level cd above the datum line fg. One

pound of water at the level cd has no kinetic energy, being at rest, and no pressure energy; hence, its total energy is its potential energy with reference to fg, and this energy is $1 \times h_0 = h_0$ foot-pounds. The energy of this pound on the surface must be the same as that of a pound passing through the pipe at the section s_1; hence,

$$h_0 = \frac{v_1^2}{2g} + y_1 + e_1 \qquad (1)$$

But $h_0 = e_1 + h_1$; hence, $\frac{v_1^2}{2g} + y_1 + e_1 = e_1 + h_1$, or $\frac{v_1^2}{2g} = h_1 - y_1$. In general,

$$\frac{v^2}{2g} = h - y; \text{ or } y = h - \frac{v^2}{2g} \qquad (2)$$

That is, *in frictionless flow, the velocity head at any section is equal to the difference between the hydrostatic head and the pressure head.* Evidently, therefore, the distances of the water levels below cd in the tubes t_1, t_2, and t_3, Fig. 1, are the velocity heads for the sections s_1, s_2, and s_3, respectively.

EXAMPLE 1.—In Fig. 1, $h_1 = 15$ feet, $h_2 = h_3 = 20$ feet, and the areas at s_1, s_2, and s_3 are $A_1 = 3$ square inches, $A_2 = 5$ square inches, and $A_3 = 1$ square inch. The velocity of the water as it passes s_1 is 8 feet per second. Find the velocity heads and pressure heads for the three sections; also the absolute pressure of the water as it passes each section.

SOLUTION.—Using the formula of Art. 2, $A_1 v_1 = A_2 v_2$, $3 \times 8 = 5 \times v_2$; whence,

$$v_2 = \frac{3 \times 8}{5} = 4.8 \text{ ft. per sec.;}$$

and, similarly, $$v_3 = \frac{3 \times 8}{1} = 24 \text{ ft. per sec.}$$

The velocity heads are:

$$\frac{v_1^2}{2g} = \frac{8^2}{2 \times 32.16} = .995 \text{ ft. at } s_1,$$

$$\frac{v_2^2}{2g} = \frac{4.8^2}{2 \times 32.16} = .358 \text{ ft. at } s_2,$$

and $$\frac{v_3^2}{2g} = \frac{24^2}{2 \times 32.16} = 8.95 \text{ ft. at } s_3.$$

The pressure heads are now found by formula 2.

$$y_1 = h_1 - \frac{v_1^2}{2g} = 15 - .995 = 14.005 \text{ ft.;}$$

likewise, $$y_2 = 20 - .358 = 19.642 \text{ ft.;}$$

$$y_3 = 20 - 8.95 = 11.05 \text{ ft.}$$

The absolute pressure at the section s_1 is

$$p_1 = .434\,y_1 + 14.7 = .434 \times 14.005 + 14.7 = 20.778 \text{ lb. per sq. in.}$$

At s_2,

$$p_2 = .434 \times 19.642 + 14.7 = 23.225 \text{ lb. per sq. in.}$$

And at s_3,

$$p_3 = .434 \times 11.05 + 14.7 = 19.496 \text{ lb. per sq. in.} \quad \text{Ans.}$$

EXAMPLE 2.—If the hydrostatic head is 20 feet and the pressure head at a given section is 12 feet, what is the velocity at the section?

SOLUTION.—By formula **2**, $\frac{v^2}{2g} = h - y = 20 - 12 = 8$ ft.

Therefore,

$$v^2 = 2g \times 8, \text{ or } v = \sqrt{2g \times 8} = \sqrt{2 \times 32.16 \times 8}$$
$$= 22.684 \text{ ft. per sec.} \quad \text{Ans.}$$

7. Application of Bernoulli's Law.—The following example shows how Bernoulli's law may be used in the solution of actual problems.

EXAMPLE.—Water flows from a vessel a and through an inclined pipe p, Fig. 3. At the section s_1, the pipe diameter is 5 inches, while at the end s_2 the diameter is 3 inches. The tube inserted at s_1 shows a pressure head y_1 of 22 feet, and the vertical distance between sections s_1 and s_2 is e_1 = 14 feet. Required, the flow in cubic feet per second.

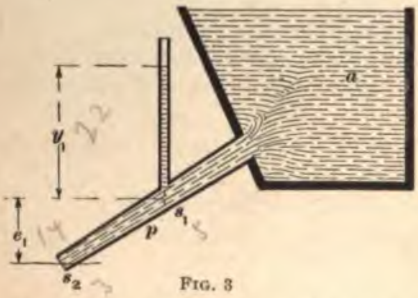


FIG. 3

SOLUTION.—Friction is neglected. Taking the line through s_2 as the datum line, the potential head for the section s_1 is 14 ft. and for the section s_2 is 0. Since, at s_2, the water is subjected only to atmospheric pressure, the pressure head is 0, while for the section s_1 it is 22 ft. Denoting by v_1 and v_2 the velocities at s_1 and s_2, the velocity heads are, respectively, $\frac{v_1^2}{2g}$ and $\frac{v_2^2}{2g}$. Applying formula **4** of Art. 5 to the sections s_1 and s_2,

$$\frac{v_1^2}{2g} + y_1 + e_1 = \frac{v_2^2}{2g} + y_2 + e_2,$$

or

$$\frac{v_1^2}{2g} + 22 + 14 = \frac{v_2^2}{2g} + 0 + 0;$$

whence,

$$\frac{v_2^2}{2g} - \frac{v_1^2}{2g} = 36,$$

or

$$v_2^2 - v_1^2 = 36 \times 2 \times 32.16 = 2{,}315.5$$

By the formula of Art. **2**, $A_1 v_1 = A_2 v_2$, or

$$v_2 = v_1 \times \frac{A_1}{A_2} = v_1 \times \frac{.7854 \times 5^2}{.7854 \times 3^2} = \frac{25}{9} v_1$$

Hence,
$$v_2^2 = \left(\frac{25}{9} v_1\right)^2 = \frac{625}{81} v_1^2$$

Substituting this value of v_2^2 in the preceding equation,

$$\frac{625}{81} v_1^2 - v_1^2 = 2{,}315.5,$$

or
$$v_1^2 = \frac{2{,}315.5}{\frac{625}{81} - 1} = \frac{2{,}315.5 \times 81}{625 - 81} = 344.77,$$

and
$$v_1 = \sqrt{344.77} = 18.57 \text{ ft. per sec.}$$
$$v_2 = \frac{25}{9} v_1 = 51.583 \text{ ft. per sec.}$$

By formula **1** of Art. **1**,

$$Q = A_1 v_1 = \frac{.7854 \times 5^2}{144} \times 18.57 = 2.532 \text{ cu. ft. per sec.} \quad \text{Ans.}$$

8. Piezometers.—A gauge or tube inserted in a pipe to show the pressure of the water is called a **piezometer.** When a piezometer is to be placed on a pipe through which water is flowing, the tube should always be so inserted as to be at right angles to the current in the pipe, as shown at *a*, Fig. 4. If the tube is so inclined that the current flows against the end, as shown at *b*, the action of the current will force the water into the tube and cause it to rise higher than the head due to the pressure; and if inclined in the opposite direction, as at *c*, the action of the current will reduce the indicated pressure.

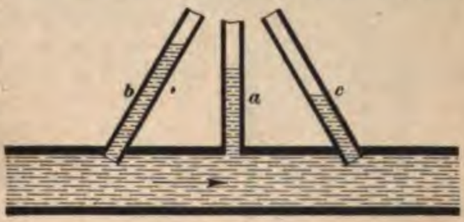


FIG. 4

If a pipe running full of water is equipped with a number of piezometers at various points along its length, the line joining the tops of the columns of water in the several piezometers is called the *hydraulic grade line*. In the case of a pipe of uniform diameter discharging from one reservoir into another, the hydraulic grade line is a straight line

drawn from a point on the surface of the water vertically above the inlet to a point on the surface vertically above the outlet.

EXAMPLES FOR PRACTICE

1. How much water per minute will pass any section of an 8-inch water main if the velocity is $6\frac{1}{2}$ feet per second? Ans. 136.14 cu. ft.

2. In example 1, what will be the velocity through a part of the main that is contracted to a diameter of $6\frac{1}{2}$ inches?

Ans. 9.846 ft. per sec.

3. The hydrostatic head on a given section of a pipe is 13 feet, the velocity of the water passing through the section is 12 feet per second, and friction is neglected. (*a*) What is the velocity head? (*b*) What is the pressure head? (*c*) What is the absolute fluid pressure at the section?

Ans. $\begin{cases} (a)\ 2.239 \text{ ft.} \\ (b)\ 10.761 \text{ ft.} \\ (c)\ 19.37 \text{ lb. per sq. in.} \end{cases}$

4. In Fig. 3 suppose that the height of water shown by the piezometer is 15 feet and the vertical distance between the sections s_1 and s_2 is 10 feet. Let the pipe at s_1 be 6 inches in diameter, and at s_2 4 inches in diameter. Compute the discharge Q in cubic feet per second. Ans. 3.9 cu. ft.

5. When there is no flow, the piezometer inserted at a certain point of a pipe shows a height of 39 inches, but when the flow is started this height drops to 23 inches. Neglecting friction, what is the velocity of the flow at the section? Ans. 9.26 ft. per sec.

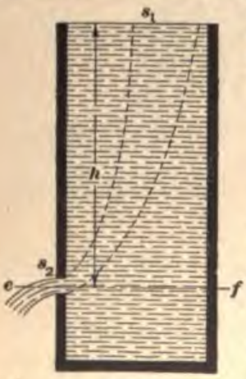


FIG. 5

FLOW OF WATER THROUGH ORIFICES

9. Theoretical Velocity of Efflux. Let a small aperture be made in the side of a vessel, Fig. 5, containing water and let the distance of the aperture from the upper level of the water be denoted by h. To determine the velocity with which the water will flow from the aperture, Bernoulli's law may be applied. A small mass of water may be imagined as starting at the surface s_1 and moving downwards to the orifice s_2 just as though enclosed in a

frictionless tube, as shown by the dotted outlines. At s_1, the velocity is practically zero, and the velocity head is therefore zero; the pressure head is likewise zero. If the line ef is taken as the datum line for potential heads, the potential head for s_1 is h. For s_2, let v denote the velocity with which the water flows through the aperture, called the **velocity of efflux**; then $\frac{v^2}{2g}$ is the velocity head. The pressure head is zero, the orifice being open to the atmosphere, and the potential head is also zero. Hence, applying Bernoulli's law,

$$0 + 0 + h = \frac{v^2}{2g} + 0 + 0;$$

whence, $v^2 = 2gh$, and

$$v = \sqrt{2gh}$$

The theoretical velocity of efflux is the same as the velocity the water would attain in falling through the height h.

EXAMPLE 1.—A small orifice is made in a pipe 50 feet below the water level; what is the velocity of the issuing water?

SOLUTION.—By the formula,

$$v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 50} = 56.71 \text{ ft. per sec.} \quad \text{Ans.}$$

If the velocity of efflux is known, the head h required to produce the velocity can be found as shown by the following example:

EXAMPLE 2.—An issuing jet of water has a velocity of 60 feet per second; what is the head that causes it to flow with this velocity?

SOLUTION.—By transposing the formula,

$$h = \frac{v^2}{2g} = \frac{60^2}{2 \times 32.16} = 55.97 \text{ ft.} \quad \text{Ans.}$$

10. Flow Under Pressure.—Suppose that the surface of the water in the vessel of Fig. 5 is subjected to a pressure in addition to the pressure of the atmosphere. This extra pressure may be due to a loaded piston placed on the surface of the water, or perhaps to steam pressure.

Let p denote this extra pressure in pounds per square inch. The height of a column of water that will cause the pressure p is $h_1 = \frac{p}{.434} = 2.304\,p$; hence, under these conditions, the water at the surface s_1 has a pressure head $h_1 = 2.304\,p$.

If water issues from an orifice h feet below the surface that sustains the pressure h_1, Bernoulli's law gives the equation

$$0 + h_1 + h = \frac{v^2}{2g} + 0 + 0;$$

whence, $$v = \sqrt{2g(h_1 + h)} \qquad (1)$$

The total head, $h_1 + h$, is called the **equivalent head,** and must, in all cases, be reduced to feet before substituting in the formula.

EXAMPLE 1.—The area of a piston fitting a vertical vessel filled with water is 27.36 square inches. The total pressure on the piston is 80 pounds; the weight of the piston is 25 pounds; and the head of the water at the level of the orifice is 6 feet 10 inches; what is the velocity of the efflux, assuming that there are no resistances?

SOLUTION.— $80 + 25 = 105$ lb., the total pressure on the upper surface of the liquid.

$$105 \div 27.36 = 3.8377 \text{ lb. per sq. in.}$$

Then, $h_1 = 3.8377 \div .434 = 8.8426$ ft.,

the head, in feet, due to the pressure of 105 lb.;

$$h = 6 \text{ ft. } 10 \text{ in.} = 6.8333 \text{ ft.}$$

Using formula **1**,

$$v = \sqrt{2g(h_1 + h)} = \sqrt{2g(8.8426 + 6.8333)} = \sqrt{2 \times 32.16 \times 15.6759}$$
$$= 31.75 \text{ ft. per sec. Ans.}$$

Because the atmospheric pressure is the same at both the upper surface and the point of efflux, its effect at these two points is neutralized and pressures above the atmosphere are used instead of absolute pressures. When the fluid is discharged into a vacuum, however, absolute pressures must be used.

In deriving the formulas of Art. **9** and formula **1** of this article, it is assumed that the orifice opens to the atmosphere. If the orifice is subjected to an external pressure, which may be denoted by p_2, the corresponding pressure head on the orifice is $h_2 = 2.304\ p_2$. Using this pressure head for s_2, Bernoulli's law gives the equation

$$0 + h_1 + h = \frac{v^2}{2g} + h_2 + 0;$$

whence, $\frac{v^2}{2g} = h + h_1 - h_2$, and

$$v = \sqrt{2g(h + h_1 - h_2)} \qquad (2)$$

EXAMPLE 2.—Water under a head of 175 feet flows through an orifice into a tank containing compressed air having a gauge pressure of 35 pounds per square inch; find the velocity of efflux.

SOLUTION.—Use formula **2**, $v = \sqrt{2g(h + h_1 - h_2)}$. $h = 175$, $h_1 = 0$, and $h_2 = 2.304 \times 35 = 80.64$ ft. Then, substituting,

$$v = \sqrt{2 \times 32.16 \times (175 - 80.64)} = 77.9 \text{ ft. per sec. Ans.}$$

11. Flow Through a Large Orifice.—If, in Fig. 5, the area of the orifice is greater than about one-twentieth of the area of the cross-section of the vessel, the velocity of the water at the surface becomes appreciable and must be taken into account.

Let a = area of orifice in any unit, as square feet or square inches;
A = area of surface, in the same unit as a;
v = velocity of efflux, in feet per second;
V = velocity with which surface s_1 sinks, in feet per second.

By the formula of Art. **2**, $A\,V = a\,v$, or $V = \frac{a}{A}v$. The velocity head at s_1 is $\frac{V^2}{2g}$, and, using this instead of zero, Bernoulli's law gives

$$\frac{V^2}{2g} + 0 + h = \frac{v^2}{2g} + 0 + 0$$

Transposing, $\frac{v^2}{2g} - \frac{V^2}{2g} = h$; but $V^2 = \frac{a^2}{A^2}v^2$; hence

$$\frac{v^2}{2g} - \frac{a^2 v^2}{2g A^2} = h,$$

or

$$\frac{v^2}{2g}\left(1 - \frac{a^2}{A^2}\right) = h$$

Solving for v,

$$v = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}}$$

When the area at the office is more than one-twentieth of the area of the cross-section of the vessel, this formula should be used; when less than one-twentieth of that area, the formula of Art. **9** gives results that are sufficiently accurate.

EXAMPLE.—An orifice 4 inches square is cut in the bottom of a vessel having a rectangular cross-section 11 inches by 14 inches; the water level is 14 feet above the bottom. Compute: (*a*) the velocity of efflux; (*b*) the discharge per second.

SOLUTION.—(*a*) Area of cross-section is $14 \times 11 = 154$ sq. in. Area of orifice is $4 \times 4 = 16$ sq. in. Since $154 \div 16 = 9\frac{5}{8}$, the area of the surface is less than twenty times the area of the orifice; hence, using the above formula and substituting,

$$v = \sqrt{\frac{2gh}{1-\frac{a^2}{A^2}}} = \sqrt{\frac{2 \times 32.16 \times 14}{1-\frac{16^2}{154^2}}} = 30.17 \text{ ft. per sec. Ans.}$$

(*b*) Using the formula of Art. **1**,

$$Q = Av = \frac{16}{144} \times 30.17 = 3.352 \text{ cu. ft. per sec. Ans.}$$

If the formula of Art. **9** is used instead of the more exact formula just given, the velocity is found to be

$$v = \sqrt{2 \times 32.16 \times 14} = 30.008 \text{ ft. per sec.},$$

which is slightly less than the value obtained by the formula of this article.

STANDARD ORIFICES

12. Edges of Standard Orifices.—An orifice in the side or bottom of a vessel or reservoir, and at a distance below the surface of the water, is called a **standard orifice** when the flow through it takes place in such a manner that the jet touches the opening on the inside edge only. A hole in a thin plate, as shown in Fig. 6, is such an orifice, as is also a square-edged hole in the side of a vessel, as shown in Fig. 7, when the thickness of the side is not so great that the jet touches it beyond the inner edge. If the sides of the reservoir are very thick, a standard orifice can be made by beveling the outer edges, as shown in Fig. 8.

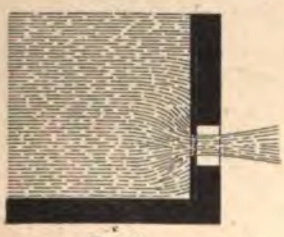


FIG. 6

FIG. 7

13. Contraction of the Jet.—When a jet issues from a circular orifice, it contracts so that the diameter is least at a distance from the edge equal to about one-half the diameter of the orifice. Beyond this point, the jet gradually enlarges and becomes broken by the resistance of the air. Orifices that are not circular also cause contractions.

FIG. 8

The **coefficient of contraction** is the number by which the area of the orifice is multiplied in order to obtain the least cross-section of the jet. Experiments on jets from standard orifices have given values for this coefficient varying from .57 to .71. The probable mean value is about .62.

14. Coefficient of Velocity.—The number by which the theoretical velocity must be multiplied in order to obtain the actual maximum velocity, or the velocity where the cross-section of the jet is least, is called the **coefficient of velocity.**

If v = theoretical velocity;
v' = actual velocity;
c' = coefficient of velocity;

the formula of Art. 9 becomes,

$$v' = c' v = c' \sqrt{2gh}$$

It is found that c' is greater for high heads than for low, and values ranging from .975 to nearly 1 have been obtained by different experimenters. An average value usually taken is .98.

EXAMPLE 1.—What is the actual velocity of discharge from a small standard orifice in the side of a vessel, if the head is 20 feet?

SOLUTION.—

$v' = c' \sqrt{2gh} = .98 \sqrt{2 \times 32.16 \times 20} = 35.15$ ft. per. sec. Ans.

Most of the problems occurring in hydraulics involve the operations of multiplication, division, involution, and evolution in such a way that they are most readily solved by the

use of logarithms. *Logarithms*, therefore, should be reviewed very carefully and thoroughly in order that its principles may be readily applied to the solutions of the problems here given.

EXAMPLE 2.—Solve example 1 by means of logarithms.

SOLUTION.—First find the logarithm of the product of the numbers under the radical sign from the table of logarithms by adding the logarithms of the individual numbers, as follows:

$$\begin{aligned} \log 2 &= .30103 \\ \log 32.16 &= 1.50732 \\ \log 20 &= 1.30103 \\ & \overline{3.10938} = \log (2 \times 32.16 \times 20) \end{aligned}$$

The logarithm of the square root of this product is found by dividing its logarithm by 2; thus,

$$\log \sqrt{2 \times 32.16 \times 20} = 3.10938 \div 2 = 1.55469$$

Finally, the logarithm of the product of .98 multiplied by the quantity under the radical sign is the sum of the logarithm of .98 and 1.55469, or $\bar{1}.99123 + 1.55469 = 1.54592$. From the table of logarithms, the number corresponding to this logarithm is found to be 35.15. Ans.

15. Coefficient of Discharge.—It is evident that the contraction of the jet issuing from an orifice and the reduction of the theoretical velocity at the smallest cross-section both tend to reduce the quantity of water flowing through an orifice as calculated from the formulas $Q = a v$ and $v = \sqrt{2 g h}$. Let Q' denote the theoretical discharge and Q the actual discharge; then the ratio $\frac{Q}{Q'}$ is called the **coefficient of discharge.** Denoting this coefficient by k,

$$Q = k Q'$$

The values of k vary with the shape of the orifice, the head, and the velocity of discharge. These values have been determined by experiment, and it is found that they vary between .59 and .63 for the most practical cases. For ordinary computations, an average value of k may be taken as .61. Therefore,

$$Q = k Q' = k A v = k A \sqrt{2 g h}$$

or

$$Q = .61 A \sqrt{2 g h}$$

where A = area of orifice, in square feet;
Q = discharge, in cubic feet per second;
h = head on center of orifice, in feet.

EXAMPLE.—What will be the actual discharge from a circular standard orifice 3 inches in diameter under a head of 25 feet?

SOLUTION.—The area of a 3-in. circle is

$$.7854 \times 3^2 = 7.0686 \text{ sq. in.} = 7.0686 \div 144 = .049 \text{ sq. ft.}$$

$$Q = k A \sqrt{2 g h} = .61 \times .049 \sqrt{2 \times 32.16 \times 25} = 1.1986 \text{ cu. ft. per sec.}$$

Ans.

EXAMPLES FOR PRACTICE

1. What is the discharge, in cubic feet per minute, from a standard circular orifice whose diameter is $2\frac{1}{2}$ inches, if the head is 20 feet?

Ans. 44.75 cu. ft. per min.

2. A square orifice in the side of a reservoir measures .2 foot on each side, and the head on the center is 22 feet; what is the discharge in cubic feet per second? Ans. .9178 cu. ft. per sec.

3. What is the discharge from a rectangular orifice 1 foot wide, if the head on the upper edge is $2\frac{1}{2}$ feet and the depth of the orifice $10\frac{1}{2}$ inches? Ans. 7.337 cu. ft. per sec.

WEIRS

16. Use of Weirs.—A **weir** is a vertical obstruction placed across a stream or channel and containing a notch (or a number of them) in its upper edge through which water is allowed to flow for purposes of measurement. It has been found that, when properly constructed and carefully managed, a weir forms one of the most convenient and accurate devices for measuring the discharge of streams. Many careful experiments* have been made to determine the quantity of water that will flow through different forms of weirs under varying conditions. As a result of these experiments, rectangular weirs have come to be generally used, and the amount of flow in any particular case may be calculated by simple formulas with tabulated coefficients that depend on observed conditions. Triangular weirs are occasionally used for experimental purposes.

17. Rectangular Weirs.—There are two types of rectangular weirs; those with and those without end contractions. A **weir with end contractions** is shown in Fig. 9. The notch is narrower than the channel

through which the water flows and this causes a contraction at the bottom and at the two sides of the issuing stream. A **weir without end contractions** is shown in Fig. 10. In

FIG. 9

FIG. 10

this case, the notch is as wide as the channel leading to it; consequently, the issuing stream is contracted at the bottom only.

The edge *a* of the notch, in either Fig. 11 or Fig. 12, is called the **crest of the weir.** The inner edges of the notch are made sharp, so that the water in passing through it touches only along a line. For very accurate work, the edges, both vertical and horizontal, should be made with a thin plate of metal having a sharp inner edge, as shown in Fig. 11; but for ordinary work, the edges of the board in which the notch is cut may be chamfered off to an angle of about 30°, as shown in Fig. 12.

FIG. 11

The bottom edge of the notch must be straight and set perfectly level, and the sides must be set carefully at right angles to the bottom.

The head H producing the flow, Figs. 11 and 12, is the vertical distance from the crest to the surface of the water. It must be measured at a point so far from the crest that the curvature of the flowing water will not affect the measurement.

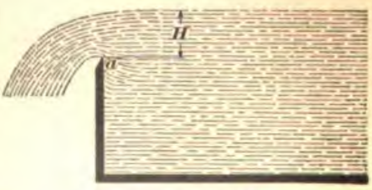


FIG. 12

The distance from the crest of the weir to the bottom of the feeding canal or reservoir should be at least three times the head; and, with a weir having end contractions, the distance from the vertical edges to the sides of the canal should also be at least three times the head.

The water must approach the weir quietly, and with little velocity. It is sometimes necessary to provide means for reducing the velocity of the water as it approaches the weir.

18. Discharge of Weirs.—When the dimensions of the notch and the head on the crest of a weir are known, the discharge can be computed by means of the following formulas and tables of coefficients:

Let l = length of weir in feet;
H = measured head, in feet;
v = velocity, in feet per second, with which water approaches weir;
h = head equivalent to velocity with which water approaches weir;
k = coefficient of discharge;
Q' = theoretical discharge, in cubic feet per second;
Q = actual discharge, in cubic feet per second.

The formula for the theoretical discharge per second is obtained by the use of higher mathematics and is

$$Q' = \tfrac{2}{3}\sqrt{2g}\,l(H+h)^{\frac{3}{2}} \tag{1}$$

If there is no velocity of approach, this becomes

$$Q' = \tfrac{2}{3}\sqrt{2g}\,l\,H^{\frac{3}{2}} \tag{2}$$

The actual discharge for weirs with end contractions, and considering the velocity of approach, is given by the formula

$$Q = \tfrac{2}{3}k\sqrt{2g}\,l(H+1.4\,h)^{\frac{3}{2}} = 5.347\,k\,l(H+1.4\,h)^{\frac{3}{2}} \tag{3}$$

If there is no velocity of approach, this becomes

$$Q = \tfrac{2}{3}k\sqrt{2g}\,l H^{\frac{3}{2}} = 5.347\,k\,l\,H^{\frac{3}{2}} \tag{4}$$

For weirs without end contractions, and considering the velocity of approach, the formula is

$$Q = \tfrac{2}{3}k\sqrt{2g}\,l(H+\tfrac{4}{3}h)^{\frac{3}{2}} = 5.347\,k\,l(H+\tfrac{4}{3}h)^{\frac{3}{2}} \tag{5}$$

If there is no velocity of approach, this becomes

$$Q = \tfrac{2}{3}k\sqrt{2g}\,l\,H^{\frac{3}{2}} = 5.347\,k\,l\,H^{\frac{3}{2}} \tag{6}$$

EXAMPLE.—What is the discharge of a stream, if the length of the weir is 5 feet, the head $10\frac{1}{2}$ inches, the coefficient of discharge .603, and the velocity of approach = 0, the weir having end contractions?

SOLUTION.—Applying formula **4**, and substituting,

$$Q = 5.347 \times .603 \times 5 \times .875^{\frac{3}{2}} = 13.195 \text{ cu. ft. per sec. Ans.}$$

19. The value of the coefficient k varies with the effective head and width of the weir. Tables I and II give fairly accurate values for these coefficients for the given conditions.

TABLE I

COEFFICIENTS FOR WEIRS WITH END CONTRACTIONS

Effective Head Feet	Length of Weir, in Feet						
	.66	1	2	3	5	10	19
.1	.632	.639	.646	.652	.653	.655	.656
.15	.619	.625	.634	.638	.640	.641	.642
.20	.611	.618	.626	.630	.631	.633	.634
.25	.605	.612	.621	.624	.626	.628	.629
.30	.601	.608	.616	.619	.621	.624	.625
.40	.595	.601	.609	.613	.615	.618	.620
.50	.590	.596	.605	.608	.611	.615	.617
.60	.587	.593	.601	.605	.608	.613	.615
.70		.590	.598	.603	.606	.612	.614
.80			.595	.600	.604	.611	.613
.90			.592	.598	.603	.609	.612
1.00			.590	.595	.601	.608	.611
1.2			.585	.591	.597	.605	.610
1.4			.580	.587	.594	.602	.609
1.6				.582	.591	.600	.607

NOTE.—The head given is the effective head, $H + 1.4\,h$. When the velocity of approach is small, h is neglected.

Table I, Coefficients for Weirs with End Contractions, gives values of the coefficient of discharge k for weirs with end contractions and different values of H and l. Table II, Coefficients for Weirs Without End Contractions, gives values for k for weirs without end contractions. Weirs with end

contractions are more often used and are to be recommended in most cases. Values of k for values of H and l between those given in the tables can be found by interpolating, assuming that the variation is uniform between the values given.

TABLE II

COEFFICIENTS FOR WEIRS WITHOUT END CONTRACTIONS

Effective Head Feet	Length of Weir, in Feet						
	19	10	7	5	4	3	2
.10	.657	.658	.658	.659			
.15	.643	.644	.645	.645	.647	.649	.652
.20	.635	.637	.637	.638	.641	.642	.645
.25	.630	.632	.633	.634	.636	.638	.641
.30	.626	.628	.629	.631	.633	.636	.639
.40	.621	.623	.625	.628	.630	.633	.636
.50	.619	.621	.624	.627	.630	.633	.637
.60	.618	.620	.623	.627	.630	.634	.638
.70	.618	.620	.624	.628	.631	.635	.640
.80	.618	.621	.625	.629	.633	.637	.643
.90	.619	.622	.627	.631	.635	.639	.645
1.00	.619	.624	.628	.633	.637	.641	.648
1.2	.620	.626	.632	.636	.641	.646	
1.4	.622	.629	.634	.640	.644		
1.6	.623	.631	.637	.642	.647		

NOTE.—The head given is the effective head, $H + \frac{4}{3}h$. When the velocity of approach is small, h may be neglected.

20. The **velocity of approach** is the mean velocity with which the water flows through the canal leading to the weir. If A is the area of the cross-section of the water in this canal, $v = \frac{Q}{A}$, and the equivalent head is

$$h = \frac{v^2}{2g} = .01555\ v^2$$

The velocity v may be measured approximately by means of a float on the water in the canal or stream, but a better

method is to compute the flow Q by formula **4** or formula **6** of Art. **18**, assuming that $v = 0$, and then calculate an approximate value of v from $v = \frac{Q}{A}$. However, since v is small with a properly constructed weir, it is usually neglected, unless great accuracy is required.

EXAMPLE 1.—What is the discharge from a weir with end contractions under the following conditions: the length of the weir is 4 feet $1\frac{1}{2}$ inches, and the measured head $10\frac{1}{16}$ inches? Assume that there is no velocity of approach.

SOLUTION.—The length l of the weir is 4 ft. $1\frac{1}{2}$ in. = 4.125 ft., and the head H is $10\frac{1}{16}$ in. = .84 ft., nearly. From Table I, the coefficient k = .600 for a weir 3 ft. long and a head of .8 ft. and k = .604 for a weir 5 ft. long with the same head. There is an increase in the coefficient of $(.604 - .600) \div 2 = .002$ for each increase of 1 ft. in length. The coefficient for a weir 4.125 ft. long is, therefore,

$$.600 + (1.125 \times .002) = .60225$$

The coefficient k = .603 for a weir 5 ft. long with a head of .9 ft. and k = .598 for a weir 3 ft. long with the same head. There is an increase in the coefficient of $(.603 - .598) \div 2 = .0025$ for each increase of 1 ft. in length.

The coefficient for a weir 4.125 ft. long with a head of .9 ft. is, therefore, $.598 + (1.125 \times .0025) = .60081$

The decrease in coefficient for an increase in head of .1 ft. is

$$.60225 - .60081 = .00144$$

and for an increase in head of .04 ft. the decrease is

$$.00144 \times \frac{.04}{.1} = .000576$$

This subtracted from the coefficient for .8 ft. gives $.60225 - .000576 = .601674$ as the coefficient of discharge for a weir 4.125 ft. long and a head of .84 ft. Using but four decimal places, the discharge, by formula **4** of Art. **18**, is

$$Q = 5.347 \times .6017 \times 4.125 \times .84^{\frac{3}{2}} = 10.22 \text{ cu. ft. per sec.} \quad \text{Ans.}$$

EXAMPLE 2.—If the canal leading to the weir of example 1 is 10 feet wide and 3 feet deep below the crest of the weir, what is the head equivalent to the velocity of approach?

SOLUTION.—The depth of water in the canal is the depth below the crest plus the head = 3.84 ft. The area of the cross-section of the water in the canal is

$$A = 3.84 \times 10 = 38.4 \text{ sq. ft.},$$

and the velocity is

$$v = 10.22 \div 38.4 = .266 \text{ ft. per sec.}$$

The head h equivalent to the velocity v is

$$h = \frac{v^2}{2g} = \frac{.266^2}{64.32} = .0011 \text{ ft.} \quad \text{Ans.}$$

NOTE.—This value of h is so small that its influence on the discharge is much less than the probable errors in measuring the head H, and so need not be considered in finding the discharge.

EXAMPLES FOR PRACTICE

1. What is the discharge per second from a weir with end contractions when the length of the weir is 24 inches, and the measured head is $10\frac{1}{2}$ inches? Assume that there is no velocity of approach.
Ans. 5.188 cu. ft.

2. With the same conditions as in example 1, except that the weir is without end contractions, what is the discharge per second?
Ans. 5.641 cu. ft.

3. What is the discharge per second from a weir with end contractions 36 inches long, with a head of 9 inches and a velocity of approach of 3 feet per second?
Ans. 8.80 cu. ft.

FLOW THROUGH TUBES

21. The Standard Tube.—A **standard tube,** or an **adjutage,** is a tube whose length is two and one-half or three times its diameter. When water flows from a reservoir through such a tube, as shown in Fig. 13, the jet contracts when it first leaves the reservoir, then expands again until it fills the tube near its outer end, this contraction and expansion resembling that of the jet from a standard orifice.

FIG. 13

Owing to the contraction of the jet, there is a space between the jet and the tube at the point where the jet is smallest. When the tube extends far enough beyond the contraction, the jet again fills the tube; the swiftly moving water of the jet carries some of the air, from the space, along with it, thus producing a partial vacuum around the contracted portion of the jet.

If a small branch tube as shown at *b*, Fig. 13, is carried down into a cup of mercury *a*, the pressure of the atmosphere will force mercury into *b* to a height *h* that depends on the vacuum in the space around the jet, and if a small hole is made in the tube it will be found that air is drawn in through the hole.

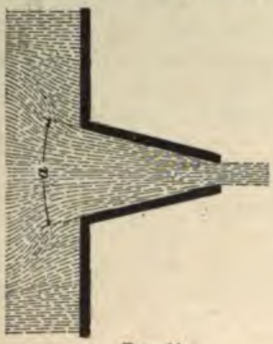


FIG. 14

FIG. 15

On account of the difficulty in maintaining uniform conditions, which makes the value of the coefficient of discharge uncertain, tubes are seldom used for measuring the flow of water.

The coefficient of discharge for a standard tube is greater than for a standard orifice. An average value is $k = .82$.

The coefficient of velocity for cylindrical tubes is the same as the coefficient of discharge.

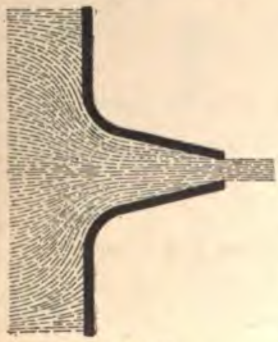


FIG. 16

22. Conical Tubes.—For **conical tubes**, as shown in Figs. 14 and 15, the coefficient of discharge reaches a maximum value of .946 when the angle *a* of the cone is 13° 24′. If the inner edge of the tube is well rounded, as shown in Fig. 16, the coefficient of discharge is still further increased and may be made nearly 1.

The coefficient of velocity for conical tubes increases with the angle of the cone until at an angle of about 40° it becomes

approximately the same as the coefficient of velocity for the standard orifice, which, it will be remembered, is .98.

23. Compound Tubes.—Examples of **compound tubes** are shown in Figs. 17 and 18. Experiments have shown that the velocity through the minimum section *a* is greater

FIG. 17

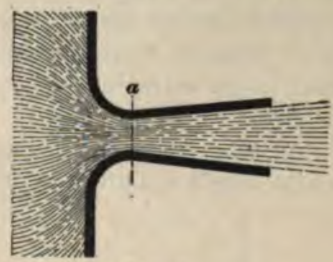


FIG. 18

than the theoretical velocity due to the head. The values of the coefficient of discharge for the section *a* vary greatly under different conditions of head and proportions of tubes. Under certain conditions, values as high as 2.43 have been obtained.

24. Inward Projecting Tubes.—It has been demonstrated by experiment that when a tube projects into a reservoir, as shown in Figs. 19 and 20, the contraction is increased and

FIG. 19

FIG. 20

the discharge is greatly reduced. For the tube shown in Fig. 19, the coefficient of discharge is about .50; and for that shown in Fig. 20, about .72.

FLOW OF WATER IN PIPES

LOSSES OF HEAD

25. Preliminary Statement.—In considering the flow of water through pipes of considerable length, one important factor hitherto neglected must receive attention; namely, the friction between the water and the interior surface of the pipe.

Referring to Fig. 21, the level of the water in the reservoir is at a height h above the end of the pipe. If the pipe discharges into the atmosphere, h is the head that causes the flow; but if the pipe discharges into a second reservoir, the head h is the difference between the water levels in the two reservoirs. If G pounds of water are discharged per second, the theoretical work of this quantity of water falling through the height h is $G h$ foot-pounds. Now, if v is the velocity of discharge, $\frac{G v^2}{2 g}$ is the kinetic energy of the outflowing water; and if there is no work done in overcoming friction, the two expressions are equal, or $G h = G\frac{v^2}{2 g}$, or $h = \frac{v^2}{2 g}$, as in the case of a theoretical flow from an orifice. In the case of long pipes, it is always found that $\frac{G v^2}{2 g}$ is less than $G h$, the difference being the work done against the various frictional resistances. Hence, the head $\frac{v^2}{2 g}$ is less than the total head h, and the difference $h - \frac{v^2}{2 g}$ is the loss of head due to frictional resistance. In the case of very long pipes, the

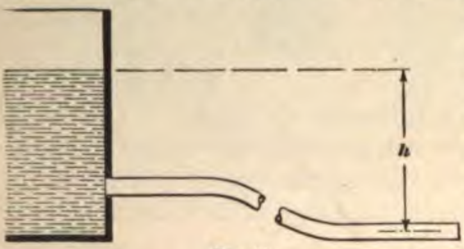


FIG. 21

velocity v may become very small, showing that nearly all the head h has been lost in overcoming resistance.

In the following paragraphs, the losses of head occasioned by resistances of various kinds will be considered.

26. Loss of Head From Friction in the Pipe.—Experiments have shown that the friction of water flowing through a pipe depends, approximately, on the following laws:

I. *The loss in friction is proportional to the length of the pipe.*
II. *It varies nearly as the square of the velocity.*
III. *It varies inversely as the diameter of the pipe.*
IV. *It increases with the roughness of the pipe.*
V. *It is independent of the pressure in the pipe.*

The following formula is based on experimental data, and is in accord with the above laws:

Let l = length of pipe, in feet;
d = diameter of pipe, in feet;
v = velocity, in feet per second, of water flowing in pipe;
f = a coefficient depending on roughness of pipe; the value .02 is usually taken where very accurate results are not required;
h_f = head lost by friction;
g = 32.16.

Then,
$$h_f = f\frac{l v^2}{d 2 g}$$

EXAMPLE.—What is the loss of head due to friction in a 10-inch pipe 1,000 feet long, if the mean velocity of flow is 8 feet per second and f = .0197?

SOLUTION.—Using the formula and substituting,
$$h_f = f\frac{l v^2}{d 2 g} = .0197 \times \frac{1{,}000}{.83\frac{1}{3}} \times \frac{8^2}{2 \times 32.16} = 23.522 \text{ ft.} \quad \text{Ans.}$$

27. Loss of Head at Entrance.—Water, on entering a pipe from a reservoir, meets with resistances due to friction and contraction, and there is a loss of head similar to

the loss due to friction. The usual form of the equation expressing this loss of head is

$$h_e = m\frac{v^2}{2g}$$

where v = velocity, in feet per second;
m = coefficient depending on form of end of pipe;
h_e = head lost at entrance, in feet.

For long water mains the value of m is usually taken as .5.

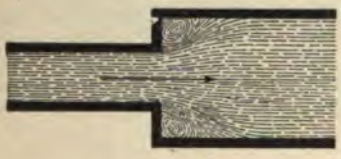


FIG. 22

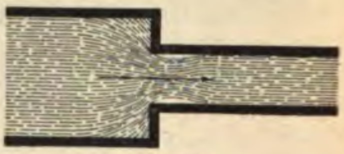


FIG. 23

28. Loss of Head Due to Change of Section.—When water flows from a small section to a larger one, as shown in Fig. 22, energy is absorbed in producing eddies among the water particles just at the enlargement. The change from a large to a smaller section, as shown in Fig. 23, causes a contraction in the mouth of the smaller section. The result in both cases is a loss of head. If the change in section in the pipe is made gradually, as in Figs. 24 and 25, the loss is small and may be neglected when computing the flow. In practice, a change in section is usually made by means of a **reducer**, like that shown in Fig. 26.


FIG. 24


FIG. 25


FIG. 26

29. Loss of Head Due to Bend.—When there are sudden bends in the pipe, there will be a loss, due partly to shock and eddies, and partly to the contraction in the flow,

as shown in Figs. 27 and 28. Experiments made with bends like that shown in Fig. 27 show that the loss of

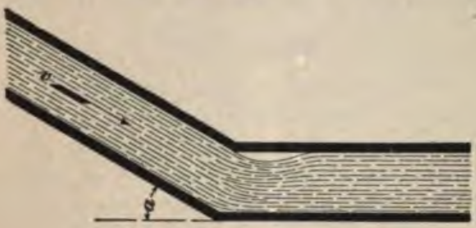


FIG. 27

head h_b may be expressed in terms of the mean velocity by the formula

$$h_b = c\frac{v^2}{2g} \qquad (1)$$

Table III gives values of c for different values of the angle a.

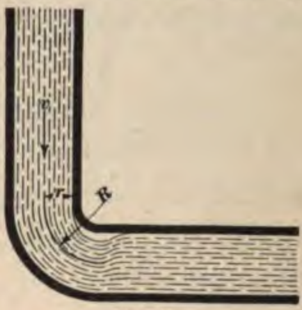


FIG. 28

For a 90° bend like that shown in Fig. 28, the loss of head h_b' is expressed by the formula

$$h_b' = c'\frac{v^2}{2g} \qquad (2)$$

in which c' depends on the ratio between the radius r of the pipe and the radius R of the bend.

TABLE III

VALUES OF c FOR ANGLES a

$a =$	10°	20°	40°	60°	80°	90°	100°	110°	120°	130°	140°	150°
$c =$	.017	.046	.139	.364	.74	.984	1.26	1.56	1.86	2.16	2.43	2.81

Table IV gives values of c' corresponding to various values of the ratio $\frac{r}{R}$.

From Table IV, it is seen that when R is made large in comparison to r, the value of c', and hence the loss in head, is small.

There may be other resistances, such as valves, that change the direction of flow of the water or suddenly change the area through which the water flows. If the pipe is carefully designed and laid, however, these losses may be made so small in comparison with the other losses just named that they may be neglected in the formulas for head and velocity.

TABLE IV

VALUES OF c' FOR RATIOS $\frac{r}{R}$

$\frac{r}{R}$ =	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
c' =	.131	.138	.158	.206	.294	.440	.661	.977	1.408	1.978

GENERAL FORMULAS FOR FLOW IN PIPES

30. Formulas for Velocity of Flow.—As stated in Art. **25**, the difference $h - \frac{v^2}{2g}$, between the total head and velocity head, is the loss of head due to the various frictional resistances; hence, if a pipe has n bends, like that shown in Fig. 28,

$$h - \frac{v^2}{2g} = h_f + h_e + n\,h_b';$$

whence,

$$v = \sqrt{2g\left[h - (h_f + h_e + n\,h_b')\right]} \qquad (1)$$

Using the values of h_f, h_e, and h_b' from the formulas of Arts. **26** and **27**, and formula **2** of Art. **29**,

$$h - \frac{v^2}{2g} = f\frac{l}{d}\frac{v^2}{2g} + m\frac{v^2}{2g} + n\,c'\frac{v^2}{2g}$$

Solving for v, the following formula is obtained for the velocity of flow:

$$v = \sqrt{\frac{2gh}{1 + f\frac{l}{d} + m + n\,c'}} = 8.02\sqrt{\frac{h}{1 + f\frac{l}{d} + m + n\,c'}} \qquad (2)$$

When the form of bend shown in Fig. 27 is used, the coefficient c from Table III must be used instead of c'.

Giving m the value .5 for the common case of a pipe with a bell end, and assuming that there are no sharp bends or similar resistances, the formula for v becomes

$$v = \sqrt{\frac{2gh}{1.5 + f\frac{l}{d}}} = 8.02\sqrt{\frac{h}{1.5 + f\frac{l}{d}}} \qquad (3)$$

31. Flow in Long Pipes.—When the pipe is very long, compared with its diameter, say when l exceeds 4,000 d, the term $f\frac{l}{d}$ becomes so much larger than the 1.5 to which it is added in formula **3** of Art. **30**, that the 1.5 may be disregarded, and the formula becomes

$$v = \sqrt{\frac{2gh}{f\frac{l}{d}}} = \sqrt{\left(\frac{2g}{f}\right)\left(\frac{hd}{l}\right)} \qquad (1)$$

The factor $\frac{2g}{f}$ may be replaced by a new symbol $\frac{1}{c}$, in which case the formula takes the form

$$v = \sqrt{\frac{hd}{cl}} \qquad (2)$$

This is D'Arcy's formula for the flow in long pipes.

Since the quantity Q discharged is given by the formula $Q = Av$, the following formula is obtained for the discharge, in cubic feet per second:

$$Q = A\sqrt{\frac{hd}{cl}} \qquad (3)$$

or, since $A = .7854\,d^2$ for circular pipes, $Q = .7854\,d^2\sqrt{\frac{hd}{cl}}$, which may also be written

$$Q = \sqrt{\frac{.617\,d^5h}{cl}} \qquad (4)$$

32. Table V gives the values of the coefficient c based on the experiments of D'Arcy.

It will be observed that the coefficient for smooth pipes is in all cases half that of rough ones. As all pipes, no matter how clean and smooth they may be when first laid

become in process of time more or less coated and foul, it is safer in practice to always use the coefficient for rough pipes when a permanent system is being laid down.

TABLE V

TABLE OF COEFFICIENTS

Diameters Inches	Value of c for Rough Pipes	Value of c for Smooth Pipes
3	.00080	.00040
4	.00076	.00038
6	.00072	.00036
8	.00068	.00034
10	.00066	.00033
12	.00066	.00033
14	.00065	.00033
16	.00064	.00032
24	.00064	.00032
30	.00063	.00032
36	.00062	.00031
48	.00062	.00031

33. By carefully studying Table V, it will be seen that the coefficients for pipes from 8 to 48 inches in diameter do not vary greatly. Moreover, from formula **3** of Art. **31**, it appears that, all other conditions being equal, the quantity discharged is affected by only the square root of the coefficient, so that slight differences in its value are insignificant in reference to the volume of water discharged. Formula **4** of Art. **31** contains the factor .617; hence, by taking .000617 as an approximate coefficient for pipes within the limits of 8 and 48 inches in diameter, the formula becomes

$$Q = \sqrt{\frac{.617\, d^5 h}{.000617\, l}};$$

whence,

$$Q = \sqrt{\frac{1{,}000\, d^5 h}{l}} \qquad (1)$$

Now, let H denote the head per thousand feet, so that $\frac{h}{l} = \frac{H}{1{,}000}$. Substituting this in the formula just given,

$$Q = \sqrt{d^5 H} \qquad (2)$$

In this formula, it must be borne in mind that H is the fall in feet per thousand feet.

For pipes of smaller diameter, from 3 to 8 inches, it is well to assume a coefficient of .000785. Then for such pipes, from formula **4** of Art. **37**,

$$Q = \sqrt{\frac{.617\, d^5 h}{.000785\, l}} = .887\, \sqrt{d^5 H} \qquad \textbf{(3)}$$

That is to say, for these smaller diameters the delivery will be, in round numbers, 90 per cent. of that given by formula **2**.

34. Formulas for Smooth Pipes.—While, in practice, the formulas for rough pipes should always be used, it is sometimes useful to know the probable discharge through smooth ones. Since the coefficients for the latter are always one-half of those for the former, for smooth pipes formula **2** of Art. **33** may be written,

$$Q = \sqrt{2\, d^5 H} = 1.4\, \sqrt{d^5 H}$$

In general, the discharge through a smooth pipe is 1.4 times that through a rough pipe of the same diameter; and, reciprocally, the discharge through a rough pipe is .7 that through a smooth one of the same diameter; these factors represent the practical limits between which the extremes of roughness and smoothness can affect the flow through long pipes.

35. Diameter of Pipe for Given Flow.—Formulas giving the diameter of a pipe for a desired discharge per second are readily obtained from the formulas in Arts. **33** and **34**. Solving formula **2** of Art. **33** for d gives for rough pipes,

$$d = \sqrt[5]{\frac{Q^2}{H}} \qquad \textbf{(1)}$$

For smooth pipes, by solving the formula in Art. **34**,

$$d = \sqrt[5]{\frac{Q^2}{2\, H}} \qquad \textbf{(2)}$$

For pipes of small diameters, up to about 8 inches, multiply by 1.05; that is, add 5 per cent. to the diameters given by these formulas.

36. Formulas for Velocity.—From the formula, $v = \frac{Q}{A}$, and as $A = .7854\, d^2$ for circular pipes $v = \frac{Q}{.7854\, d^2}$; from

formula **2** of Art. **33**, $Q = \sqrt{d^5 H}$; hence, for large rough pipes,

$$v = \frac{\sqrt{d^5 H}}{.7854\, d^2} = 1.27\sqrt{d H} \qquad (1)$$

Similarly, for small rough pipes,

$$v = 1.13\sqrt{d H} \qquad (2)$$

For large smooth pipes,

$$v = 1.78\sqrt{d H} \qquad (3)$$

and for small smooth pipes

$$v = 1.6\sqrt{d H} \qquad (4)$$

37. Head Required for a Given Flow.—The head required per 1,000 feet to produce a flow of Q cubic feet per second in a pipe d feet in diameter is found from formula **2** of Art. **33.** Squaring both members of the formula, it becomes,

$$Q^2 = d^5 H;$$

whence,

$$H = \frac{Q^2}{d^5} \qquad (1)$$

For pipes of small size, from formula **3** of Art. **33,**

$$H = \frac{Q^2}{.785\, d^5} = 1.27\frac{Q^2}{d^5} \qquad (2)$$

The following examples show the application of the preceding formulas:

Example 1.—A rough pipe, 16 inches in diameter and 3,700 feet long, connects two reservoirs, the difference of elevation between the two being 187 feet. With what velocity does the water flow through the pipe?

Solution.—Substituting in formula **2** of Art. **31,** d = 16 in. = $1\frac{1}{3}$ ft., h = 187 ft., l = 3,700 ft., and, from Table V, c = .00064. Hence,

$$v = \sqrt{\frac{187 \times \frac{4}{3}}{.00064 \times 3{,}700}} = 10.26 \text{ ft. per sec.} \quad \text{Ans.}$$

Example 2.—What is the velocity through the pipe in example 1, calculated by formula **1** of Art. **36?**

Solution.— $H = \dfrac{187 \times 1{,}000}{3{,}700} = 50.5$; $d = \frac{4}{3}$; hence, substituting,

$$v = 1.27\sqrt{\frac{4}{3} \times 50.5} = 10.42 \text{ ft. per sec.} \quad \text{Ans.}$$

Note.—In approximate formulas, such as all those that apply to the flow of water through the pipes necessarily are, the results obtained in examples 1 and 2 are equivalent to an agreement, and in practice one might happen to be as nearly right as the other. It is obvious that, when the character of the pipe may vary so widely as to interior surface, a very close result can never be hoped for, and all that can be done is to keep within probable limits.

Example 3.—A rough pipe, 10 inches in diameter, is laid with a fall of $7\frac{1}{2}$ feet per 1,000; what is the discharge?

Solution.—Applying formula **2** of Art. **33**,

$$Q = \sqrt{d^5 H} = \sqrt{\left(\frac{10}{12}\right)^5 \times 7.5}$$

and using logarithms,

log 10	1.00000
log 12	1.07918
Subtracting, log $\frac{10}{12}$ . .	$\bar{1}.92082$
	5
log of fifth power . . .	$\bar{1}.60410$
log 7.5	.87506
	2)0.47916
log of square root . . .	0.23958
Corresponding number = 1.736	

Therefore, the discharge is 1.736 cu. ft. per sec. Ans.

Example 4.—It is desired to discharge 3 cubic feet per second from a pipe line having a fall of 5 feet per 1,000; what diameter of rough cast-iron pipe will be required?

Solution.—Inserting the data in formula **1** of Art. **35**,

$$d = \sqrt[5]{\frac{Q^2}{H}} = \sqrt[5]{\frac{9}{5}}$$

log 9	.95424
log 5	.69897
	5) .25527
log of fifth root	0.05105
Corresponding number = 1.125	

Therefore, the diameter is 1.125 ft. = $13\frac{1}{2}$ in. Ans.

As cast-iron pipes are made to standard sizes, and there are no $\frac{1}{2}$ inches, the nearest appropriate size would be a 14-in. pipe.

Example 5.—It is desired to discharge $\frac{1}{2}$ cubic foot per second from a 4-inch pipe; what head per 1,000 feet of length is necessary to accomplish this?

Solution.—From formula **2**, $H = 1.27\,\frac{Q^2}{d^5}$, in which $Q = \frac{1}{2}$ and $d = \frac{1}{3}$; hence, substituting, it becomes,

$$H = 1.27 \times \frac{(\frac{1}{2})^2}{(\frac{1}{3})^5} = 77.15 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1. A rough pipe 20 inches in diameter connects two reservoirs 2 miles apart; the difference in level is 375 feet. With what velocity will the water flow through the pipe? Ans. 9.617 ft. per sec.

2. What should be the diameter of a rough pipe 1 mile long, with a uniform drop of 52.8 feet, to discharge 5 cubic feet per second?
Ans. 14.41 in. A 15-in. pipe would be used

3. What is the discharge per second from a pipe 14 inches in diameter with a fall of 9 feet per 1,000? Ans. 4.41 cu. ft.

FLOW OF GASES

FLOW THROUGH ORIFICES AND SHORT TUBES

38. Preliminary Statement.—Owing to the compressibility of gases, the laws governing their flow through orifices and in pipes are not so simple as those relating to the flow of liquids. Different writers and experimenters give different formulas, and the results obtained by the use of these formulas frequently disagree; hence, in practical problems, considerable judgment is required in selecting the formula that suits most closely the conditions of the case under consideration.

It is believed that the formulas given in the following paragraphs are as reliable as any. In each case, the conditions under which the formula is applicable are stated.

39. Water Formula.—Suppose that a gas that has been enclosed in a vessel is allowed to flow through an orifice into the atmosphere. Let p_1 denote the pressure per square inch of the gas in the reservoir, and p_a the atmospheric pressure. Now, if p_1 is but slightly greater than p_a, the density of the gas changes but little during the flow, and in consequence the same formula as that used for water may be used without serious error, namely, $v^2 = 2gh$.

In the case of the flow of water, h denotes the head, in feet, on the orifice; likewise, in the case of a gas, h denotes the head or height, in feet, of a column of gas that corresponds to the difference of pressure $p_1 - p_a$. To illustrate this point, suppose that the gas in the reservoir is air and that the pressure is 16 pounds per square inch, absolute, and the temperature 60° F. At this pressure and temperature, 1 cubic foot

of air weighs .08316 pound. The difference in pressure, $p_1 - p_a$, is $16 - 14.7 = 1.3$ pounds per square inch, or $1.3 \times 144 = 187.2$ pounds per square foot. Evidently, a column having an area of 1 square foot and weighing 187.2 pounds will produce this pressure and the column will have to contain $187.2 \div .08316 = 2{,}251$ cubic feet; the column will, therefore, be 2,251 feet high. Hence, in this case, $h = 2{,}251$, and the velocity of flow will be

$$v = \sqrt{2\,g\,h} = \sqrt{2 \times 32.16 \times 2{,}251} = 380.5 \text{ feet per second.}$$

In general, if w_1 is the weight of 1 cubic foot of gas in the reservoir at the pressure p_1,

$$h = \frac{144\,(p_1 - p_a)}{w_1} \qquad (1)$$

Substituting the value of h in the formula $v^2 = 2\,g\,h$,

$$v^2 = \frac{2\,g \times 144\,(p_1 - p_a)}{w_1};$$

whence,
$$v = 96.24\sqrt{\frac{p_1 - p_a}{w_1}} \qquad (2)$$

This formula is termed the **water formula**, and holds good only when the pressure difference is small.

40. Formula for Weights Discharged.—Just as in the case of water, $Q' = A\,v$, where Q' denotes the theoretical volume discharged in cubic feet per second and A is the area of the orifice. In the case of air, as with water, there is a coefficient of discharge by which the quantity Q' must be multiplied to obtain the actual discharge Q.

Denoting the coefficient by k, $Q = k\,Q' = k\,A\,v$; and by substituting the value of v in formula **2** of Art. **39**,

$$Q = 96.24\,k\,A\sqrt{\frac{p_1 - p_a}{w_1}} \qquad (1)$$

If a represents the area of the orifice, in square inches, $A = \frac{a}{144}$, and the formula becomes

$$Q = \frac{96.24}{144}\,k\,a\sqrt{\frac{p_1 - p_a}{w_1}}$$

or
$$Q = .668\,k\,a\sqrt{\frac{p_1 - p_a}{w_1}} \qquad (2)$$

If G denotes the weight of the Q cubic feet of air, that is, the number of pounds discharged per second,

$$G = w_1 Q = .668\ k\,a\,w_1\sqrt{\frac{p_1 - p_a}{w_1}}$$

or
$$G = .668\ k\,a\sqrt{w_1\,(p_1 - p_a)} \qquad \textbf{(3)}$$

Now, from the general formula $p\,v = G\,R\,T$, from *Pneumatics*, and the weight w_1 of 1 cubic foot of air at a pressure of p_1 and an absolute temperature T_1, the following formula is obtained, in which v_1 is the volume of 1 cubic foot of air and R is .37.

$$p_1 v_1 = w_1 R\,T_1$$

and
$$w_1 = \frac{p_1 v_1}{R\,T_1} = \frac{p_1 \times 1}{.37\ T'}$$

or
$$w_1 = 2.7\,\frac{p_1}{T_1} \qquad \textbf{(4)}$$

Inserting this value of w_1 in formula **3**,

$$G = .668\ k\,a\sqrt{\frac{2.7\,p_1\,(p_1 - p_a)}{T_1}} = 1.1\ k\,a\sqrt{\frac{p_1\,(p_1 - p_a)}{T_1}}$$

From numerous experiments, Fliegner found that the coefficient k has the value .964; therefore, $1.1\ k = 1.06$. The final equation is, therefore,

$$G = 1.06\ a\sqrt{\frac{p_1\,(p_1 - p_a)}{T_1}} \qquad \textbf{(5)}$$

Fliegner states that this formula may be used when the pressure in the reservoir is less than twice the atmospheric pressure, that is, when p_1 is less than $2\,p_a$.

EXAMPLE.—Air flows from a reservoir in which the pressure is 5 pounds per square inch, gauge, into the atmosphere; the temperature in the reservoir is 69° F. and the diameter of the orifice is 1 inch. Compute the flow.

SOLUTION.—Here $p_1 = 5 + 14.7 = 19.7$ lb., which is less than twice 14.7 lb.; $a = .7854 \times 1^2 = .7854$ sq. in.; $T_1 = 69 + 460 = 529°$. Using formula **5**, and substituting,

$$G = 1.06 \times .7854 \times \sqrt{\frac{19.7 \times (19.7 - 14.7)}{529}} = .3593 \text{ lb.} \quad \text{Ans.}$$

41. Fliegner's Second Formula.—For cases in which the reservoir pressure is more than double the atmospheric

pressure, Fliegner gives the following formula, in which the symbols have the same meanings as in formula **5** of Art. **40**:

$$G = .53\, a \frac{p_1}{\sqrt{T_1}}$$

This formula and formula **5** of Art. **40** were deduced from experiments made on the flow of air into the atmosphere; however, it is probable that they may be used for the flow from one reservoir into a second reservoir in which the pressure is different from atmospheric pressure.

EXAMPLE 1.—Air having a temperature of 60° F. and an absolute pressure of 63 pounds per square inch flows through an orifice $\frac{1}{2}$ inch in diameter into the atmosphere; what is the flow per second?

SOLUTION.—Since 63 is greater than 2×14.7, the formula just given is used. $a = .7854 \times (\frac{1}{2})^2$; $p_1 = 63$; and $T_1 = 460 + 60 = 520$. Substituting these values,

$$G = .53 \times \frac{63 \times .7854 \times (\frac{1}{2})^2}{\sqrt{520}} = .2875 \text{ lb. Ans.}$$

EXAMPLE 2.—Compressed air flows from a reservoir in which the pressure is 60 pounds per square inch, gauge, into a second reservoir containing air at a pressure of 45 pounds per square inch, gauge; the temperature is 65° F. What is the flow per second if the area of the orifice is $\frac{1}{4}$ square inch?

SOLUTION.—Since the higher pressure is less than twice the lower pressure, formula **5** of Art. **40** is used, replacing p_a by the lower pressure p_2. $p_1 = 60 + 14.7 = 74.7$; $p_2 = 45 + 14.7 = 59.7$; $T_1 = 460 + 65 = 525$; and $a = \frac{1}{4}$. Then, substituting,

$$G = 1.06 \times \frac{1}{4} \times \sqrt{\frac{74.7 \times (74.7 - 59.7)}{525}} = .387 \text{ lb. Ans.}$$

FLOW OF AIR IN PIPES

42. Phenomena of Flow in Pipes.—When air or any other compressible gas flows in a pipe, the same weight must pass any cross-section of the pipe in a given interval of time. It does not follow, however, from this that the same volume passes every cross-section; for since the gas is compressible, the volume of a given weight depends on the pressure, and, in general, this is different at different sections.

Suppose that the gas flows into a pipe from a reservoir in which the pressure is p_1 pounds per square inch. The velocity at the start is v_1 feet per second. The temperature is supposed to remain constant during the flow. As in the case of liquids, the friction of the gas against the sides of the pipe causes a loss of pressure, the amount of which depends on the diameter and length of pipe and the velocity of flow.

After traveling a distance of l feet, the pressure of the gas is p_2 pounds per square inch, which is less than the initial pressure p_1. Now, as the temperature has not changed, the drop of pressure must result in an increase of volume per pound of gas; for, according to Boyle's law, $p_1 V_1 = p_2 V_2$, or $V_2 = V_1 \frac{p_1}{p_2}$; hence, since the same weight is passing each cross-section, the gas must have a greater velocity when its pressure is p_2 than at the start, when its pressure was p_1.

It is found that, other conditions being equal, the drop of pressure is greater the higher the initial velocity v_1. It is advisable therefore to keep v_1 as low as possible consistent with a reasonable diameter of pipe. In long mains, v_1 should not exceed 20 to 25 feet per second.

43. General Formula.—The fundamental formula for the flow of air in a long pipe is derived by the use of higher mathematics. It is

$$v_1 = \sqrt{\frac{12\,g\,R\,T\,d}{4\,f\,l} \times \left(\frac{p_1^2 - p_2^2}{p_1^2}\right)} \qquad (1)$$

in which g = acceleration of gravity = 32.16;
R = .37, from *Pneumatics;*
T = absolute temperature of air, which is assumed constant;
d = diameter of pipe, in inches;
l = length of pipe, in feet;
f = coefficient of friction of air against pipe walls;
p_1 = initial absolute pressure, in pounds per square inch;
p_2 = final absolute pressure, in pounds per square inch;
v_1 = initial velocity of air, in feet per second.

Substituting the numerical values of g and R, the formula takes the simpler form

$$v_1 = 5.975\sqrt{\frac{Td}{fl} \times \left(\frac{p_1^2 - p_2^2}{p_1^2}\right)} \qquad (2)$$

Ordinarily, the temperature of the air is approximately 70° F., that is, $T = 460° + 70° = 530°$. Using this value of T, the formula becomes

$$v_1 = \frac{137.6}{p_1}\sqrt{\frac{d}{fl} \times (p_1^2 - p_2^2)} \qquad (3)$$

EXAMPLE.—Air at an initial gauge pressure of 60 pounds per square inch flows through a pipe 6 inches in diameter and 9,000 feet long; assuming that $f = .0045$, with what velocity must the air enter the pipe so that the drop of pressure shall not exceed 8 pounds per square inch?

SOLUTION.—Here, $p_1 = 60 + 14.7 = 74.7$; $p_2 = 74.7 - 8 = 66.7$; and $p_1^2 - p_2^2 = 1{,}131.2$; $d = 6$; $f = .0045$; $l = 9{,}000$. Then, substituting in formula 3,

$$v_1 = \frac{137.6}{74.7}\sqrt{\frac{6 \times 1{,}131.2}{.0045 \times 9{,}000}} = 23.85 \text{ ft. per sec.}$$

Hence, the velocity should not exceed 23.85 ft. per sec. Ans. In solving examples of this kind, time may be saved by using logarithms.

44. Quantity of Air Delivered.—Let Q_1 denote the volume of air, in cubic feet, entering the pipe per second at the pressure p_1, and Q the volume of an equal weight of free air, that is, air at atmospheric pressure; then, since the volumes of equal weights of air are inversely as the absolute pressures,

$$\frac{Q_1}{Q} = \frac{14.7}{p_1}, \text{ or } Q = \frac{Q_1 p_1}{14.7}$$

Let a denote the area of the pipe in square inches, which, for a pipe of circular section, equals $.7854\ d^2$, when d is in inches; then,

$$Q_1 = \frac{a}{144} \times v_1$$

and
$$Q = \frac{Q_1 p_1}{14.7} = \frac{a\, v_1 p_1}{144 \times 14.7} = \frac{.7854\, d^2 v_1 p_1}{144 \times 14.7}$$

Usually, the delivery is expressed in cubic feet of free air per minute, and, using the minute instead of the second, the formula becomes

$$Q = \frac{60 \times .7854}{144 \times 14.7} d^2 v_1 p_1 \qquad (1)$$

For v_1, substitute the value in formula **3** of Art. **43**, and the formula becomes

$$Q = \frac{60 \times .7854 \times 137.6}{144 \times 14.7} d^2 \sqrt{\frac{d}{fl}(p_1^2 - p_2^2)}$$

or

$$Q = 3.063\sqrt{\frac{d^5}{fl}(p_1^2 - p_2^2)} \qquad (2)$$

45. Values of Coefficient of Friction for Air.—The coefficient f is not constant, but varies with the diameter of the pipe. Values agreeing well with experiments are given by the formula

$$f = .003\left(1 + \frac{4}{d}\right)$$

EXAMPLE.—Air at an initial gauge pressure of 80 pounds per square inch flows through a pipe 8 inches in diameter and 8,000 feet long; the drop in pressure is 5 pounds per square inch. Compute the discharge in cubic feet of free air per minute.

SOLUTION.—From the formula

$$f = .003\left(1 + \frac{4}{8}\right) = .0045$$

Substituting in formula **2** of Art. **44**,

$$Q = 3.063\sqrt{\frac{8^5}{.0045 \times 8{,}000}(94.7^2 - 89.7^2)} = 2{,}806 \text{ cu. ft.} \quad \text{Ans.}$$

46. Formulas for Diameter, Length, and Drop in Pressure.—By transforming formula **2** of Art. **44**, the following formulas are readily derived:

$$d = .64\sqrt[5]{\frac{Q^2 fl}{p_1^2 - p_2^2}} \qquad (1)$$

$$l = 9.38\frac{d^5}{Q^2 f}(p_1^2 - p_2^2) \qquad (2)$$

$$p_2 = p_1\sqrt{1 - \frac{.107\, Q^2 fl}{d^5 p_1^2}} \qquad (3)$$

When the drop in pressure is small, it may be calculated by the simple approximate formula

$$p_1 - p_2 = .0535\frac{Q^2 fl}{d^5 p_1} \qquad (4)$$

If the drop, as calculated by formula **4**, is relatively large, say 5 per cent. of p_1, or more, formula **3**, which is more exact, should be used to find the lower pressure, p_2.

From formula **3** of Art. **43**, the following formula is also obtained:

$$p_2 = p_1\sqrt{1 - \frac{v_1^2 f l}{18{,}934\,d}} \qquad (5)$$

This formula may be used to compute the pressure p_2 when the initial velocity v_1 is known.

EXAMPLE 1.—A pipe 5 miles long is required to deliver the equivalent of 4,000 cubic feet of free air per minute with a final pressure of 230 pounds per square inch, gauge, and the drop in pressure is not to exceed 20 pounds per square inch; what must be the diameter of the main?

SOLUTION.—Since d is unknown, f must be assumed and afterwards corrected if necessary; therefore, assume $f = .0045$; $p_2 = 230 + 14.7 = 244.7$; $p_1 = 244.7 + 20 = 264.7$; $p_1^2 - p_2^2 = 10{,}188$; $Q = 4{,}000$; and $l = 5 \times 5{,}280$. Now, using formula **1** and substituting,

$$d = .64\sqrt[5]{\frac{4{,}000^2 \times .0045 \times 5 \times 5{,}280}{10{,}188}} = 7.25 \text{ in. Ans.}$$

For this value of d, $f = .003\left(1 + \frac{4}{7.25}\right) = .0047$, which is so near the assumed value, .0045, that a correction is unnecessary. The next larger commercial size of pipe, 8 in., should be used.

EXAMPLE 2.—Using the 8-inch pipe in example 1, compute the actual drop in pressure.

SOLUTION.—For an 8-inch pipe,

$$f = .003\left(1 + \frac{4}{8}\right) = .0045$$

From formula **3**,

$$p_2 = 264.7\sqrt{1 - \frac{.107 \times 4{,}000^2 \times .0045 \times 5 \times 5{,}280}{8^5 \times 264.7^2}} = 252.7 \text{ lb.}$$

Hence, the drop is $p_1 - p_2 = 264.7 - 252.7 = 12$ lb. Ans.

By the approximate formula **4**, the drop in pressure would be

$$p_1 - p_2 = \frac{.0535 \times 4{,}000^2 \times .0045 \times 5 \times 5{,}280}{8^5 \times 264.7} = 11.72 \text{ lb.}$$

EXAMPLE 3.—Through what length of 6-inch pipe can the equivalent of 1,500 cubic feet of free air per minute be discharged with a final pressure of 75 pounds per square inch, gauge, and a drop in pressure of 4 pounds per square inch?

SOLUTION.—For a 6-in. pipe, $f = .003\,(1 + \frac{4}{6}) = .005$; $p_2 = 75 + 14.7 = 89.7$; and $p_1 = 89.7 + 4 = 93.7$; hence, $p_1^2 - p_2^2 = 93.7^2 - 89.7^2 = 733.6$. Now, substituting in formula **2**,

$$l = 9.38 \times \frac{6^5 \times 733.6}{1{,}500^2 \times .005} = 4{,}756 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1. With what theoretical velocity will air flow from a reservoir in which the pressure is 20 pounds per square inch, absolute, into the atmosphere? Assume the temperature to be 60°. Ans. 687.5 ft. per sec.

NOTE.—First use formula **4** of Art. **40** for finding the weight of 1 cubic foot of air, and then use formula **2** of Art. **39**.

2. Air flows from a reservoir, in which the pressure is 150 pounds per square inch, gauge, into the atmosphere through an orifice $\frac{3}{4}$ inch in diameter; the temperature of the air on the reservoir is 65° F. Compute the weight flowing per minute. Ans. 101 lb., nearly

NOTE.—Use the formula in Art. **41** and remember that the formula gives pounds per second.

3. A main to carry compressed air is 20,000 feet long and 10 inches in diameter; the initial pressure is 125 pounds per square inch, absolute. Compute the drop in pressure for the following initial velocities: (*a*) 20 feet per second; (*b*) 30 feet per second.

Ans. $\begin{cases} (a) \text{ 11.63 lb. per sq. in.} \\ (b) \text{ 28.12 lb. per sq. in.} \end{cases}$

NOTE.—Use the formula in Art. **45** and then use formula **5** of Art. **46**.

4. What diameter of pipe is required for transmission of air under the following conditions: length, 1 mile; final pressure, 80 pounds per square inch, gauge; drop in pressure not to exceed 5 pounds per square inch; quantity discharged per minute, the equivalent of 7,000 cubic feet of free air? Ans. 10.4 in.; hence, use 11-in. pipe

NOTE.—Assume f = .0042 and use formula **1** of Art. **46**.

5. With an initial pressure of 300 pounds per square inch, absolute, and a drop of 20 pounds per square inch, what will be the discharge of free air per minute from a main 10 miles long and 15 inches in diameter? Ans. 20,295 cu. ft. in round numbers

NOTE.—Use the formula in Art. **45** to find f, and then use formula **2** of Art. **44**.

HYDRAULICS

(PART 2)

ENERGY OF FALLING WATER

1. Methods of Utilizing the Energy of Water. Water, in passing from a higher to a lower level, can do work, and the amount of work that can thus be done by a given weight G of water is the available energy of that weight of water. This energy may be used in one of three ways, as illustrated in Fig. 1: (1) The water may be per-

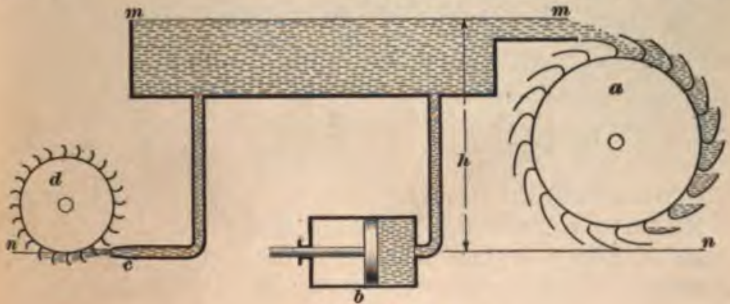


FIG. 1

mitted to flow from the upper level $m\,m$ into the buckets of an overshot waterwheel a; the weight of the water acts as a motive force and causes the wheel to turn. (2) The water may enter a cylinder b placed at the lower level $n\,n$; the water in the cylinder has a pressure due to the hydrostatic head h, the distance between the two levels, and by virtue of this pressure may push a piston back and forth and thus do work. (3) The water may be allowed to escape from a

nozzle c at the level $n n$, and the jet of water may be made to impinge on the buckets of an impulse wheel d.

. Corresponding to these three ways of utilizing the energy of the falling water, hydraulic motors are divided into three classes: (1) gravity motors, in which the weight of the water is used; (2) pressure motors, in which the pressure of the water is used; (3) velocity motors, in which the impulse of a moving jet is the motive force.

2. Available Energy.—In each case, let G be the weight of water in pounds used in some given interval of time. In case the water is used to turn the wheel a, the weight G simply descends a vertical distance h and the work done is $G h$ foot-pounds. Let the water used per stroke in the cylinder b be G pounds, and let A denote the piston area in square feet and l the length of stroke in feet. Then $A l$ is the volume of water used per stroke, in cubic feet, and $P A l$ is the work per stroke, where P denotes the pressure on the piston, in pounds per square foot. If, now, H denotes the weight of a cubic foot of water and G pounds of water are used per stroke, $G \div H$ is the volume used. Then,

$$\frac{G}{H} = A l; \text{ also, } P = H h$$

Hence,

$$\text{work per stroke} = P \times A l = H h \times \frac{G}{H} = G h \text{ foot-pounds}$$

Consequently, in discharging G pounds of water from the higher to the lower level through the cylinder b, the work done is also $G h$ foot-pounds. Finally, the water issues from the nozzle c with a velocity v, and the kinetic energy of G pounds moving at this speed is $\frac{G v^2}{2 g}$ foot-pounds. But in the case of frictionless flow through the tube and nozzle, the theoretical velocity is $v = \sqrt{2 g h}$, whence $\frac{v^2}{2 g} = h$ and the energy is again $G h$ foot-pounds. This leads, therefore, to the following important principle: *The available energy of a weight of water G falling through a height h is the product $G h$, whether the water acts by weight, pressure, or impulse.*

3. Power of a Fall of Water.—Let Q denote the quantity of water, in cubic feet, flowing in 1 second, and let h denote the fall, in feet. The work done in 1 second is $G h$ foot-pounds = 62.5 $Q h$ foot-pounds, as water weighs approximately 62.5 pounds per cubic foot. Now, 1 horsepower is the performance of 33,000 foot-pounds of work per minute, or 33,000 ÷ 60 = 550 foot-pounds per second; hence, the theoretical horsepower of a given fall of water may be expressed by the formula

$$\text{H. P.} = \frac{62.5\,Q\,h}{550} = .1136\,Q\,h$$

EXAMPLE.—A flume leading from a dam has a fall of 35 feet, and discharges 210 cubic feet of water per minute; what is the theoretical horsepower?

SOLUTION.— $Q = 210 \div 60 = 3.5$, and $h = 35$; hence,
H. P. = .1136 $Q h$ = .1136 × 3.5 × 35 = 13.916. Ans.

4. Efficiency.—No motor, however, can utilize all the power in the fall of a given weight of water. Part of the energy is lost in overcoming the resistances due to the friction of the water as it flows through the gates and channels leading to the motor; part is absorbed in shocks and eddies, and in the friction of the water as it passes through the motor; and part is lost in the form of velocity as the water leaves the motor, or as it falls from the motor to the lower level of the water. Besides the above losses, due to resistances to the motion of the water, the mechanical losses due to the friction of the motor itself must be taken into account.

The **efficiency** of a motor is the ratio of the actual work it will do to the theoretical work in the water used. Thus, if the actual work done by a waterwheel is equal to 750 horsepower, when the theoretical work that the water would do is equal to 1,000 horsepower, the efficiency of the wheel is 750 ÷ 1,000 = .75 = 75 per cent.

EXAMPLE.—What is the efficiency of a waterwheel that delivers 24 horsepower when using 660 pounds of water per second with a head of 25 feet?

SOLUTION.—The theoretical power is $\frac{660 \times 25}{550} = 30$ H. P.; therefore, the efficiency is 24 ÷ 30 = .80 = 80 per cent. Ans.

5. Power Transmitted by a Water Main.—When water flows through a main, part of the head h is lost because of the friction. If this lost head is denoted by h_f, the net head is $h - h_f$; hence, the horsepower delivered at the end of the main is

$$\text{H. P.} = .1136\,Q(h - h_f)$$

Without friction, the horsepower is that given by the formula of Art. **3**; hence, the efficiency of the transmission is

$$\frac{h - h_f}{h} = 1 - \frac{h_f}{h}$$

6. Energy of a Jet.—As stated in Art. **2**, the theoretical energy of G pounds of water moving with a velocity of v feet per second is $\frac{G v^2}{2g}$ foot-pounds. If h denotes the head on the orifice, Fig. 1, theoretically $v = \sqrt{2 g h}$; actually, however, $v = c\sqrt{2 g h}$, where c denotes a coefficient known as the coefficient of velocity; hence, $\frac{v^2}{2g} = c^2 h$, and $\frac{G v^2}{2g} = c^2 G h$.

Let A denote the area of the cross-section of the jet in square feet, and H the weight of a cubic foot of water; then the weight of water discharged per second is $G = H A v$ and the energy E of this quantity per second is $\frac{G v^2}{2g}$, or $c^2 G h$. Substituting the value of G, the formulas become

$$E = \frac{H A v^2}{2g} \qquad (1)$$

and

$$E = c^2 H A v h \qquad (2)$$

Since E is the work the jet is capable of doing in 1 second, the horsepower of the jet is $\frac{c^2 G h}{550}$; and by substituting the value of G the formula becomes

$$\text{H. P.} = \frac{c^2 H A v h}{550} \qquad (3)$$

It should be noted that the area of the jet is not necessarily the area of the orifice from which the jet issues, as explained in *Hydraulics*, Part 1.

The efficiency of the jet, that is, the ratio of the actual energy to the theoretical energy, is $\frac{c^2 G h}{G h} = c^2$.

EXAMPLE.—In a test of an impulse waterwheel, the discharge from the nozzle was found to be 2.819 cubic feet per second and the effective head was 384.7 feet. (*a*) Taking $c = .98$, calculate the horsepower of the jet. (*b*) The horsepower developed by the wheel was found to be 107.4; what was the efficiency of the wheel?

SOLUTION.—(*a*) Substituting in formula **3**,

$$\text{H. P.} = \frac{.98^2 \times 62.5 \times 2.819 \times 384.7}{550} = 118.36. \quad \text{Ans.}$$

(*b*) $\text{Efficiency} = \frac{\text{actual H. P}}{\text{theoretical H. P.}} = \frac{107.4}{118.36} = .9074 = 90.74$ per cent. Ans.

7. Pressure Due to Impact of a Jet.—Let a jet of water strike a surface inclined at an angle a with the original direction of the jet, as shown in Fig. 2. The surface is supposed to be perfectly smooth, so that there is no loss from shocks or friction, and the water is prevented from spreading sidewise. Under these conditions, the velocity v with which the water leaves the surface is equal to the velocity v that it had when it struck.

FIG. 2

The impact of the water produces a pressure on the surface that tends to move it. Let P denote the component of this pressure in the direction of the jet, and let M be the mass of the water that issues in 1 second. On leaving the nozzle, the momentum of this mass is Mv; but on leaving the surface the velocity component in the direction of the jet is $v \cos a$, and the momentum in the same direction is therefore $Mv \cos a$; the mass, therefore, has its momentum in the direction of the jet decreased by the amount $Mv - Mv \cos a = Mv(1 - \cos a)$. This change of momentum must be caused by the reaction of the surface against the jet in the direction of the jet. This reaction is of course equal to and opposite to the pressure P. From the principles of mechanics, the change of momentum is equal to the impulse,

which is Pt, in which t is the time in seconds through which the force acts. Then, $Pt = Mv(1 - \cos a)$. The time t is taken as 1 second so that $P = Mv(1 - \cos a)$; but $M = \frac{G}{g}$;

hence, $$P = \frac{Gv}{g}(1 - \cos a)$$

EXAMPLE.—A jet whose cross-section is 1 square inch flows with a velocity of 75 feet per second, and strikes a surface that changes its direction 35°; what pressure is exerted on the surface in the direction of the jet before striking?

SOLUTION.—From the above formula, by substituting,

$$P = \frac{75 \times 1 \times 62.5}{144} \times \frac{75}{32.16} \times (1 - .81915) = 13.73 \text{ lb. Ans.}$$

8. Pressure on a Flat Surface at Right Angles to the Jet.—When the surface is at right angles to the jet, as shown in Fig. 3, $a = 90°$ and $\cos a = \cos 90° = 0$; in this case, therefore, the formula of Art. 7 reduces to

FIG. 3

$$P = \frac{Gv}{g} \qquad (1)$$

As the jet issues from the orifice, there is a reaction on the vessel from which it issues, which is just equal to the pressure that is produced by the jet as it strikes the vertical surface. The effect of the reaction and pressure of a jet may be shown by experiment

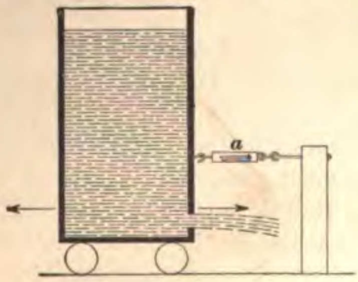


FIG. 4

FIG. 5

as follows: Let the vessel be placed on rollers, as shown in Fig. 4, in such a way that a very slight pressure will produce motion. When the water issues from the orifice, as shown,

the vessel will begin to move in the opposite direction. If there were no friction, a spring balance attached to the vessel, as shown at a, would show a pull equal to $\frac{G v}{g}$. Now, if a plate is fastened to the vessel as shown in Fig. 5, so that the jet strikes it, the pressure exerted by the jet on the plate will equal the reaction of the jet on the vessel, and there will be no motion.

If the plate is perfectly smooth, so that there is no loss from friction, the velocity of the water as it leaves the plate will be the same as the velocity with which it struck, and there will be no change in the energy contained in the water. The velocity in the direction of the jet has been entirely overcome and changed to pressure, but since this pressure produces no motion, no work is done.

As in Art. **6**, let $G = H A v$, and $\frac{v^2}{2g} = c^2 h$; then,

$$P = \frac{G v}{g} = \frac{H A v^2}{g} = 2 c^2 H A h \qquad (2)$$

The hydrostatic pressure exerted on an area A by a head h is equal to $H A h$; it appears therefore that, with c equal to one, the reaction of a jet, whose area is a and whose velocity of flow is produced by a head h, is twice the hydrostatic pressure that would be produced on the same area by the same head.

EXAMPLE.—The area of a jet from the side of a vessel is 2 square inches, the head on the center of the orifice is 10 feet, and the coefficient of velocity is .98. (a) What pressure will the jet exert when it impinges on a vertical plane surface? (b) What is the pressure on the vessel due to the reaction of the jet?

SOLUTION.—Using formula **2**,

(a) $P = 2 \times .98^2 \times 62.5 \times \frac{2}{144} \times 10 = 16.67$ lb. Ans.

(b) The reaction is equal to the pressure P. Ans.

9. Pressure of a Jet on a Hemispherical Cup. If the water strikes into a hemispherical cup, as shown in Fig. 6, the direction in which it leaves the cup makes an angle of 180° with the direction of motion of the jet. The cup is supposed to be smooth, so that there is no loss of velocity

or energy. Then, since $a = 180°$, $\cos a = -1$ and $1 - \cos a = 1 - (-1) = 2$, which, substituted in the formula of Art. **7**, gives for the hemispherical cup

$$P = \frac{2\,G\,v}{g}$$

that is, *the pressure is twice as great as when the jet strikes a flat plate at right angles to the direction of its motion.*

FIG. 6

EXAMPLE.—If the jet in the example of Art. 7 strikes a hemispherical cup, so that its direction is changed 180°, what is the pressure exerted?

SOLUTION.—Substituting in the formula,

$$P = 2 \times \frac{75 \times 1 \times 62.5}{144} \times \frac{75}{32.16} = 151.83 \text{ lb. Ans.}$$

10. Pressure on Moving Surfaces Due to Jets. Suppose the plate, Fig. 3, to be moving away from the nozzle with a velocity of u feet per second. The absolute velocity of the jet, or its velocity relative to the fixed nozzle, being v, its velocity relative to the moving plate is $v - u$. The one plate considered will soon move beyond the influence of the jet. If, however, the plate is one of a series of plates that follow each other in rapid succession, as in waterwheels, the pressure of the water acts continuously. The effect then is the same as though the plates were stationary and the jet had the velocity $(v - u)$. The pressure on the plate in the direction of the jet is obtained by merely substituting the relative velocity $(v - u)$ for the velocity v in the formula of Art. **7**, thus:

$$P = \frac{G}{g}(v - u)(1 - \cos a) \qquad (1)$$

For a series of flat plates at right angles to the jet, $a = 90°$, $\cos a = 0$, and the formula becomes

$$P = \frac{G}{g}(v - u) \qquad (2)$$

For a series of hemispherical cups like that shown in Fig. 6, $a = 180°$, $\cos a = -1$, $1 - \cos a = 2$, and the formula becomes

$$P = \frac{2\,G}{g}(v - u) \qquad (3)$$

11. Work Done by Jets Impinging on Moving Surfaces.—The surface moves u feet per second and the force acting on it in the line of its motion is P. The work done per second is therefore $P u$ foot-pounds. Substituting for P the expressions given in formulas **2** and **3** of Art. **10,** and denoting by W the work done per second,

$$W = \frac{G u}{g}(v - u) \text{ for flat plates}$$

and $$W = \frac{2 G u}{g}(v - u) \text{ for hemispherical cups}$$

From Art. **6,** the energy of the jet is $\frac{G v^2}{2 g}$ foot-pounds per second. If all this energy is utilized,

$$W = \frac{G v^2}{2 g};$$

but if the work done on the plate is less than the energy,

$$W = e \frac{G v^2}{2 g},$$

where e, the efficiency, is some proper fraction. The highest value of e for each of the two cases just mentioned can be found by taking $e \frac{G v^2}{2 g}$ equal to the values of the work done.

For a jet striking a flat plate at right angles, $\frac{G u}{g}(v - u) = e \frac{G v^2}{2 g}$, or $u v - u^2 = \frac{e v^2}{2}$. Multiplying the last expression by 4, $4 u v - 4 u^2 = 2 e v^2$, or $4 u^2 - 4 u v = -2 e v^2$. Completing the square, $4 u^2 - 4 u v + v^2 = v^2 - 2 e v^2$, and taking the square root, $2 u - v = \sqrt{v^2 - 2 e v^2} = v \sqrt{1 - 2 e}$.

In order that the quantity under the radical shall not be negative, $2 e$ must not exceed 1, or e must not exceed $\frac{1}{2}$. Hence, when a jet impinges on a series of flat plates, not more than one-half of the energy of the jet is utilized, or, in other words, the efficiency cannot exceed 50 per cent. When

$$e = \tfrac{1}{2},$$

$$v \sqrt{1 - 2 e} = 0;$$

and, therefore, $$2 u - v = 0;$$

or, $$u = \tfrac{1}{2} v.$$

For hemispherical cups, $\frac{2Gu}{g}(v-u) = e\frac{Gv^2}{2g}$, or $2uv - 2u^2 = \frac{ev^2}{2}$. Multiplying by 2, $4uv - 4u^2 = ev^2$; completing the square, $4u^2 - 4uv + v^2 = v^2 - ev^2 = v^2(1-e)$; extracting the square root, $2u - v = v\sqrt{1-e}$. With hemispherical cups, therefore, the highest possible efficiency is when $e = 1$, which makes $v\sqrt{1-e} = 0$; in which case, $2u - v = 0$, or $u = \frac{1}{2}v$. Hence, *when the velocity of the cups is one-half the velocity of the impinging jet, the efficiency is unity, that is, the entire energy of the jet is utilized in moving the cups.*

If the whole energy is thus utilized, the absolute velocity of the water leaving the cup must be zero. That this is true is readily seen; for the water leaves the cups with the velocity $v - u$ backwards and the cup is moving forwards with the velocity u; the absolute velocity of the water is therefore $u - (v - u) = 2u - v$, and this is zero when $u = \frac{1}{2}v$. Of course, in the actual case, there is friction, so that these high values cannot be fully attained.

EXAMPLES FOR PRACTICE

1. If a stream discharges 120 cubic feet of water per minute with a fall of 50 feet, what work is it theoretically able to do?
Ans. 375,000 ft.-lb. per min.

2. What is the horsepower corresponding to the work in example 1?
Ans. $11\frac{4}{11}$ H. P.

3. If 450 pounds of water is discharged each minute from an orifice under a head of 80 feet and the coefficient of velocity is .98, what is the horsepower equivalent to the energy in the jet? Ans. 1.048 H. P.

4. If the jet in example 3 impinges on a plane surface at right angles to its direction of motion, what pressure does it exert?
Ans. 16.4 lb.

5. A jet of water flows from a nozzle under a head of 100 feet with a coefficient of velocity of .98, and impinges on a series of moving hemispherical cups; what must be the velocity of a cup in order that the water will leave it with no absolute velocity? Ans. 39.3 ft. per sec.

6. If .25 cubic foot of water is discharged in each second from the nozzle in example 5, (*a*) what is the pressure exerted on the cup? (*b*) What is the work done by this pressure in one second?

Ans. $\begin{cases} (a) \text{ 38.19 lb.} \\ (b) \text{ 1,500.9 ft.-lb.} \end{cases}$

HYDRAULIC MACHINES AND MOTORS

WATERWHEELS

12. Overshot Wheels.—**Overshot waterwheels** are most often applied to falls of from 10 to 50 feet. Higher falls are sometimes used, however. In an overshot wheel,

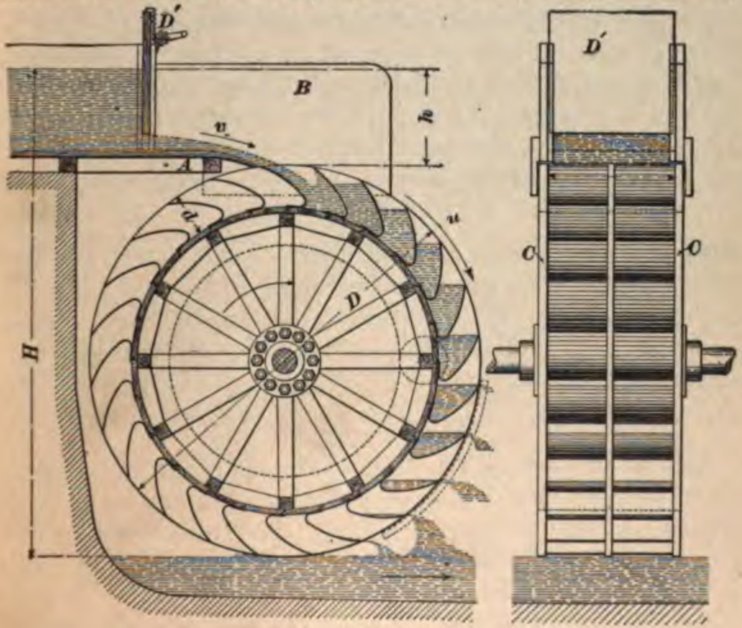


Fig. 7

a small amount of the work is done by the impact of the water as it enters the buckets, but much the greater part is done by the weight of the water as it descends in the buckets.

Fig. 7 shows two views of an overshot wheel with curved iron buckets. The water is brought out to the crown *C* by a

trough, or sluice, A, which may be curved toward the wheel. It should be so placed that the water will enter the first, second, or third bucket from the vertical center line of the wheel. The thickness of the sheet of water in the trough should not exceed 6 or 8 inches. The sides B of the trough are extended far enough beyond the vertical center line to insure the filling of several buckets when the wheel is to be started.

The supply of water to the wheel is regulated by a gate in the sluice, as shown at D'. This gate is generally operated by hand, but may be operated by an automatic governor.

An example will best illustrate the losses to which the overshot wheel is subject. Suppose the total fall H, Fig. 7, to be 20 feet. It may be assumed, as a first approximation, that the diameter D is 16 feet. In order that the centrifugal force acting on the water in the buckets will not cause too much spilling, the circumferential speed u should not be too great. A good value of u is given by the formula

$$u = \sqrt{2\,D};$$

whence, in this case, $u = \sqrt{2 \times 16} = 5.6$ feet per second. The velocity v of the water entering the buckets should be about double the velocity u of the buckets; hence, $v = 2 \times 5.6 = 11.2$ feet per second. The head to produce this velocity is

$$h = \frac{v^2}{2g} = \frac{11.2^2}{2 \times 32.16} = 1.95 \text{ feet}$$

Adding 10 per cent. for frictional losses in the sluice gate, $h = 2.15$ feet. Now, according to Art. **11**, not more than 50 per cent. of the kinetic energy of the stream due to the head h can be utilized; hence, the loss of head at this point is at least $\frac{1}{2}\,h$, or 1.08 feet.

The distance from the middle of the stream to the center of gravity of the buckets, which may be taken as 1 foot, is a further loss. The water discharges from the buckets before reaching the lower level, which is a third source of loss. The magnitude of this loss depends on the form of the buckets, but it varies from .12 D to .2 D. Taking this as .15 D, the loss of head is $.15 \times 16 = 2.4$ feet.

Finally, there is usually a clearance between the wheel and the water in the tailrace, and this constitutes a fourth loss of head. Assuming this clearance to be 6 inches, or .5 foot, the total loss of head is 1.08 feet + 1 foot + 2.4 feet + .5 foot = 4.98 feet, say 5 feet. The effective head is, therefore, 20 feet − 5 feet = 15 feet, and the

$$\text{efficiency} = \frac{\text{net head}}{\text{total head}} = \frac{15 \text{ feet}}{20 \text{ feet}} = .75 = 75 \text{ per cent.}$$

The efficiency of the overshot waterwheel ranges from 70 to 90 per cent. in well-constructed wheels. When the supply of water is small, as during a drought, the buckets are only partly filled; hence, the loss from the water leaving the buckets too early is reduced, and the efficiency of the wheel is increased.

13. Breast Wheels.—The **breast wheel** is used where

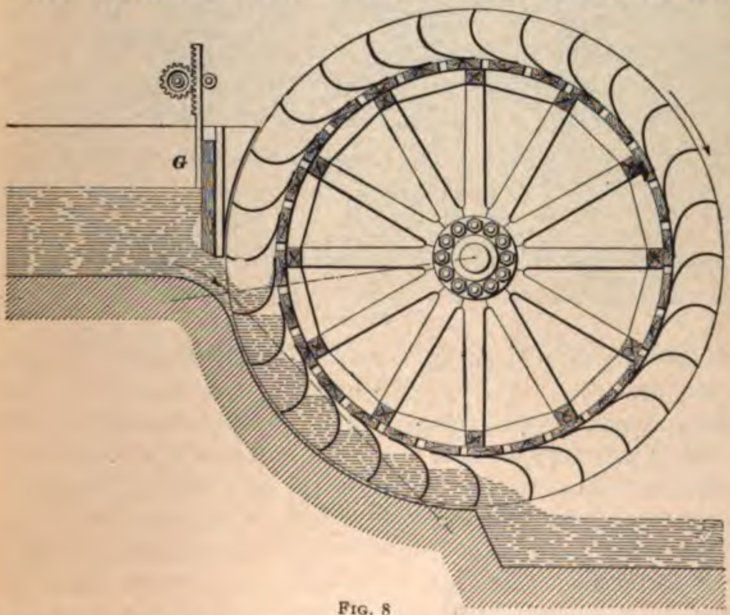


Fig. 8

the fall is too low for an overshot wheel. As shown in Fig. 8, the water flows through an opening in a reservoir or sluice and enters the wheel somewhat below a horizontal

plane through the center of the wheel. The size of the opening is regulated by the gate *G*. In breast wheels, the water acts more largely by its impulse than in overshot wheels, but generally the greater part of the action is due to the weight of the water. The efficiency of a breast wheel ranges from 50 to 70 per cent., the smaller value applying to the smaller sizes.

14. Undershot Wheels.—The **undershot wheel** is used for falls of 6 feet or less. The least efficient form has straight radial floats, as shown in Fig. 9, that are acted on

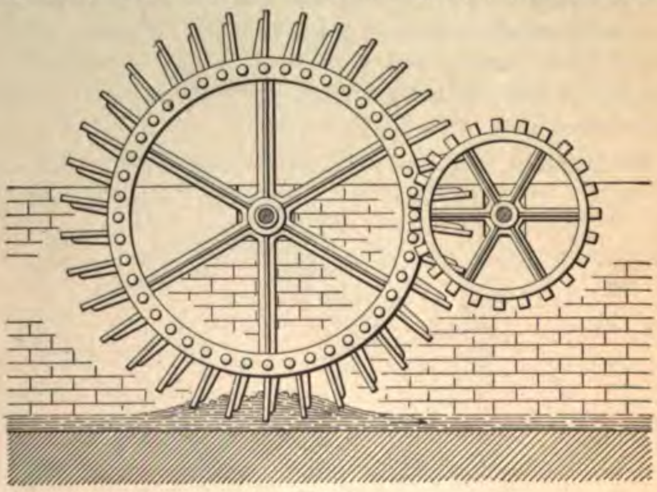


FIG. 9

directly by the current of a swiftly flowing stream. In this case the water acts only by impulse, and the efficiency is seldom greater than 25 per cent. The dimensions of these wheels may vary from 12 feet to 24 feet in diameter and they may have from twenty-four to forty-eight floats. The depth of the floats for best effect should be at least three times the depth of the stream. The velocity of the circumference of the wheel should be about one-half the velocity of the water in the stream; the depth of the stream should be from 4 to 6 inches, and the depth of the floats 12 to 20 inches. There should be as little clearance as possible between the floats and the bottom and sides of the race.

A formula for the horsepower that may be developed by the use of an undershot wheel, such as has been described, is easily derived.

By Art. **11**, the work of the stream on the series of floats is

$$W = \frac{G u}{g}(v - u)$$

where v = velocity of water in race, in feet per second;
u = velocity of circumference of wheel, in feet per second.

If Q denotes the cubic feet of water flowing per second, then, from the experiments of Smeaton and Bossut, about .61 Q is the quantity striking the floats under usual conditions; hence, $G = 62.5 \times .61\, Q$, and

$$\text{H. P.} = \frac{W}{550} = \frac{62.5 \times .61\, Q u (v - u)}{550 \times 32.16}$$
$$= .00216\, Q u (v - u) \qquad (1)$$

For a **paddle wheel** suspended in an unconfined current, the horsepower may be computed from the formula

$$\text{H. P.} = .00282\, A v u (v - u) \qquad (2)$$

where v = velocity of current, in feet per second;
u = velocity of circumference of wheel, in feet per second;
A = area of immersed portion of the float, in square feet.

MACHINES UTILIZING THE PRESSURE OF WATER

15. Losses of Efficiency in Water Motors.—Machines in which water pressure is employed as the motive force are much used, especially in modern shops. There are hydraulic jacks, cranes, and elevators for hoisting, hydraulic presses for baling, flanging, forging, and many other purposes; and in the machine shop, hydraulic punches, shears, riveters, rail benders, etc. There are also engines and pumps driven by high-pressure water instead of steam. In all these machines, the water is introduced into a closed vessel or cylinder and acts on a movable piston. The theoretical work done is the

product of the weight G of the water used and the head h corresponding to the pressure; that is, work $= G h$.

The sources of loss of efficiency are as follows: (1) Friction of the water in mains and passages; (2) losses due to shock when the water changes direction; (3) waste of water when the same quantity is used for light loads as for heavy loads; (4) friction of the mechanism.

16. Water-Pressure Engines.—An oscillating cylinder engine driven by water is shown in Fig. 10. This form is much used in Europe for domestic purposes and for small manufacturing, where only a small amount of power is

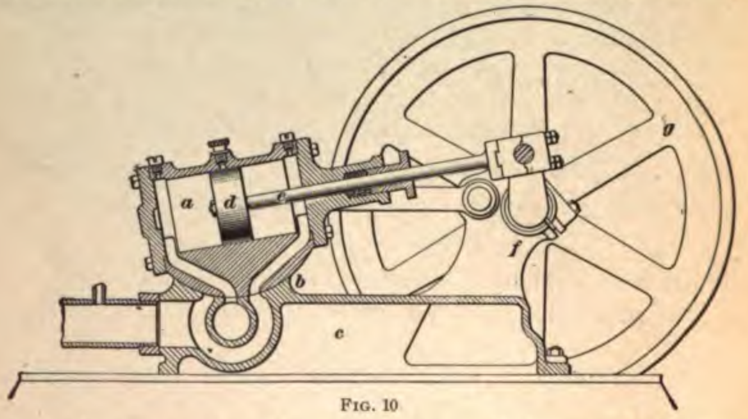


FIG. 10

desired. It consists of an oscillating cylinder a, which turns about the center of the cylindrical valve face b in the bed c as the engine runs. The piston d is a solid piston and may be made water-tight with leather packing. The rod e is guided by a long stuffingbox in the end of the cylinder cover. The bearings f, one on each side of the rod e, carry the crankshaft, on one end of which the flywheel g is fastened.

In this engine, the cylinder moves while the valve remains stationary. There are trunnions on each side of the cylinder with their centers at the center of the arc forming the valve face and provided with screws for holding the cylinder to the valve face. There are three ports in the valve; the middle one admits the water first to one end and then to the other

end of the cylinder. The exhaust or outlet ports are connected together around the inlet port and discharge the water into a waste pipe.

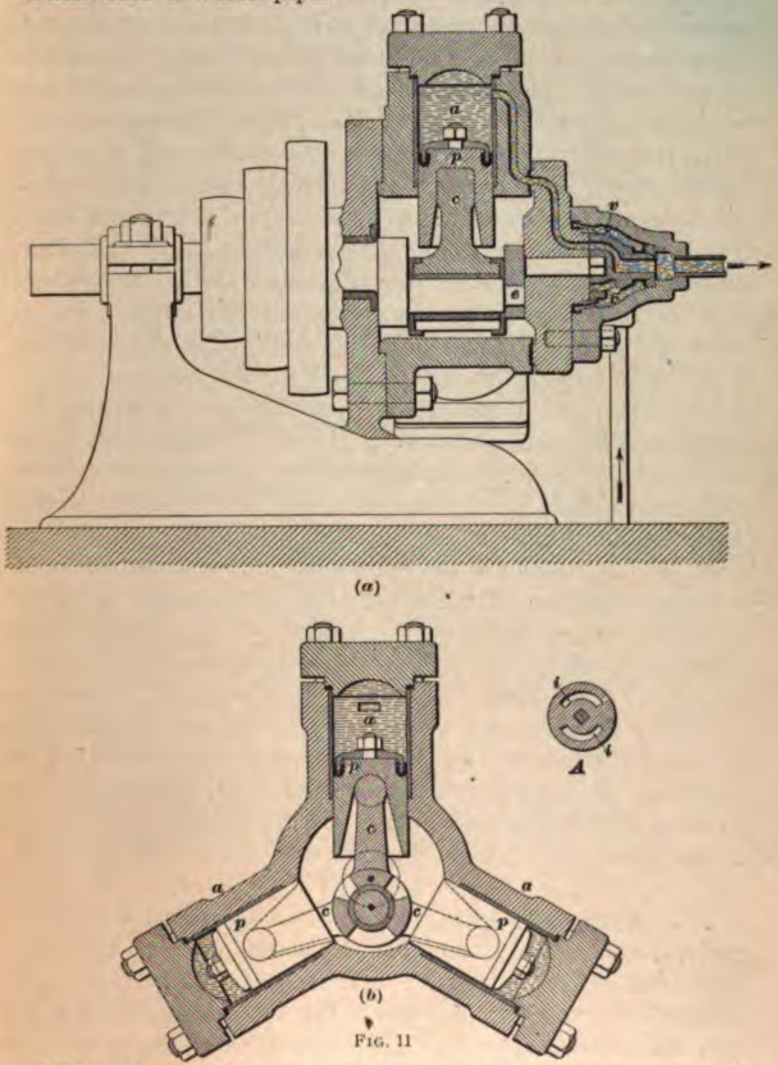


FIG. 11

17. A type of engine that has proved satisfactory in the use of water pressure for producing rotary motion is the

Brotherhood three-cylinder engine, two sectional views of which are shown in Fig. 11 (*a*) and (*b*), the one being taken parallel, and the other at right angles, to the shaft. It consists of three cylinders *a* in which work single-acting pistons *p*. These pistons are all connected to a single crankpin by means of the rods *c*. The water is admitted to the cylinders one after the other by the circular valve *v*, which also controls the exhaust. This valve has a lignum-vitæ seating and is rotated by the eccentric-pin *e* on the end of the crankpin. A view of the face of the valve, showing the ports *i*, *i*, is given at *A*. These ports pass over the passages leading to the cylinders, alternately admitting and exhausting the water. The arrangement of the three cylinders at angles of 120° secures a constant and nearly uniform turning force on the crankpin and makes it possible to start the engine in any position.

When the load is constant and the engine is designed for the load, a hydraulic-pressure engine is an efficient machine. As ordinarily constructed, however, the efficiency with variable loads cannot be high, owing to the fact that the cylinders must be filled with water at each stroke. Throttling the water in its passage to the cylinders reduces the pressure in them, since a part of the energy due to the pressure of the water in the supply pipe is expended in overcoming resistance to flow through the partly closed valves. This energy does no useful work and is therefore lost. A number of hydraulic pressure engines have been designed in which the stroke of the piston is varied to correspond with the work to be done. In this way, the water used is proportional to the work, while the pressure is kept constant, and the efficiency, consequently, is more nearly constant for all loads.

18. Hydraulic Flanging Press.—A press for flanging boiler heads, ship plates, etc. is shown in Fig. 12. The plate *p* is placed on a table *k* that is carried by two small plungers or rams working inside the small cylinders *h*, *h*, so that the table can be lowered when required, as shown by the dotted lines. A heavy adjustable crosshead carries a ring-shaped die *g*, made in the form in which the plate is to

be pressed. A large ram *b* works inside the cylinder *a* and carries a crosshead *c*, to which the annular die *d* is attached by means of the small columns *e*. The plate to be flanged is placed on the table *k* and pressed firmly against the die *g* by admitting water under pressure from a pump or accumulator to the cylinders *h*, *h*; then the die *d* is raised by the

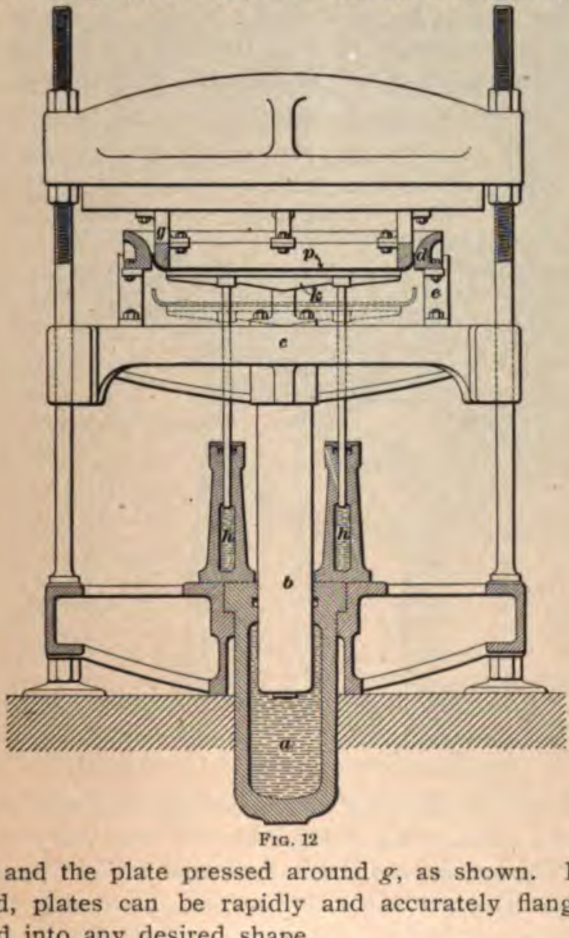


FIG. 12

ram *b* and the plate pressed around *g*, as shown. By this method, plates can be rapidly and accurately flanged, or pressed into any desired shape.

19. Hydraulic Riveter.—Fig. 13 shows a stationary hydraulic riveter of a form much used in boiler work. It

consists of a heavy frame G that carries the ram for pressing down the rivet. To this frame is bolted the **stake** V, which carries the die n' against which the rivet is held while being subjected to pressure. The rivet is headed by being compressed between the movable die n and the fixed die n'; m is a ring-shaped die that holds the plates to be joined against n' while the rivet is being headed. Water for operating the machine is brought in through the pipe r and is admitted through the valve v to the cylinder A by means of the

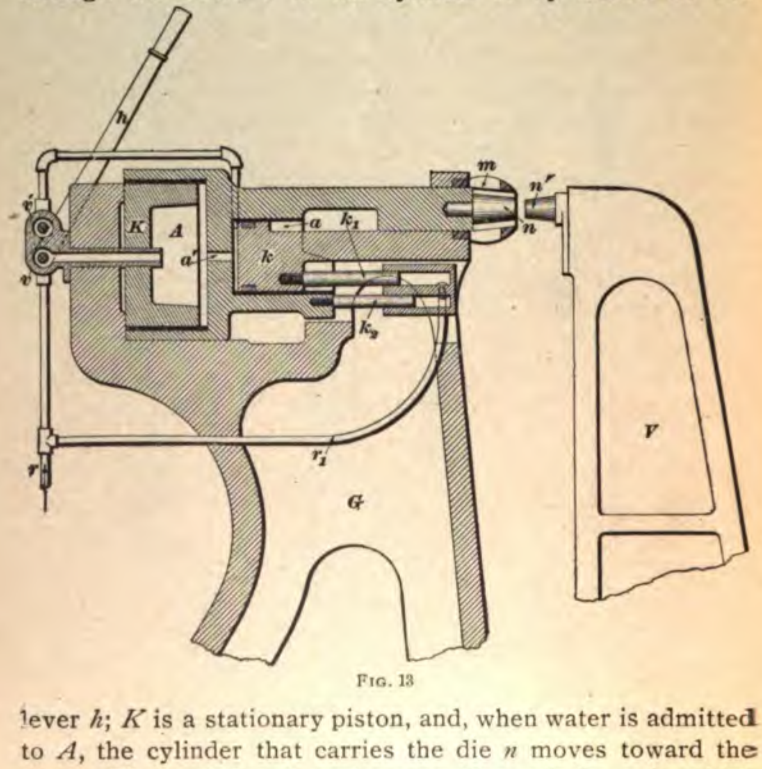


FIG. 13

lever h; K is a stationary piston, and, when water is admitted to A, the cylinder that carries the die n moves toward the rivet. The cylinder A carries a smaller cylinder a, in which works the piston k to which the ring die m is attached, and when water is admitted to A it also passes into a through the small hole a' and thus forces the die m against the plate. After the rivet head has been formed, the lever h is moved so

as to close the valve v and open v'. The water is then discharged from A and a, and the piston k and cylinder A are forced back by means of the two small plungers k_1 and k_2, which are constantly acted on by the water brought in through the branch pipe r_1.

20. General Formula for Pressure Machines.—The principal loss in slow-moving hydraulic machinery is that due to the friction of the packing required to keep the water from leaking past the ram or piston. This friction varies considerably with different methods of packing and the condition of the packing. For hemp packing in good condition, the loss from friction may be taken at from 3 to 8 per cent. of the total pressure. When the packing is new and very tight, the loss from friction will be greatly increased and may be as high as 25 per cent. under specially unfavorable conditions. Table I gives the results of experiments on the friction of leather packing on rams of different sizes.

TABLE I

Diameter of Ram Inches	Friction Per Cent.	Diameter of Ram Inches	Friction Per Cent.
2	2.00	12	.33
3	1.33	13	.30
4	1.00	14	.28
5	.80	15	.26
6	.66	16	.25
7	.57	17	.23
8	.50	18	.22
9	.44	19	.21
10	.40	20	.20
11	.38		

21. To find the net pressure exerted by the ram or plunger of a hydraulic press:

Let d = diameter, in inches, of a hydraulic piston or ram;
G = weight, in pounds, of the ram and attachments that must be lifted by the water;

p = pressure of the water, in pounds per square inch;
f = percentage of friction;
P = net pressure exerted by the ram.

Then, $$P = .7854 \times d^2 \times p \times \left(1 - \frac{f}{100}\right) - G$$

EXAMPLE.—What pressure will be exerted by the ram of a hydraulic press if its diameter is 10 inches, weight 1,500 pounds, pressure of water 550 pounds per square inch, and the friction .4 per cent.?

SOLUTION.—Applying the formula,

$$P = .7854 \times 10^2 \times 550 \times (1 - .004) - 1{,}500 = 41{,}524 \text{ lb. Ans.}$$

22. To find the pressure per square inch required to exert a given net pressure, when the diameter and weight of the ram and the percentage of friction are given, solve for p in the formula of Art. **21.**

Then, $$p = \frac{P + G}{.7854\, d^2 \times \left(1 - \frac{f}{100}\right)}$$

EXAMPLE.—What must be the pressure per square inch on the horizontal ram of a hydraulic riveting machine if the diameter of the ram is 12 inches, the pressure required to head the rivet 80,000 pounds, and the friction $1\frac{1}{4}$ per cent.?

SOLUTION.—Since the ram is horizontal, its weight does not act against the pressure. Then, from the formula, and neglecting the factor G,

$$p = \frac{80{,}000}{.7854 \times 12^2 \times (1 - .0125)} = 716 \text{ lb. per sq. in. Ans.}$$

23. To find the diameter of the piston or ram required to exert a given net pressure, solve for d in the formula of Art. **21.**

$$d = \sqrt{\frac{P + G}{.7854 \times p \times \left(1 - \frac{f}{100}\right)}}$$

EXAMPLE.—What must be the diameter of the ram of a hydraulic crane intended to lift a weight of 40,000 pounds, the weight of the moving parts of the crane being 12,750 pounds, the pressure 750 pounds per square inch, and the friction 2 per cent.?

SOLUTION.—Applying the formula,

$$d = \sqrt{\frac{40{,}000 + 12{,}750}{.7854 \times 750 \times (1 - .02)}} = 9.56 \text{ in. Ans.}$$

24. Accumulators.—It is not often that a natural supply of high-pressure water is available; and, therefore, when it is not available, power-driven pumps are used to produce the extra pressure needed. Sometimes the water is used direct from the pumps; but when machines are working intermittently, some arrangement is needed by means of which the work of the pumps may be stored. The device most used for this purpose is the **accumulator.**

Fig. 14 shows an accumulator as usually constructed. There is a hydraulic cylinder *B* in which works a ram *C*. At its upper end, this ram carries a head *D*, from which a heavy weight *E*, *E* is suspended. The weight may consist of an annular cylinder filled with some heavy material, such as iron ore or scrap iron, or it may be made up of rings of cast iron. Water from the pumps enters the cylinder through the pipe *A*, and the hydraulic machines draw their supply through the pipe *F*. When the pressure of the water is great enough, the ram rises and lifts the weight. If the pumps supply more water than is used by the machines, the weight is lifted until it strikes the lever *G*. When *G* is raised, it operates the rods *r*, *r* and bell-crank lever *l*, and so closes the valve that admits steam to the pumps, thus stopping them until enough water is used to allow the weight to fall, when the steam valve again opens and sets the pumps in operation.

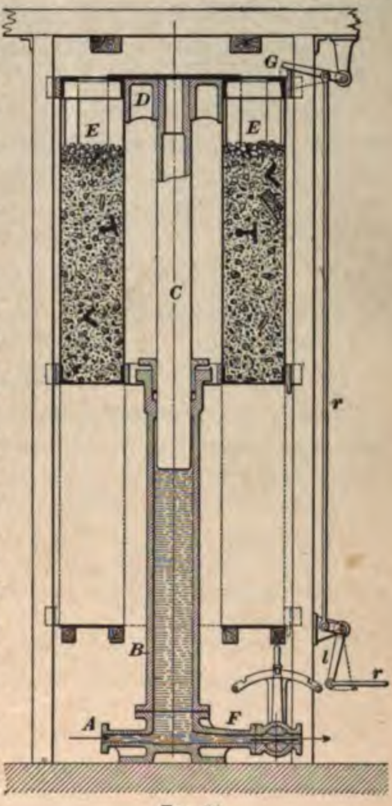


Fig. 14

25. Energy Stored in an Accumulator.—

Let p = pressure of water, in pounds per square inch;
G = total weight of ram with load;
d = diameter of ram, in inches;
l = stroke of accumulator, in feet;
Q = volume of water used, in cubic feet;
m = number of minutes in which Q is used;
H. P. = horsepower.

Neglecting the friction, which is small, the total weight of the ram and its load equals the total pressure on the ram, or

$$G = .7854\,d^2 p \qquad (1)$$

From which, by solving for the pressure,

$$p = \frac{G}{.7854\,d^2} \qquad (2)$$

Again, solving formula **1** for d,

$$d = \sqrt{\frac{G}{.7854\,p}} \qquad (3)$$

To raise the ram and load through the stroke l requires the performance of $G\,l$ foot-pounds of work; hence, the stored energy is

$$E = G\,l, \text{ or } E = .7854\,d^2 p\,l \qquad (4)$$

The volume of water in the cylinder when the ram is at its highest point is $\frac{.7854\,d^2}{144} \times l$ cubic feet; hence, the energy stored per cubic foot of water is

$$.7854\,d^2 p\,l \div \frac{.7854\,d^2\,l}{144} = 144\,p \text{ foot-pounds.}$$

The number of cubic feet required per minute for each horsepower is therefore $\frac{33{,}000}{144\,p} = \frac{229.2}{p}$. Then the number of cubic feet for any horsepower for m minutes is found by the formula

$$Q = \text{H. P.}\,\frac{229.2\,m}{p} \qquad (5)$$

EXAMPLE 1.—An accumulator is required to supply machines having a total of 45 horsepower for an interval of 15 seconds; the pressure is 600 pounds per square inch. If the ram is 12 inches in diameter, how far will the accumulator fall?

SOLUTION.—For 1 H. P. per min. $\frac{229.2}{p}$ cu. ft. is required; hence, for 45 H. P. for $\frac{1}{4}$ min., the volume required is found, by substituting in formula **5**, to be

$$Q = 45 \times \frac{229.2 \times \frac{1}{4}}{600} = 4.3 \text{ cu. ft.}$$

The area of the ram is $.7854 \times 1^2 = .7854$ sq. ft.; hence, to deliver 4.3 cu. ft., the ram must fall $4.3 \div .7854 = 5.47$ ft. Ans.

EXAMPLE 2.—In example 1, what energy can be stored in the accumulator if its stroke is 12 feet?

SOLUTION.—Applying formula **4**,

$$E = .7854 \times 12^2 \times 600 \times 12 = 814{,}300 \text{ ft.-lb., nearly. Ans.}$$

EXAMPLE 3.—A 10-inch accumulator ram weighs 6,340 pounds; with what weight must it be loaded in order that the pressure in the cylinder shall be 2,000 pounds per square inch?

SOLUTION.—Applying formula **1**,

$$G = .7854\, d^2\, p = .7854 \times 10^2 \times 2{,}000 = 157{,}080 \text{ lb.}$$

Since the ram weighs 6,340 lb., the load must be

$$157{,}080 \text{ lb.} - 6{,}340 \text{ lb.} = 150{,}740 \text{ lb. Ans.}$$

EXAMPLES FOR PRACTICE

1. What pressure will be exerted by the ram of a hydraulic press if it is 12 inches in diameter, the weight 2,000 pounds, the pressure of water 600 pounds per square inch, and the friction .33 per cent.?
Ans. 65,635 lb.

2. An 8-inch accumulator ram weighs 4,320 pounds; with what weight must it be loaded in order that the pressure in the cylinder may be 1,800 pounds per square inch? Ans. 86,158 lb.

3. What is the energy that can be stored in an accumulator if the diameter of the ram is 10 inches, the stroke is 8 feet, and the pressure is 500 pounds per square inch? Ans. 314,160 ft.-lb.

MOTORS IN WHICH THE VELOCITY OF WATER IS UTILIZED

IMPULSE WHEELS

26. An **impulse wheel**, or **hurdy-gurdy**, is a waterwheel that has a number of vanes or buckets, against which a jet of water is made to impinge in such a way that the velocity and direction of motion of the water are changed in its passage over the moving vanes. This causes the water

to press against the vanes, and the kinetic energy of the jet is changed to work according to the principles stated in Art. **11**.

The simplest form of impulse wheel consists of a wheel provided with a series of flat radial vanes around its circumference, similar to the paddle wheel of a steamboat. A wheel of this kind can never have a high efficiency, since the water must leave the vanes with an absolute velocity nearly equal to the relative velocity with which it strikes. Experiments have shown a maximum efficiency of a little more than 40 per cent. for this kind of wheel.

27. The Pelton Waterwheel.—The **Pelton waterwheel,** shown in Fig. 15, is an impulse wheel that is used

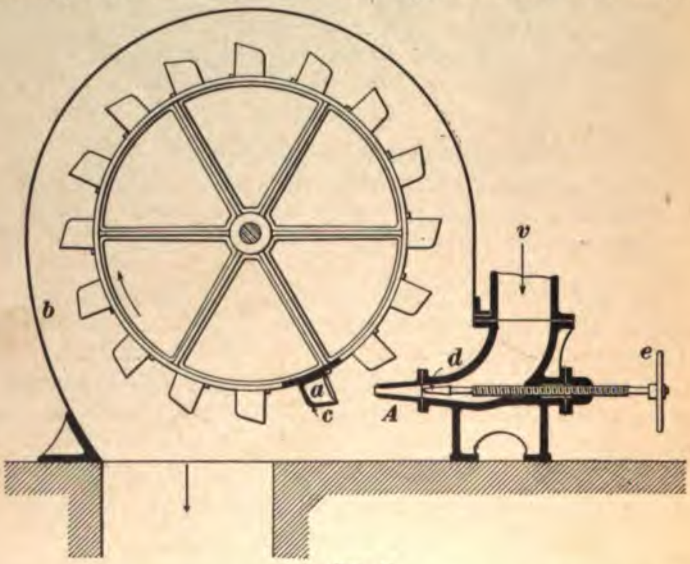


FIG. 15

for very high heads and comparatively small volumes of water. The wheel is covered by the casing *b*. The water enters at *v*, and the flow is regulated by the valve *d*, operated by the hand wheel *e*. The jet from the nozzle *A* impinges on the raised center *a* of the cups *c*, is deflected to both sides, and finally leaves the cups in a direction tangent to their

outer edges. In this way, the direction of the motion of the jet is changed nearly 180°; and when the velocity of the cup is equal to one-half the velocity of the jet, the theoretical efficiency of the wheel is 100 per cent. Experiments have shown that the actual efficiency is sometimes nearly 90 per cent. and that the best efficiency is obtained when the actual velocity of the cups corresponds nearly to the theoretical velocity.

The loss of efficiency is due to the friction of the water in passing through the cups and the energy that is lost in the absolute velocity of the water when it leaves them.

28. Fig. 16 shows two sections of the cups, and the common method of fastening them to the rim of a cast-iron wheel. The inclination of the outer edges *a* is such that the water, as it leaves them, flows clear of the wheel; this is done so that the water will offer no resistance to the motion of the wheel after leaving the cups. The faces of the cups are also inclined to the radius of the wheel, as shown, in order to give the water a slight tendency to flow away from the center of the wheel as it reacts from the cups. The outer edges of the cups are made sharp, so as to offer as little resistance to the water as possible, and the inside surface is sometimes finished for the purpose of reducing the loss by friction.

FIG. 16

A wheel that acts on the same principle as the Pelton wheel is the **Leffel cascade wheel.** The Pelton and Leffel cascade wheels are seldom used for heads of less than 50 feet, but are applicable to falls of any greater height. A number of wheels are in use under heads of more than 2,000 feet. They are specially applicable to mountain streams with high heads and small quantities of water, conditions that exist in the mining districts of Western America.

29. Calculations for Impulse Wheels.—The circumferential velocity of an impulse wheel, that is, the actual velocity of the cups, depends on the head, and hence on the velocity of the jet. With a properly designed nozzle, the velocity of the jet will be nearly that due to the pressure head in the end of the pipe, and the best efficiency is obtained when the velocity of the cups is about one-half the velocity of the jet.

The number of revolutions, with a given velocity at the circumference, varies inversely as the diameter of the wheel; it is therefore possible to make the number of revolutions correspond to the speed of the machinery to be driven, within certain limits. In accordance with this principle, wheels are often designed so as to run at a speed that enables them to be connected directly to the shafts of dynamos, centrifugal pumps, or similar machinery without the use of belts or gearing.

EXAMPLE 1.—Required, the diameter of an impulse wheel that is to be directly connected to the shaft of a dynamo; the pressure head is 275 feet, the dynamo is required to make 850 revolutions per minute, and the coefficient of velocity of the jet is .98.

SOLUTION.—The velocity of the jet is $.98\sqrt{2gh} = .98 \times 8.02\sqrt{275} = 130.34$ ft. per sec. The circumferential velocity of the wheel, therefore, should be $130.34 \div 2 = 65.17$ ft. per sec., or (65.17×60) ft. per min., and the diameter required for 850 rev. per min. is

$$d = \frac{65.17 \times 60}{850 \times 3.1416} = 1.464 \text{ ft., say 18 in. Ans.}$$

Let e = efficiency of the wheel;
G = weight of water used per second;
h = head, in feet.

The work done per second is $e\,G\,h$ and the horsepower is

$$\text{H. P.} = \frac{e\,G\,h}{550}$$

EXAMPLE 2.—In example 1, suppose that the jet has a diameter of $1\frac{1}{4}$ inches and the efficiency is .85; what is the horsepower developed?

SOLUTION.—The weight $G = \frac{.7854 \times (1\frac{1}{4})^2}{144} \times 130.34 \times 62.5$; hence, by substituting in the above formula,

$$\text{H. P.} = \frac{.85 \times .7854 \times (1\frac{1}{4})^2 \times 130.34 \times 62.5 \times 275}{144 \times 550} = 29.5. \quad \text{Ans.}$$

REACTION WHEELS

30. Barker's Mill.—In the simple reaction wheel, commonly known as **Barker's mill**, and shown in Fig. 17, the pressure that produces the motion is caused by the reaction of a jet of water that issues from an orifice under a head.

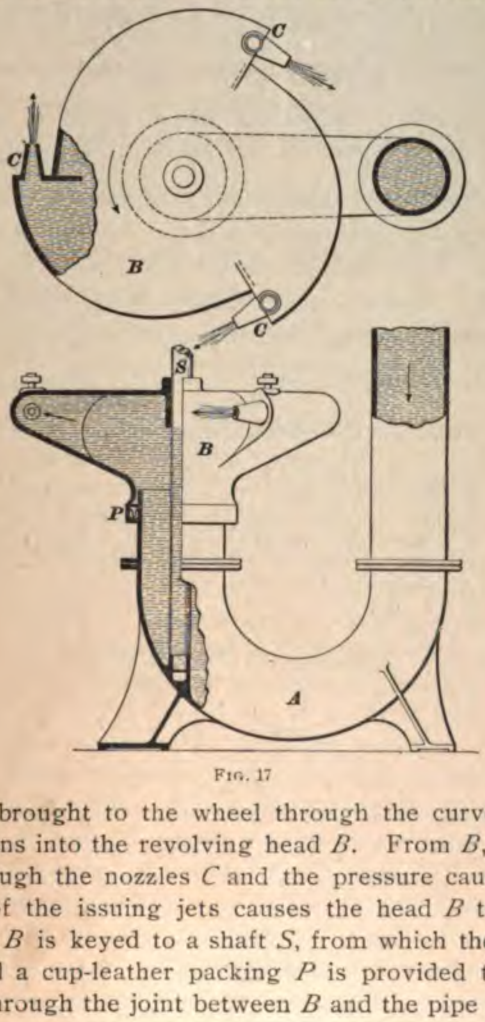


FIG. 17

Water is brought to the wheel through the curved pipe *A*, which opens into the revolving head *B*. From *B*, the water flows through the nozzles *C* and the pressure caused by the reaction of the issuing jets causes the head *B* to revolve. The head *B* is keyed to a shaft *S*, from which the power is taken, and a cup-leather packing *P* is provided to prevent leakage through the joint between *B* and the pipe *A*.

The efficiency of the simple reaction wheel can be but little more than 50 per cent., even under very favorable conditions, and for that reason it is not now used as a motor. A familiar example of the simple reaction wheel is the revolving lawn sprinkler.

31. Efficiency of Reaction Wheels.—In Art. **8**, it was shown that the reaction of the jet on the vessel from which it issues is $\frac{G v}{g}$ pounds, provided that the vessel is at rest. If, however, the orifice has a speed of u feet per second, while the speed of the jet relative to the orifice is v feet per second, the reaction is $\frac{G}{g}(v-u)$ pounds, according to Art. **10**, and the work done per second is $\frac{G}{g}u(v-u)$ foot-pounds. The absolute velocity of the issuing water is $(v-u)$ feet per second, and the work wasted per second is therefore the kinetic energy $\frac{G}{2g}(v-u)^2$. Hence,

$$\begin{aligned}
\text{total work} &= \text{useful work} + \text{lost work} \\
&= \frac{G}{g}u(v-u)+\frac{G}{2g}(v-u)^2 \\
&= 2\frac{G}{2g}uv-2\frac{G}{2g}u^2+\frac{G}{2g}v^2-2\frac{G}{2g}vu+\frac{G}{2g}u^2 \\
&= \frac{G}{2g}(v^2-u^2)
\end{aligned}$$

$$\begin{aligned}
\text{Efficiency} &= \frac{\text{useful work}}{\text{total work}} \\
&= \frac{\frac{G}{g}u(v-u)}{\frac{G}{2g}(v^2-u^2)} \\
&= \frac{2u(v-u)}{(v+u)(v-u)} \\
&= \frac{2u}{v+u}
\end{aligned}$$

This efficiency becomes theoretically 100 per cent. when $u = v$, so that the water drops without velocity, but with

such a high value of u, the friction of the water through the nozzles and orifices is relatively large. In general, the efficiency is little more than 50 per cent. under the most favorable conditions.

TURBINES

32. Classification of Turbines.—The name **turbine** is given to a waterwheel that utilizes the energy of water by causing it to flow through curved vanes or buckets on which the water acts either by impulse alone, or by both impulse and pressure. Turbines may be divided into two chief classes:

1. **Impulse turbines,** in which the whole available energy of the water is converted into kinetic energy before it acts on the moving parts of the turbine, the water striking the turbine blades in the form of jets.
2. **Reaction turbines,** in which part of the energy of the water is in the form of kinetic energy and part is in the form of pressure energy.

In an impulse turbine, the water flows freely into the wheel from the guide buckets in the form of a jet, whose velocity is produced by the head of water on the buckets, and it passes over the wheel vanes without filling the space between them; the passages through the wheel are always open to the air, and consequently the pressure in the space between the wheel vanes is always nearly equal to the atmospheric pressure; the acting force is almost entirely the pressure due to the impulse of the jets issuing from the guides.

In a reaction turbine, the passages between the wheel vanes are always completely filled, so that the flow is said to be continuous. The pressure and the velocity of the water as it enters the wheel may, under different conditions, be equal to, greater, or less than the pressure and the velocity due to the head on the wheel; and the forces that act on the wheel vanes are: (1) a certain amount of static pressure; (2) the pressure caused by the change in direction of the moving water; and (3) a pressure due to the reaction of the

water as it issues from the vanes of the wheel. In most cases, the greatest of these forces is the pressure caused by the change in direction of the moving water in its passage through the wheel. If a reaction turbine is working open to the air, and the flow from the guides is restricted so that the passages between the wheel vanes are only partly filled, it becomes an impulse turbine; hence, the same wheel, under different conditions, may work either as a reaction turbine or as an impulse turbine.

FIG. 18

33. Types of Turbines. Fig. 18 shows a cross-section of an **outward-flow turbine,** also called a **Fourneyron turbine,** from its inventor. The water is brought in at the center, passes outwards between the curved guide vanes B, B to the wheel vanes C, C and is discharged at the circumference of the wheel. The flow of water is regulated by a cylindrical gate that can be raised or lowered in an annular space between the wheel and guides.

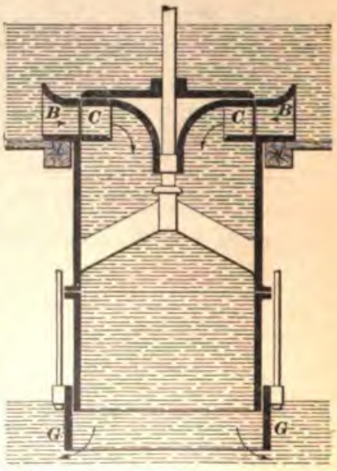


FIG. 19

34. In Fig. 19 is shown a vertical section, and in Fig. 20 a cross-section, through the vanes, of an **inward-flow,** or **Francis, turbine.** Here the water enters the guides B from the outside, passes inwards to the wheel vanes C, and is discharged near the center of the wheel. These wheels are often placed some distance above

the level of the tail-water, as shown, and discharge into an air-tight tube, commonly called a *draft tube*. The wheel is thus in a position that makes it possible to inspect and repair it easily, while at the same time it utilizes the total fall.

FIG. 20

The supply of water in the wheel shown in the figure is regulated by a gate G at the outlet of the draft tube.

Outward-flow and inward-flow turbines are also called **radial turbines,** since the general direction in which the water moves through the wheel is radial.

35. In Fig. 21 is shown a **downward-flow,** or **Jonval, turbine.** Here the general direction of the water is always parallel to the shaft A, or axis; hence, wheels of this class are also known as *parallel-flow* and *axial turbines*.

The water usually enters the guides B from above and is discharged downwards through the wheel C into a draft tube D, as shown in the figure. The discharge may also take place into the air or tail-water without the use of a draft tube.

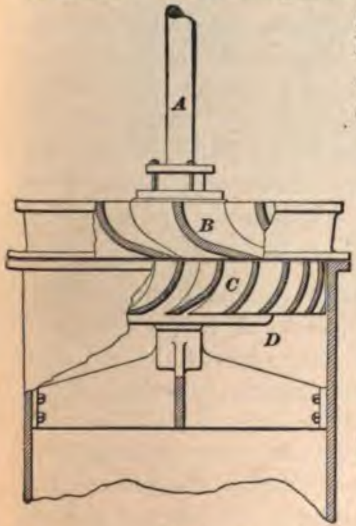


FIG. 21

FIG. 22

36. Many American turbines are made with the wheel vanes so curved that the water enters the wheel in a radial

direction, like an inward-flow turbine, and is discharged in a downward, or axial, direction. These are called *mixed-flow turbines*.

Fig. 22 shows the wheel of a **Risdon mixed-flow turbine** with the double curvature of the vanes. This wheel is cast in one piece. The band *a* serves the double purpose of strengthening the wheel and of making the proper form for the passage of the water through the lower part of the wheel, confining it on all sides.

PUMPS

RECIPROCATING PUMPS

37. Pumps as Hydraulic Machines.—In the hydraulic machines and motors so far described, the water has acted as the motive force, that is, it has done work on the moving parts. Another class of hydraulic machines will now be considered, in which the moving parts do work on the water, either in raising it from a lower to a higher level or in forcing it through pipes. These machines are called **pumps.**

38. Lifting and Force Pumps.—The ordinary **lifting pump** is shown in Fig. 23, and the **force pump** in Fig. 24. The action of the lifting pump is as follows: Suppose the piston to be at the bottom of its stroke; then, as it is raised by the rod S it tends to leave a vacuum in the space below it and the atmospheric pressure on the surface of the water in the well forces the water up the pipe P and through the valve V. When the piston has reached the top of its stroke, the space below it is filled with water. When the piston starts to descend, the weight of the water in the cylinder forces the valve V shut, and the enclosed water passes up through the valves u, u. On the next upward stroke, the valves u, u close and the water above the piston is forced into the pipe P' through the valve c. At each upward stroke, therefore, a quantity of water equal in bulk

to the volume swept through by the piston is raised from the well.

The action of the force pump is easily seen from Fig. 24. During the upward stroke of the piston, the water rises in the pipe *P*, passes through the valve *V*, and fills the cylinder. When the piston starts to return, the pressure exerted on the water by the piston causes the valve *V'* to open, and the valve *V* to close, so that the water is forced from the cylinder into the pipe *P'*. With this pump, the water may be forced to almost any height.

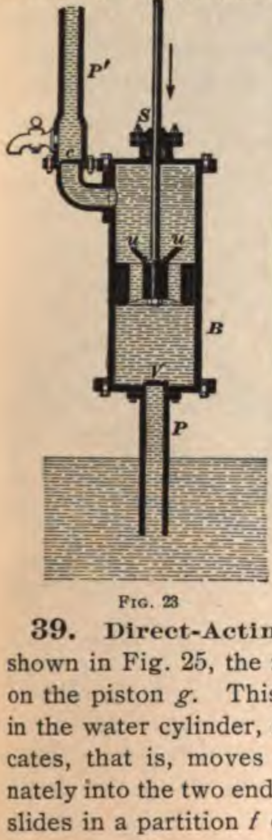


Fig. 23

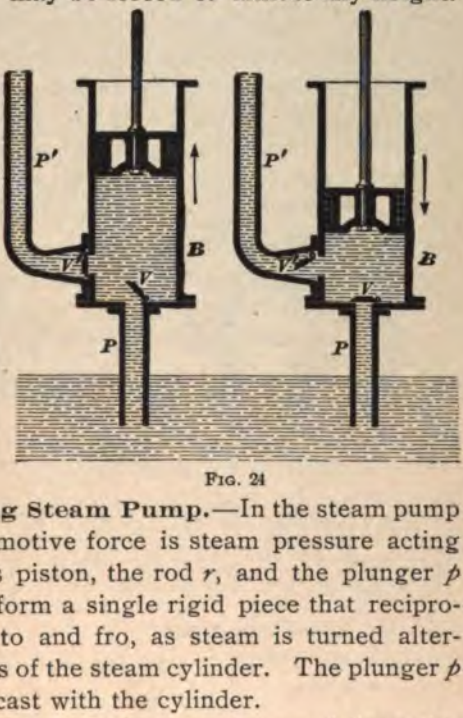


Fig. 24

39. Direct-Acting Steam Pump.—In the steam pump shown in Fig. 25, the motive force is steam pressure acting on the piston *g*. This piston, the rod *r*, and the plunger *p* in the water cylinder, form a single rigid piece that reciprocates, that is, moves to and fro, as steam is turned alternately into the two ends of the steam cylinder. The plunger *p* slides in a partition *f* cast with the cylinder.

Suppose the piston and plunger to move toward the right. The displacement of the plunger to the right increases the volume to the left of the partition *f* and decreases that to the right of *f* by the same amount. In consequence, water will flow through the suction pipe *c* into the chamber *k*; and

because of the partial vacuum to the left of *f*, will lift the valves *s'* and fill the space left vacant by the motion of the plunger. At the same time, the water in the right end will be forced through the valves *v'* out into the delivery pipe *h*. During the stroke to the left, the action is reversed; water flows into the right end of the cylinder through valves *s* and is forced out of the left end through the valves *v*. The hand-hole *d* is provided for examining the valves.

This pump is *double-acting*, as it forces water during both strokes. The pumps shown in Figs. 23 and 24 are *single-*

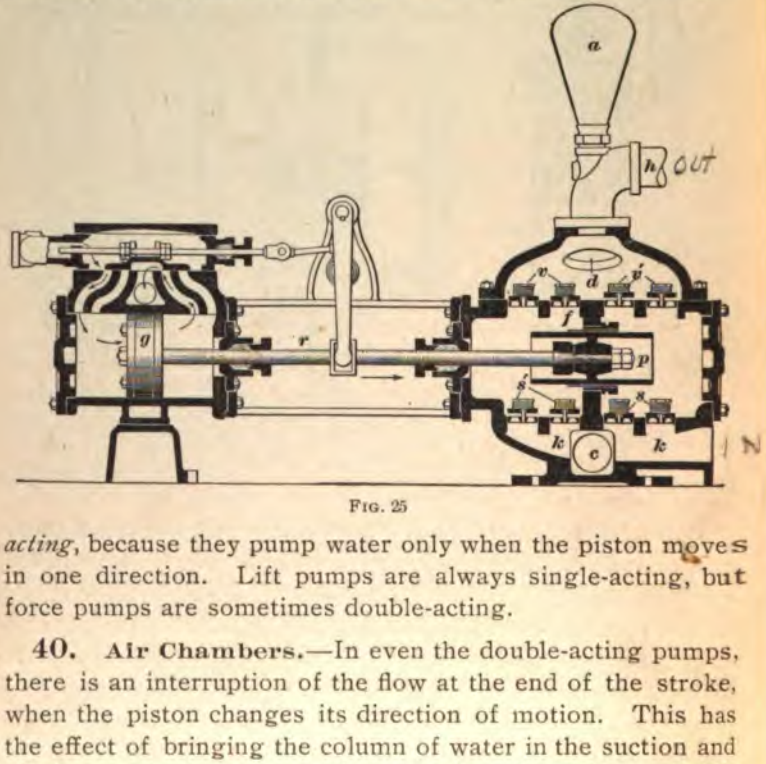


Fig. 25

acting, because they pump water only when the piston moves in one direction. Lift pumps are always single-acting, but force pumps are sometimes double-acting.

40. Air Chambers.—In even the double-acting pumps, there is an interruption of the flow at the end of the stroke, when the piston changes its direction of motion. This has the effect of bringing the column of water in the suction and discharge pipes to rest at the end of each stroke, and this column of water must be set in motion again as the next stroke is made. If the pipes are long, the force required to stop and start the water will be very great, and there will be

a severe shock at the end of every stroke that will absorb power and subject the pump and pipes to great pressures.

This difficulty is removed and the flow through the pipes is made more continuous and steady by the use of an **air chamber.** An air chamber is a vessel containing air, and is attached either to the pump just outside of the discharge valves or to the discharge pipe near the pump, as shown at *a*, Fig. 25.

The water, after being drawn in through the valves *s* is forced by the plunger *p* through the valves *v'* into the discharge pipe *h*; but part of it flows into the air chamber *a* and compresses the air therein. When the plunger reaches the end of its stroke and no more water is being forced into the discharge pipe, the compressed air in the air chamber expands and forces the extra water out through the discharge pipe. In this way, the air chamber acts as a reservoir that receives its supply during the motion of the plunger, and gives it out again when the plunger comes to a pause. The air in the air chamber acts as a spring or cushion that absorbs some of the force of each stroke of the plunger and gives it out while the plunger is at rest at the end of the stroke. The pump and pipe are thus relieved of shocks and a nearly constant rate of flow from the discharge pipe is insured.

41. Displacement and Discharge.—The **displacement** of a pump for a single stroke is the volume of water that would be displaced, that is, the volume swept through by the piston or plunger, during that stroke. The **theoretical discharge** of a pump is equal to the displacement. The **actual discharge** is generally less than the displacement, owing to leakage past the valves and piston, and also to the return of water through the valves while they are closing.

The difference between the displacement and the actual discharge, expressed as a percentage of the displacement, is called the **slip** of a pump. When the column of water in the suction and discharge pipes of a pump is long and the lift moderate, the energy imparted by the piston during the

discharge stroke may be sufficient to keep the column in motion during all or a part of the return stroke. Under these conditions, the actual discharge will be greater than the displacement, in which case the slip is said to be *negative*.

42. Let Q' = displacement or theoretical discharge, in cubic feet per minute;
Q = actual discharge, in cubic feet per minute;
N = actual discharge, in gallons per minute;
d = diameter of piston or plunger, in inches;
l = stroke of piston, in feet;
n = number of discharging strokes per minute;
s = the slip $= \frac{Q' - Q}{Q'}$.

The volume swept through by the piston in one stroke is $\frac{.7854\, d^2}{144} \times l$ cubic feet; hence,

$$Q' = \frac{.7854\, d^2\, l\, n}{144} = .005454\, d^2\, l\, n \qquad (1)$$

As $s = \frac{Q' - Q}{Q'} = 1 - \frac{Q}{Q'}$, $Q = Q'(1 - s)$; hence,

$$Q = .005454\, d^2\, l\, n\, (1 - s) \qquad (2)$$

Since there are 7.48 gallons per cubic foot, $N = 7.48\, Q$, and hence,

$$N = .0408\, d^2\, l\, n\, (1 - s) \qquad (3)$$

The diameter of the piston for a given discharge, in cubic feet per minute, is found by solving formula **2** for d; this gives

$$d = 13.54 \sqrt{\frac{Q}{l\, n\, (1 - s)}} \qquad (4)$$

If the discharge is taken in gallons per minute, solve formula **3** for d; then,

$$d = 4.95 \sqrt{\frac{N}{l\, n\, (1 - s)}} \qquad (5)$$

The slip varies from 0 to 40 per cent., depending on the tightness of valves, piston, etc.

EXAMPLE 1.—A single-acting plunger pump with a plunger 8 inches in diameter and 36 inches stroke discharges 33.5 cubic feet of water per minute when making thirty-five discharging strokes; what is the slip?

SOLUTION.—Using formula **1**, and substituting the given valves, $Q' = .005454 \times 8^2 \times 3 \times 35 = 36.65$ cu. ft. per min. The slip, therefore, is

$$\frac{36.65 - 33.5}{36.65} = .086 = 8.6 \text{ per cent., nearly.} \quad \text{Ans.}$$

EXAMPLE 2.—A pump making thirty discharging strokes per minute is required to discharge 450 gallons per minute; the length of stroke is 36 inches. Assuming the slip to be .25, compute the diameter that must be given the plunger.

SOLUTION.—From formula **5**, by substituting,

$$d = 4.95\sqrt{\frac{450}{3 \times 30 \times (1 - .25)}} = 12.78 \text{ in., say 13 in.} \quad \text{Ans.}$$

43. Work Done by a Pump.—The theoretical work done by a pump may be computed in either of two ways. In addition to the symbols given in Art. **42**, let p be the pressure on the piston, in pounds per square inch, and h be the height, in feet, through which the water is lifted. The area of the piston is $.7854\,d^2$ square inches; hence, the total pressure on it is $.7854\,d^2p$ pounds. In one stroke, the resistance is overcome through a distance of l feet, and the work done per stroke is therefore $.7854\,d^2pl$ foot-pounds. In 1 minute, the work done is $.7854\,d^2pln$ foot-pounds, and therefore the theoretical horsepower is

$$\text{H. P.} = \frac{.7854\,d^2pln}{33{,}000} = .0000238\,d^2pln \qquad (1)$$

But in 1 minute the pump raises G pounds of water through a height of h feet, requiring therefore the work Gh foot-pounds. If there is no slip, $G = 62.5\,Q'$, since a cubic foot of water weighs 62.5 pounds, and

$$\text{H. P.} = \frac{62.5\,Q'h}{33{,}000} = .001894\,Q'h \qquad (2)$$

That the two expressions for the horsepower are identical is readily shown; for, if $.434\,h$ is substituted for p in the first, and for Q' the value given in formula **1** of Art. **42**, in the second, the result is

$$\frac{.7854\,d^2ln}{33{,}000} \times .434\,h = \frac{.7854\,d^2ln}{33{,}000} \times \frac{62.5}{144}h$$

and as $.434 = \frac{62.5}{144}$, the expressions are identical.

If the discharge is given in gallons instead of cubic feet, formula **2** becomes

$$H. P. = .000253\, N h \qquad (3)$$

The actual work is always greater than the useful work, and, in practice, from 20 to 50 per cent. should be added to the results obtained from formulas **1**, **2**, and **3**. Work is required to overcome the friction of the piston or plunger in the cylinder or stuffingbox, and considerable work is also required to overcome the friction of the water in its passage through the pipes and the valves and passages of the pump. Some work must also be expended in giving the water the velocity it has when it leaves the discharge pipe.

According to the principles of hydraulics and the flow of water through pipes, it is evident that the power required to overcome the frictional resistance of the water will be reduced by making the pipes large and direct and the passages through the valves and pump of ample size and as direct as possible, so as to avoid loss from sudden change of direction of flow.

EXAMPLE.—What is the theoretical horsepower of a pump raising 375 gallons of water per minute a height of 120 feet?

SOLUTION.—Applying formula **3** and substituting,

$$H. P. = .000253 \times 375 \times 120 = 11.385. \text{ Ans.}$$

EXAMPLES FOR PRACTICE

1. A single-acting pump with a plunger 10 inches in diameter and 40 inches stroke discharges 60 cubic feet of water per minute when making thirty-six discharging strokes per minute; what is the slip?
Ans. 8.32 per cent.

2. A pump making thirty-two discharging strokes per minute is required to discharge 300 gallons per minute; the length of stroke is 30 inches. Assuming the slip to be .20, compute the diameter that must be given the plunger. Ans. 10.72, say 11 in.

3. A pump raises 900 gallons of water per minute to a height of 175 feet; what theoretical horsepower is required? Ans. 39.85 H. P.

CENTRIFUGAL PUMPS

44. Reciprocating pumps may be considered as reversed pressure motors. A pressure motor, if driven from some external source, becomes a pump; the feedpipe of the motor becomes the suction pipe of the pump, and the exhaust pipe becomes the delivery pipe.

Likewise, the **centrifugal pump** may be considered as a reversed velocity motor. In the motor, water gives up kinetic energy, due to its velocity, and from this energy is

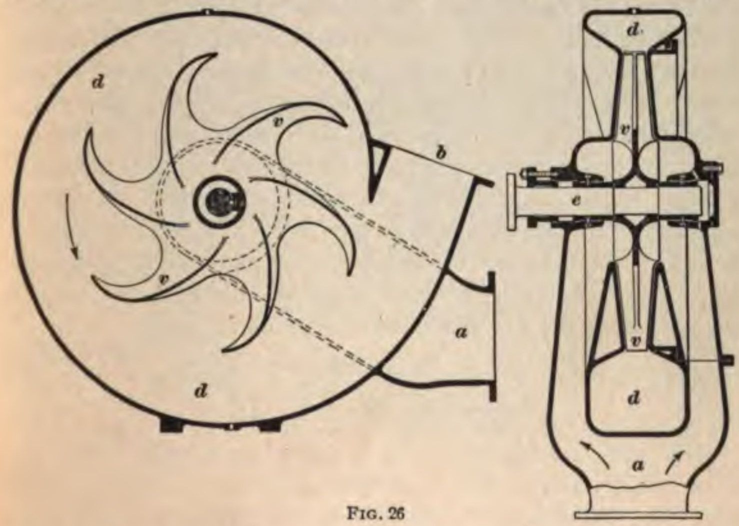


FIG. 26

obtained the work required to drive the moving parts. In the centrifugal pump, the action is reversed; the water enters with little velocity, and at atmospheric pressure work is done on it by the vanes of the *impeller* or wheel of the pump, and it leaves the pump with a higher velocity and higher pressure. By virtue of this increased velocity and pressure, the water is enabled to rise to the upper level. As the water rises, its kinetic and pressure energies are destroyed, but its potential energy is correspondingly increased.

In Fig. 26 are shown two sections of a centrifugal pump, taken at right angles to each other. The water enters

through a suction pipe *a* and is discharged at *b*. On the shaft *e* is keyed the wheel that carries the vanes *v*, *v*. By the rapid rotation of the wheel, the water is given a whirling motion as it passes outwards toward the ends of the vanes, and it is discharged into the passage *d* with considerable velocity and with a pressure in excess of atmospheric pressure.

Centrifugal pumps of this type are most efficient when working under low heads, say up to 40 feet. For low heads and large quantities of water, they give excellent results and are especially useful when the water contains grit or other impurities that would destroy the pistons and packing or prevent the closing of the valves of other pumps. Since there are no valves or other restricted passages, centrifugal pumps have been largely used in dredging machines for pumping water containing large quantities of mud, sand, and gravel; in fact, anything can be pumped that will be carried through the pump and pipes by a current of water. Recently, centrifugal pumps have been built for use with high heads, in some cases raising water several hundred feet. Such pumps are called *high-pressure centrifugal pumps*.

THE HYDRAULIC RAM

45. The **hydraulic ram** is a machine in which the pressure produced by suddenly stopping a column of moving water is used to raise a part of that water to a point above the level of the source of supply. Fig. 27 shows a section of a hydraulic ram. The pipe *a* that connects the ram with the source of supply is called the **drive pipe**. A valve *b* that closes the end of the pipe *a* has a stem that slides freely through a sleeve *c*. The sleeve *c* is provided with a regulating screw by means of which the stroke of the valve *b* may be regulated. An air chamber *f* is attached to the pipe *a* over an opening closed by a valve *d*, which opens from the pipe toward the air chamber. The action of the ram is as follows: Starting with the valve *b* open as shown, water will flow from the reservoir through the pipe *a* and out past *b* through

the opening at *c*. When the velocity of this water becomes sufficiently rapid, the current flowing past *b* will exert an upward pressure great enough to raise the valve and close it. The current is thus suddenly stopped, and the inertia of the column of water in the pipe *a* produces sufficient pressure to open the valve *d* and force some of the water into the air chamber *f*. The energy of the moving mass of water is thus absorbed in compressing the air in *f*, and the column is brought to rest. The stoppage of the column in *a* is so sudden that there is a slight recoil, which closes the valve *d* and causes a reduction in the pressure sufficient to open *b* again, when the process is repeated. The pressure of the com-

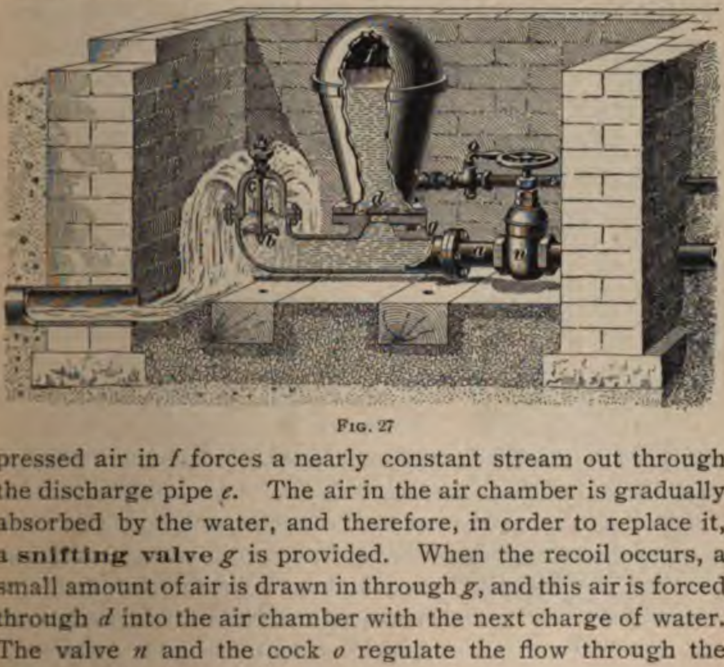


FIG. 27

pressed air in *f* forces a nearly constant stream out through the discharge pipe *e*. The air in the air chamber is gradually absorbed by the water, and therefore, in order to replace it, a **snifting valve** *g* is provided. When the recoil occurs, a small amount of air is drawn in through *g*, and this air is forced through *d* into the air chamber with the next charge of water. The valve *n* and the cock *o* regulate the flow through the inlet and discharge pipes, respectively.

The principal use of these rams is for the purpose of supplying buildings, water tanks, etc. from a source some distance below them. Tests have shown that when they are well made and adjusted their efficiency is about 50 per cent.

Rams may be used when the fall from the supply is no more than 18 inches, but the proportion of the water discharged varies almost directly as the ratio between the fall to the ram and the height to which the water must be raised. With moderate lengths of discharge pipe, the proportion of the water that can be raised is given as follows: One-seventh of the water can be raised to a level above the ram five times as high as the fall from the supply to the ram; one-fourteenth of the water can be raised to a level above the ram ten times as high as the fall to the ram; and so for other ratios between the fall and the height of discharge.

FIG. 28

AIR-LIFT PUMP

46. Another method of raising water from lower to higher levels is by the use of the **air-lift pump**, as shown in Fig. 28. This device consists of two pipes. The outer one *a*, or the casing, is suspended in the water to be pumped, so that the lower end *b* is at a distance *h* below the surface of the water. The inner pipe *c*, which is much smaller than the outer pipe, passes through a stuffingbox *d* in the elbow *e*, and extends downwards inside the casing. It is attached to an air compressor by means of the pipe *f*, and its lower end is perforated with a number of small holes, as shown.

When the casing *a* is lowered into the water, the water rises to the same level inside the casing as outside. If air under pressure is then forced into the pipe *c*, it will escape

through the holes at the lower end of this pipe, and will form bubbles in the water inside the casing *a*. As a result, the casing will soon contain a mixture of water and air bubbles instead of the solid column of water it originally held.

The weight of the mixture of air and water being less, per cubic foot, than that of water alone, the column inside the casing will weigh less than a column of the same height outside the casing. The mixture of air and water will therefore be forced upwards inside the casing. If this casing were extremely long, the mixture would rise until the pressure it exerted at the lower end *b* of the casing was just equal to the pressure due to the head *h* of the water at that point. As usually built, however, an outlet *g* is provided at a height less than that to which the mixture would otherwise rise in order to balance pressures. As a result, the mixture of air and water is discharged at *g* in a continuous flow, and the velocity of discharge is equal to that corresponding to the additional height or head through which it is capable of rising.

The force that operates the air-lift pump, then, is the difference between the pressures inside and outside the casing at the bottom. To obtain the best results, the casing should extend below the water level a distance about $1\frac{1}{2}$ times as great as the height above the water level to which the water is to be pumped.

ELEMENTARY CHEMISTRY

THEORETICAL CHEMISTRY

DEFINITIONS

1. Definition of Chemistry.—**Chemistry** is that branch of science that treats of the different kinds of matter and of the various changes that matter undergoes.

2. Matter and Energy.—The word **matter,** as used in scientific language, means that which occupies space. **Energy** may be defined as that which causes changes in matter.

3. Divisions of Matter.—Three divisions of matter are recognized in science: *masses*, *molecules*, and *atoms*.

A **mass** of matter is any portion of matter appreciable by the senses.

A **molecule** is the smallest particle of matter into which a body can be divided without losing its identity, and which can exist in the free state.

An **atom** is the smallest particle of simple matter that can enter into the composition of a molecule.

A mass of matter is made up of molecules, and a molecule is composed of atoms.

4. Bodies are composed of collections of molecules. They exist in three forms or conditions: *solid*, *liquid*, and *gaseous*.

5. Specific Gravity and Density.—The **specific gravity** of a body is the ratio of the weight of a volume of

it to that of an equal volume of some other substance selected as a standard. The standard is usually water for solids and liquids, and dry air for gases.

Density is a term frequently used with the same meaning as specific gravity. In connection with the chemistry of gases, however, it is now common to use the term specific gravity when air is taken as the standard of comparison, and the term density when the gas hydrogen is taken as the standard of comparison. In this Section, then, when speaking of the specific gravity of a gas, dry air is understood to be taken as the standard of comparison; and in speaking of the density of a gas it is understood that hydrogen gas is taken as the standard of comparison.

6. Attraction of Matter.—Matter is maintained in its normal state throughout the universe, by the action of the forces: *gravitation*, *cohesion*, *adhesion*, and *chemism*, or *affinity*.

Gravitation acts through all space. It is the power by which every particle of matter is attracted to every other particle of matter.

Cohesion and **adhesion** act only across inappreciable distances, holding together molecules of solids, and to some degree those of liquids. When these molecules are alike they are held together by cohesion. When the molecules are unlike they are held together by adhesion.

Chemism, or **affinity,** holds together the atoms that form a molecule; it modifies the molecules themselves, and brings heterogeneous substances into intimate relation, and this produces new molecules. The action of atoms on each other by virtue of their affinity is termed *chemical action* or *chemical change*.

7. Indestructibility of Matter.—By the most careful observations of all known kinds of chemical action, it has been proved that there is never any loss or gain of matter. When chemical action occurs, there is only a change of state; but never an annihilation or creation of matter.

8. Division of Molecules.—Molecules are formed by the union of atoms; they may be divided into two classes:

(1) **elemental molecules,** in which the atoms are alike; (2) **compound molecules,** in which the atoms are unlike. Matter made up of molecules containing similar atoms is called **simple,** or **elementary, matter;** matter whose molecules are composed of dissimilar atoms is called **compound matter.**

9. Analysis and Synthesis.—**Analysis** is the splitting up of compound matter into simpler matter. **Synthesis** is the building up of compound matter from simpler matter.

10. Mixtures, Compounds, and Elements.—If various kinds of matter are examined, it is found that some masses of matter are heterogeneous; that is, not uniform in composition. These are **mixtures,** and chemistry, as a science, does not deal with such, but restricts itself to dealing with the homogeneous masses of matter. Homogeneous matter may be divided into *compounds* and *elements*—**compounds** being homogeneous matter, the molecules of which are made up of unlike atoms, and **elements** being matter, the molecules of which are made up of like atoms. The number of compounds is very large, but extended observations show that there are comparatively few elements, the number known being between seventy and eighty. There are two reasons why it is impossible to state positively the exact number of elements: elements may exist that have not yet been discovered; and some that are now considered elements may in the future be proved to be compounds.

11. Size and Weight of Elemental Molecules. According to the **law of Avogadro,** *equal volumes of all bodies in the gaseous state, under the same conditions of temperature and pressure, contain the same number of molecules.* It follows, therefore, that the molecules of all bodies in the gaseous state must be of the same size, or at least occupy the same space, hence, the weight of any molecule, compared with that of a molecule of hydrogen, is proportional to any given volume of the gas compared with the same volume of hydrogen.

12. Molecular Weight.—Experiments conducted by chemists lead to the conclusion that one molecule of hydrogen is composed of two atoms. Assuming, then, that the weight of a hydrogen atom be taken as unity, the **molecular weight** of hydrogen must be two. The **vapor density** of an element or compound is the mass or quantity of a body in the gaseous state compared with the same mass or quantity of hydrogen; and, since the molecular weight of hydrogen is two, the molecular weight of an element or compound is twice its vapor density.

13. Atomicity of Elemental Molecules.—**Atomicity** refers to the number of atoms a molecule contains. Thus, by *monatomic*, *diatomic*, *triatomic*, etc. molecules is meant molecules containing one, two, three, etc. atoms. Most elemental molecules are diatomic.

ATOMS

14. An atom is the smallest particle of an element that can participate in the composition of a molecule; but atoms differ from one another in weight and in combining power.

15. Atomic Weight.—It is obviously impossible to isolate single atoms; and, if it could be done, they would be so exceedingly minute that it would be impossible to weigh them. Atomic weights are not absolute weights, as are pounds, grains, etc., but represent relative weights of atoms. Formerly, all atomic weights were based on the weight of an atom of hydrogen, which was taken as unity. At the present time, however, some chemists use as the basis of all atomic weights the weight of an atom of oxygen, to which the value 16 is given. It is obviously immaterial what number is taken as a standard, since the atomic weights are the relative weights of the atoms; but it is important that the same standard be used for all the elements.

In the case of many elements, their atomic weights have been determined with the greatest exactness; but in the case of some of the rarer ones, the values given may be proved

later to be slightly wrong. In Table I are given the latest atomic weights. In the column headed $O = 16$, the atomic weight of oxygen is taken at 16, and the other ratios calculated on this basis; in the column headed $H = 1$, the atomic weight of hydrogen is taken as unity, and the other ratios are calculated on this basis. In the column headed Approximate are given numbers with fewer decimal places; they are not as exact as those given in the other columns, but are close enough for all ordinary purposes of calculation.

16. Valence.—**Valence** signifies the combining power possessed by an atom; it expresses the number of hydrogen atoms with which an atom can combine for which it may be exchanged. Since an atom may form several compounds with the same substance, its valence is variable, and not absolutely fixed, as is the atomic weight. While there is no absolute general rule, when an element exhibits more than one valence, these are usually, but not in all cases, all odd numbers or all even numbers.

According as their valence is 1, 2, 3, 4, 5, 6, or 7, atoms are called *monads*, *dyads*, *triads*, *tetrads*, *pentads*, *hexads*, or *heptads*, which names are derived from the Greek numerals. When referred to in the adjectival form, the Latin numerals are employed, and an atom is said to be *univalent*, *bivalent*, *trivalent*, *quadrivalent*, *quinquivalent*, *sexivalent* or *septivalent*.

17. Symbols.—To prevent constant repetition of the names of the elements, and to aid in expressing the composition of substances, abbreviations, or **symbols**, are used instead of names; these symbols consist of the initial letter, or the initial letter and another letter, of its name. Sometimes they are derived from the Latin names, which are often very different from the common ones. Such is the case with the elements sodium (*Na* from *natrium*), lead (*Pb* from *plumbum*), mercury (*Hg* from *hydrargyrum*), iron (*Fe* from *ferrum*), and copper (*Cu* from *cuprum*). The names, symbols, and atomic weights of the most prominent elements are given in Table I.

TABLE I

ELEMENTS, SYMBOLS, AND ATOMIC WEIGHTS

Elements	Symbols	Atomic Weights		
		$O = 16$	$H = 1$	Approximate
Aluminum	*Al*	27.1	26.9	27.
Antimony (stibium)	*Sb*	120.2	119.3	120.
Argon	*A*	39.9	39.6	40.
Arsenic	*As*	75.0	74.4	75.
Barium	*Ba*	137.4	136.4	137.
Bismuth	*Bi*	208.5	206.9	208.
Boron	*B*	11.	10.9	11.
Bromine	*Br*	79.96	79.36	80.
Cadmium	*Cd*	112.4	111.6	112.
Cæsium	*Cs*	132.9	131.9	133.
Calcium	*Ca*	40.1	39.8	40.
Carbon	*C*	12.00	11.91	12.
Cerium	*Ce*	140.25	139.2	140.
Chlorine	*Cl*	35.45	35.18	35.5
Chromium	*Cr*	52.1	51.7	52.
Cobalt	*Co*	59.0	58.56	59.
Columbium	*Cb*	94.0	93.3	94.
Copper (cuprum)	*Cu*	63.6	63.1	63.
Erbium	*Er*	166.	164.8	166.
Fluorine	*F*	19.	18.9	19.
Gadolinium	*Gd*	156.	155.	156.
Gallium	*Ga*	70.	69.5	70.
Germanium	*Ge*	72.5	71.9	72.
Glucinum	*Gl*	9.1	9.03	9.
Gold (aurum)	*Au*	197.2	195.7	197.
Helium	*He*	4.	4.	4.
Hydrogen	*H*	1.008	1.0	1.
Indium	*In*	115.	114.08	114.
Iodine	*I*	126.85	125.90	127.
Iridium	*Ir*	193.0	191.5	193.
Iron (ferrum)	*Fe*	55.9	55.5	56.

TABLE I—(*Continued*)

Elements	Sym-bols	Atomic Weights		
		$O = 16$	$H = 1$	Approxi-mate
Krypton	*Kr*	81.8	81.2	82.
Lanthanum	*La*	138.9	137.9	139.
Lead (plumbum) . . .	*Pb*	206.9	205.35	207.
Lithium	*Li*	7.03	6.98	7.
Magnesium	*Mg*	24.36	24.18	24.
Manganese	*Mn*	55.0	54.6	55.
Mercury (hydrargyrum)	*Hg*	200.0	198.5	200.
Molybdenum	*Mo*	96.0	95.3	96.
Neodymium	*Nd*	143.6	142.5	143.
Neon	*Ne*	20.	19.9	20.
Nickel	*Ni*	58.7	58.3	58.5
Nitrogen	*N*	14.04	13.93	14.
Osmium	*Os*	191.	189.6	191.
Oxygen	*O*	16.00	15.88	16.
Palladium	*Pd*	106.5	105.7	106.
Phosphorus	*P*	31.0	30.77	31.
Platinum	*Pt*	194.8	193.3	195.
Potassium (kalium) .	*K*	39.15	38.86	39.
Praseodymium	*Pr*	140.5	139.4	140.
Radium	*Ra*	225.	223.3	225.
Rhodium	*Rh*	103.0	102.2	103.
Rubidium	*Rb*	85.4	84.4	85.
Ruthenium	*Ru*	101.7	100.9	102.
Samarium	*Sm*	150.	148.9	150.
Scandium	*Sc*	44.1	43.8	44.
Selenium	*Se*	79.2	78.6	79.
Silicon	*Si*	28.4	28.2	28.
Silver (argentum) .	*Ag*	107.93	107.12	108.
Sodium (natrium) . .	*Na*	23.05	22.88	23.
Strontium	*Sr*	87.6	86.94	87.5
Sulphur	*S*	32.06	31.83	32.
Tantalum	*Ta*	183.	181.6	183.

TABLE I—(*Continued*)

Elements	Symbols	Atomic Weights		
		$O = 16$	$H = 1$	Approximate
Tellurium	*Te*	127.6	126.6	128.
Terbium	*Tb*	160.	158.8	160.
Thallium	*Tl*	204.1	202.6	204.
Thorium	*Th*	232.5	230.8	233.
Thulium	*Tm*	171.	169.7	171.
Tin (stannum)	*Sn*	119.0	118.1	119.
Titanium	*Ti*	48.1	47.7	48.
Tungsten (wolfram)	*W*	184.	182.6	184.
Uranium	*U*	238.5	236.7	238.
Vanadium	*V*	51.2	50.8	51.
Xenon	*Xe*	128.	127.	128.
Ytterbium	*Yb*	173.0	171.7	173.
Yttrium	*Yt*	89.0	88.3	89.
Zinc	*Zn*	65.4	64.9	65.
Zirconium	*Zr*	90.6	89.9	90.

NOTE.—It is customary to divide the elements into two classes—metals and non-metals, or metalloids, a distinction first made when only a few elements were known. This distinction is now a purely arbitrary one, as it is impossible to draw a sharp line of demarcation between the two groups. The names printed in Italics are those of non-metals.

Multiplication of atoms is expressed by placing an Arabic numeral to the right of and below the symbol. Thus, S_2 stands for two atoms of sulphur. *Multiplication of molecules* is expressed by placing the multiplier, called a *coefficient*, to the left, in front of the formula. Thus, $3NaCl$ stands for three molecules of salt.

The valence of an atom is expressed by placing a Roman numeral or a corresponding number of strokes above and to the right of its symbol. Thus, Br^{I} or Br' stands for the univalent atom of bromine, and C^{IV} or C'''' stands for the quadrivalent atom of carbon. However, these Roman numerals or strokes are frequently omitted. Sometimes, in order

to make a formula more clearly understood, the symbol of the element is written and its valence indicated by lines radiating from the symbol. These lines are called **bonds.** Thus, the bivalent oxygen atom would be written $-O-$ or $O=$, and the quadrivalent carbon atom $-\overset{|}{\underset{|}{C}}-$.

18. Laws of Definite and Multiple Proportions. As a result of almost numberless experiments it has been established that *the proportions, by weight, according to which elements combine are invariable for the same compound.* This is known as the **law of definite proportions.**

Also as the result of a very large number of experiments, it has been established that *when two elements combine to form more than one compound, the different weights of the one element that unite with the same weight of the other element bear a simple ratio to each other.* This is known as the **law of multiple proportions.**

Oxygen and hydrogen may combine in two proportions; as water, H_2O, and as hydrogen peroxide, H_2O_2. In the latter compound, there is just twice as much oxygen for the same amount of hydrogen as in the former. The reason why combinations occur in multiple proportion is that, when two elements combine, one atom of one of them may combine with either one, two, or three atoms of the other, but combination with the fraction of an atom would be contrary to the atomic theory.

19. The symbols of the elements, since they stand for atoms, not only express the names of the elements, but also weights proportional to their atomic weights. Thus, the formula for salt is $NaCl$; this not only expresses that the compound is composed of the elements sodium and chlorine, but that a molecule of salt is made up of one atom of sodium having a weight of 23 and one atom of chlorine having a weight of 35.5. Since any mass of salt is composed of molecules, the weights of the two elements bear the same ratio to one another in the mass as in the single molecule.

In 58.5 pounds of salt there are 23 pounds of sodium and 35.5 pounds of chlorine.

The formula for water is H_2O. From this, it is seen that a molecule of water contains two atoms of hydrogen weighing $1 \times 2 = 2$, and one atom of oxygen weighing 16. Therefore, in 18 pounds of water, there are 2 pounds of hydrogen and 16 pounds of oxygen.

Oxygen and hydrogen also combine to form another compound, hydrogen peroxide, which for a given weight of hydrogen contains twice as much oxygen as water. The formula is H_2O_2, the molecule containing two atoms of hydrogen weighing $1 \times 2 = 2$, and two atoms of oxygen weighing $16 \times 2 = 32$. Hence, in 34 pounds of hydrogen peroxide, there are 2 pounds of hydrogen and 32 pounds of oxygen.

The following compounds of nitrogen offer a very good illustration of the law of combining proportions: N_2O contains 28 parts of nitrogen and 16 parts of oxygen. NO contains 14 parts of nitrogen and 16 parts of oxygen, being the same ratio as in 28 parts of nitrogen and $16 \times 2 = 32$ parts of oxygen. N_2O_3 contains 28 parts of nitrogen and $16 \times 3 = 48$ parts of oxygen. NO_2 contains 14 parts of nitrogen and $16 \times 2 = 32$ parts of oxygen, being the same ratio as in 28 parts of nitrogen and $16 \times 4 = 64$ parts of oxygen. N_2O_5 contains 28 parts of nitrogen and $16 \times 5 = 80$ parts of oxygen.

Chemical actions can be very simply represented by the use of equations, the substances entering into the action being placed on the left of the sign of equality, and substances resulting being placed on the right. Thus, $2H_2 + O_2 = 2H_2O$ means that two molecules of hydrogen unite with one molecule of oxygen and form two molecules of water.

COMPOUNDS

20. A compound molecule is composed of dissimilar atoms united according to the law of valence. The number of atoms that such a molecule may contain is apparently unlimited. Compound molecules are divided into two classes—*binary molecules* and *ternary molecules*.

21. Binary Molecules.—**Binary molecules** are molecules the atoms of which are directly united, and, whatever the number of their atoms may be, there can never be more than two kinds of them.

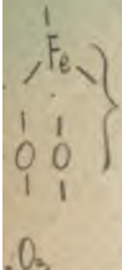
22. Nomenclature of Binaries.—The names of binary molecules are derived from their constituent atoms. The most common binaries are those composed of a metal and a non-metal; the name of the metal is written first and then that of the non-metal, one or more syllables being removed from the latter and the termination *ide* added. Thus, sodium and chlorine yield sodium chloride; zinc and sulphur yield zinc sulphide.

As it frequently happens that more than one compound of the same elements is known, it becomes necessary to distinguish these compounds. Oxygen and copper combine in two proportions, represented by the formula CuO and Cu_2O, the name oxide being common to both; but the first is called *cupric oxide* and the second *cuprous oxide*. The termination *ous* is applied to the compound containing the lower proportion of the non-metallic element, and *ic* to the one containing the higher proportion. Where there are two or more atoms of the non-metallic element present, the distinction is frequently marked by a prefix to the second name indicating the number. Thus, BaO is barium monoxide, and BaO_2 is barium dioxide.

23. Use of Prefixes.—If one of the constituents acts with more than two valences, the termination *ic* (given on the discovery of the compound) is generally arbitrarily assigned, and a further distinction is made by the use of a prefix. A compound in which the valence of the first constituent is less than the *ous* takes the prefix *hypo* (= under). When the valence is above *ic*, the prefix *per* is used, the termination of the second constituent being in all cases *ide*.

24. Formation of Binaries.—A binary molecule consists of atoms with equal or different valences. Atoms having the same valence unite in the proportion of 1 to 1, that is, the atoms mutually saturate each other and their

chemism is satisfied. When atoms with different valences unite, each atom has to furnish the same number of bonds to satisfy their chemism. This number is the least common multiple of the two valences. The absolute number of atoms of each constituent is obtained by dividing the least common multiple by the valence of each atom.



EXAMPLE.—Trivalent iron and divalent oxygen unite to form an oxide; what is the number of atoms of each?

SOLUTION.—The least common multiple of the valences (3 and 2) is 6. Then each must furnish six bonds. $6 \div 3 = 2$ atoms of iron, $6 \div 2 = 3$ atoms of oxygen. The oxide will then have the formula Fe_2O_3. Ans.

25. The groups of atoms that have been considered are called **saturated molecules**, because the bonds of their constituent atoms are all mutually engaged. There are also **unsaturated molecules**—groups of atoms that, possessing free bonds, may enter into combination like single atoms. These unsaturated groups of atoms are called **compound radicals**, or simply **radicals**. They do not exist free in nature, although in some cases, by combining with another like group, they may form a saturated molecule. Their valence is equal to the number of their unsatisfied bonds, which is the difference between the valences of their constituents.

Molecules containing compound radicals linked to an atom are usually considered as binaries and named accordingly.

26. Names of Compound Radicals.—The names of compound radicals, with the exception of the compound radicals *amidogen* (NH_2), *cyanogen* (CN), and *ammonium* (NH_4), terminate in *yl*. The roots of their names come either from the radical constituents or from some compound into which they enter. Thus, the compound radical (OH) is called *hydroxyl*, and (CH_3) is called *methyl*, from methyl alcohol, of which it is a constituent.

27. Ternary Molecules.—Ternary molecules are those whose dissimilar atoms are united by the aid of some third atom. Among the compounds higher than the binary series are those formed by the union of water with oxides,

thus forming acids and hydrates (hydroxides), and derivatives of these substances.

28. Acids.—To the chemist, the sourness of an acid is but an accidental property, as many substances that are not sour to the taste are classified as acids. An **acid** may be defined as a body containing hydrogen, which hydrogen may be replaced by a metal, the resulting compound being a salt. Most acids are sour; they are also active chemical agents; most of them are characterized by their property of changing the color of a solution of litmus (a blue dye) to red. H_2SO_4

Oxygen is a constituent of most acids, the members of the group containing oxygen being distinguished as **oxy-acids.** The oxides that, by union with water, form acids are **acid anhydrides,** or simply **anhydrides.** They are, in most cases, non-metallic oxides, but sometimes consist of metals combined with a comparatively large number of oxygen atoms. There are a few acids in which oxygen is absent; these are called **hydracids,** hydrochloric acid, HCl, being an example. According to the definition given, hydrogen is an essential constituent of all acids. It should, however, be mentioned that the term *acid* is sometimes applied to what are really anhydrides. Thus CO_2, carbon dioxide or carbonic anhydride, is sometimes spoken of as carbonic-acid gas. This name, however, is being replaced by carbon dioxide, which is more in accordance with modern chemical science.

29. Bases and Alkalies.—A **base** is an oxide or hydrate of a metal (or group of elements equivalent to a metal), which metal (or group of elements) is capable of replacing the hydrogen of an acid, forming a salt, water being formed at the same time.

An **alkali** is a base of specially active character, soluble in water, and easily recognized by the soapy taste and feel it imparts to water, and also by its ability to restore the blue color to a solution of litmus that has been reddened by an acid. The principal alkalies are sodium hydrate, $NaOH$, and potassium hydrate, KOH. **Hydrates** are mostly compounds of metallic oxides and water, and are also frequently

termed *hydroxides*. Their formation may be represented by the following equations:

Na_2O	+	H_2O	=	$2NaOH$	(1)
sodium oxide		water		sodium hydrate or sodium hydroxide	

CaO	+	H_2O	=	$Ca(OH)_2$	(2)
calcium oxide		water		calcium hydrate or calcium hydroxide	

30. Salts.—When an acid and a base react on each other, the compound produced by the replacement of the hydrogen of the acid by the metal of a base is called a **salt.** The following equations illustrate the formation of salts:

HCl	+	$NaOH$	=	$NaCl$	+	H_2O	(1)
hydrochloric acid		sodium hydrate		sodium chloride		water	

H_2SO_4	+	CaO	=	$CaSO_4$	+	H_2O	(2)
sulphuric acid		calcium oxide		calcium sulphate		water	

As a rule, salts are without action on litmus; sometimes, however, when a strong acid combines with a weak base, the resulting salt will turn litmus red; or when the base is strong and the acid weak, the salt will turn reddened litmus blue.

With acids like hydrochloric acid, where in the molecule there is only one hydrogen atom, part of the hydrogen cannot be replaced by a metal and a part be left in the molecule; but with acids containing more than one replaceable hydrogen atom in the molecule, only part of the hydrogen may be replaced by a metal, the resulting salt being called an *acid salt*. Sulphuric acid, H_2SO_4, and carbonic acid, H_2CO_3, are such acids, and from each of these, with the same base, there may be normal salts or acid salts.

H_2SO_4	+	$2NaOH$	=	Na_2SO_4	+	$2H_2O$	(3)
sulphuric acid		sodium hydrate		sodium sulphate or normal sodium sulphate		water	

$$\underset{\text{sulphuric acid}}{H_2SO_4} + \underset{\text{sodium hydrate}}{NaOH} = \underset{\text{acid sodium sulphate or sodium bisulphate}}{NaHSO_4} + \underset{\text{water}}{H_2O} \quad (4)$$

$$\underset{\text{carbonic acid}}{H_2CO_3} + \underset{\text{calcium hydrate}}{Ca(OH)_2} = \underset{\text{calcium carbonate}}{CaCO_3} + \underset{\text{water}}{2H_2O} \quad (5)$$

$$\underset{\text{carbonic acid}}{2H_2CO_3} + \underset{\text{calcium hydrate}}{Ca(OH)_2} = \underset{\text{acid calcium carbonate or calcium bicarbonate}}{Ca(HCO_3)_2} + \underset{\text{water}}{2H_2O} \quad (6)$$

A **basic salt** is one in which only a part of oxygen or hydroxyl of the base has been removed by the acid. As an example, the base bismuth hydrate, $Bi(OH)_3$, may react with nitric acid, HNO_3, to form one normal salt and two basic salts.

$$\underset{\text{bismuth hydrate}}{Bi(OH)_3} + \underset{\text{nitric acid}}{3HNO_3} = \underset{\text{bismuth tri-nitrate (normal salt)}}{Bi(NO_3)_3} + \underset{\text{water}}{3H_2O} \quad (7)$$

$$Bi(OH)_3 + 2HNO_3 = \underset{\text{bismuth di-nitrate (a basic salt)}}{Bi(OH)(NO_3)_2} + \underset{\text{water}}{2H_2O} \quad (8)$$

$$Bi(OH)_3 + HNO_3 = \underset{\text{bismuth mono-nitrate (a basic salt)}}{Bi(OH)_2(NO_3)} + \underset{\text{water}}{H_2O} \quad (9)$$

31. Nomenclature of Acids and Salts.—The names of acids are derived from their principal constituent in the same way as those of binary compounds. The name of the principal acid of a series ends in *ic*, as chloric acid, $HClO_3$; that of the next lower ends in *ous*, as chlorous acid, $HClO_2$. For the acid next higher than the *ic* acid, the prefix *per* is used, as perchloric acid, $HClO_4$; for the acid next lower than the *ous* acid, the prefix *hypo* is used, as hypochlorous acid, $HClO$. Hydracids are distinguished by the prefix *hydro;* as, for instance, hydrochloric acid, HCl.

The names of the corresponding salts are derived from the same root by changing the endings; thus, *ic* becomes *ate* and

ous becomes *ite*. Salts of sulphuric acid are called *sulphates*, and salts of sulphurous acid are called *sulphites;* and so on.

32. Chemical Equations.—It must be remembered that a **chemical equation** is a statement of fact. The facts expressed by the equation must be the result of observation and experiment. The equation itself must balance, that is, the kind and number of atoms on the two sides of the sign of equality must be the same; but the fact that an equation balances is no proof that it is correct. For example, the two equations (1) and (2) both balance:

$$2H_2 + O_2 = 2H_2O \qquad (1)$$

$$2H_2 + 3O_2 = H_4O_6 \qquad (2)$$

but while equation (1) is correct, being an expression of facts that have been proved by experiment, equation (2) is wrong, as there is no evidence that there is any compound of hydrogen and oxygen that can be represented by the formula H_4O_6. When, however, an equation is correct, it not only tells the character of the reaction, but tells also the relative weights of the substances entering into, and resulting from, the reaction.

33. Stoichiometry.—**Stoichiometry** considers the numerical relations of quantities of matter; all calculations made from the atomic weight of elements are stoichiometrical calculations. It is the mathematics of chemistry.

Since a molecule is made up of atoms, each of which has its definite weight, the weight of the molecule, or the molecular weight, is the sum of the atomic weights of its constituents. Knowing the molecular weight of any compound, the number of atoms in the molecule, and the atomic weight of each constituent atom, it is easy to calculate the percentage composition (that is, the composition of 100 parts of the substance).

Representing the molecular weight by m, the atomic weight by a, the number of atoms of each constituent by n, and the percentage amount by x, the proportion formed is, $m : a\,n = 100 : x$; and from this proportion is derived the formula

$$x = \frac{a\,n \times 100}{m} \qquad (1)$$

EXAMPLE 1.—The formula for carbon dioxide is CO_2. The atomic weight of carbon is 12, and of oxygen is 16. The molecular weight of CO_2, then, is $12 + (16 \times 2) = 44$. Then substituting in the formula,

$$x = \frac{12 \times 1 \times 100}{44} = 27.27 \text{ per cent. } C$$

$$x = \frac{16 \times 2 \times 100}{44} = 72.73 \text{ per cent. } O$$

The same principles are applied in calculating the weight of substances entering into or resulting from chemical reactions. The reaction must first be expressed in the form of an equation. The molecular weights of all the various substances are then written below their respective formulas. Having all the necessary data, the problems may be solved by the following proportion:

As the molecular weight of the given substance is to the quantity of it (that is, its absolute weight) given in the problem, so is the molecular weight of the required substance to the quantity of it required.

Letting M = molecular weight of given substance;
m = molecular weight of required substance;
W = absolute weight of given substance;
w = absolute weight of required substance.

The proportion is $M : W = m : w$; from which follows the formula

$$w = \frac{Wm}{M} \qquad (2)$$

EXAMPLE 2.—Nitric acid is formed by the action of sulphuric acid on potassium nitrate, in accordance with the equation

$$\underset{\substack{\text{potassium}\\\text{nitrate}\\101}}{KNO_3} + \underset{\substack{\text{sulphuric}\\\text{acid}\\98}}{H_2SO_4} = \underset{\substack{\text{nitric}\\\text{acid}\\63}}{HNO_3} + \underset{\substack{\text{potassium}\\\text{acid sulphate}\\136}}{KHSO_4}$$

How much nitric acid can be prepared from 500 pounds of potassium nitrate?

SOLUTION.—Substituting in formula 2,

$$w = \frac{500 \times 63}{101} = 311.89 \text{ lb. Ans.}$$

In these problems, it has been assumed that each molecule of the factor yields one in the product. If in any given reaction this is not true, then M and m must represent the

sum of the molecular weights, as shown in the equation. As all gaseous molecules have the same volume, we are able to read every equation representing a reaction between gaseous bodies, not only quantitatively but volumetrically. Thus, the equation

$$\underset{\substack{\text{carbon}\\\text{monoxide}\\2\times(12+16)\\56}}{2CO} + \underset{\substack{\text{oxygen}\\32}}{O_2} = \underset{\substack{\text{carbon}\\\text{dioxide}\\2\times(12+32)\\88}}{2CO_2}$$

shows that two molecules of carbon monoxide and one molecule of oxygen yield two molecules of carbon dioxide, and it may also be read: two volumes of carbon monoxide and one volume of oxygen yield two volumes of carbon dioxide. It must be remembered that these volume relations only hold good when all the gases are measured at the same temperature and pressure.

34. Thermometric Scales.—Temperature is measured by an instrument called a **thermometer.** It consists of a narrow glass tube, at one end of which is a bulb filled with mercury. On being heated, the mercury expands in proportion to its temperature. Thermometers are graduated in different ways. The thermometer in general use in the United States of America is known as the **Fahrenheit thermometer.** The point that the mercury reaches in the tube when the instrument is placed in melting ice is marked 32°; the point indicated by the mercury when the thermometer is placed in water boiling in the open air at the level of the sea is marked 212°. The space between these two points is divided into 180 equal parts, called **degrees,** and the degree divisions continued above 212° and below 32°. On this instrument, then, the zero mark is 32° below the melting point of ice. The **Réaumur thermometer** has the melting point of ice marked 0° and the boiling point of water marked 80°; this instrument is in common use in Russia and some other countries. The **centigrade thermometer** is in general use in France, and is the instrument commonly used by scientific men all over the world. On this

thermometer, the melting point of ice is marked 0° and the boiling point of water 100°. In this Section, unless otherwise stated, all temperatures will be given in degrees centigrade.

METRIC SYSTEM OF WEIGHTS AND MEASURES

35. Instead of using the complicated English system of weights and measures, chemists generally use the very simple and convenient **metric system.** This system is based on the meter, which has a length of about 39.37 inches. There are three principal units: the meter, the liter, and the gram—the units of length, capacity, and weight, respectively. Multiples of these units are obtained by prefixing to the names of the principal units the Greek words *deka* (10), *hekto* (100), and *kilo* (1,000); the submultiples, or subdivisions, are obtained by prefixing the Latin words *deci* ($\frac{1}{10}$), *centi* ($\frac{1}{100}$), and *milli* ($\frac{1}{1000}$). These prefixes form the key to the entire system. In the following tables, the abbreviations of the principal units of these submultiples begin with a small letter, while those of the multiples begin with a capital letter. Chemists commonly use *cc.* instead of *cm*^{3} for cubic centimeter.

The equivalents in the common units in use in the United States are given in connection with these tables.

MEASURES OF LENGTH

10 millimeters (mm.)	= 1 centimeter	cm.
10 centimeters	= 1 decimeter	dm.
10 decimeters	= 1 meter	m.
10 meters	= 1 dekameter	Dm.
10 dekameters	= 1 hektometer	Hm.
10 hektometers	= 1 kilometer	Km.
10 kilometers	= 1 myriameter	Mm.

1 meter = 39.37 inches = 3.2808 feet = 1.0936 yards

MEASURES OF SURFACE (NOT LAND)

100 square millimeters (mm^2.)	= 1 square centimeter	cm^2.
100 square centimeters	= 1 square decimeter	dm^2.
100 square decimeters	= 1 square meter	m^2.

1 square centimeter = .15500 square inch

1 square meter = 10.7639 square feet = 1.1960 square yards

MEASURES OF VOLUME

1,000 cubic millimeters (mm^3.)	= 1 cubic centimeter	cm^3.
1,000 cubic centimeters	= 1 cubic decimeter	dm^3.
1,000 cubic decimeters	= 1 cubic meter	m^3.

1 cubic centimeter = .061023 cubic inch

1 cubic meter = 35.314 cubic feet = 1.3079 cubic yards

MEASURES OF CAPACITY

10 milliliters (ml.)	= 1 centiliter	cl.
10 centiliters	= 1 deciliter	dl.
10 deciliters	= 1 liter	l.
10 liters	= 1 dekaliter	Dl.
10 dekaliters	= 1 hektoliter	Hl.
10 hektoliters	= 1 kiloliter	Kl.

1 liter = 61.023 cubic inches = 1.0567 liquid quarts = .9078 dry quart

NOTE.—The liter is equal in volume to 1 cubic decimeter.

MEASURES OF WEIGHT

10 milligrams (mg.)	= 1 centigram	cg.
10 centigrams	= 1 decigram	dg.
10 decigrams	= 1 gram	g.
10 grams	= 1 dekagram	Dg.
10 dekagrams	= 1 hektogram	Hg.
10 hektograms	= 1 kilogram	Kg.
1,000 kilograms	= 1 ton	T.

1 gram = 15.432 grains Troy = .03527 ounce avoirdupois

1 kilogram = 2.2046 pounds avoirdupois = 2.6792 pounds Troy

1 metric ton = 1.1023 tons of 2,000 pounds

NOTE.—The gram is the weight of 1 cubic centimeter of pure distilled water at a temperature of 4° C.; the kilogram is the weight of 1 liter of water; the metric ton is the weight of 1 cubic meter of water at 4° C.

INORGANIC CHEMISTRY

GENERAL CLASSIFICATION

36. The elements are arbitrarily divided into two classes —*metals* and *metalloids*, or *non-metals*. This division was made in the latter part of the 18th century, when only a few of the elements were known. With the rapid progress of chemistry and the frequent additions to the number of elements known, it has become impossible to draw as sharp and clear a line of demarcation as formerly. In fact, opinions differ, and some elements that are termed metalloids by one authority are classed among the metals by another. The metallic properties of some metals are apparent, and it is equally apparent that some other substances are not metals, while others, lying between these, are hard to classify. However, metals as a whole form oxides which act as bases, while non-metallic oxides form acids. In the first class are placed such elements as gold, silver, lead, and copper; in the second, those that are gases at ordinary temperatures, such as oxygen, chlorine, and hydrogen, together with some solids, as sulphur.

The metals exceed in number the non-metals; at present about fifty-nine metals and nineteen non-metals are known. These seventy-eight elements are the foundation on which the whole science of chemistry is built, and every kind of matter that has been examined has been found to be made up of some of these elements, either united as a compound or in the uncombined state.

Some of the elements are very abundant and occur widely distributed in nature, while others have been found only in such minute quantities that their examination has been extremely difficult and not always entirely satisfactory.

NON-METALS OR METALLOIDS

37. Before considering the most important metals, some of the most important non-metals will be discussed; these are: *hydrogen*, *oxygen*, *nitrogen*, *chlorine*, *sulphur*, *phosphorus*, *carbon*, and *silicon*. The atomic weight given with the description of each element is the approximate value, which is the more convenient to use and is satisfactory for all ordinary calculations. The more exact value will be found in Table I.

HYDROGEN

Symbol H. Atomic weight 1. Valence I. Density 1. Molecular weight 2.

38. Occurrence.—Hydrogen occurs to some extent in the free condition, and issues from the earth in small quantities in some localities. It is, for example, a constituent of the gases that escape from some oil wells. It occurs, chiefly, however, in combination with oxygen, as water, of which it forms 11.11 per cent. It occurs also in most animal and vegetable substances. In these products of life, it is combined with carbon and oxygen or with carbon, oxygen, and nitrogen. Hydrogen also occurs in petroleum, bitumen, etc.

39. Properties.—Hydrogen is a colorless, odorless, and tasteless gas. It is the lightest matter known, being 14.43 times lighter than air and 11,000 times lighter than water. Its molecular weight, therefore, is smaller than that of any other known substance. Its refractive power on light is remarkable, being 6.614 times that of air. It is soluble to a very slight extent in water, 100 volumes of which dissolve but $1\frac{1}{2}$ volumes of hydrogen. Hydrogen is the standard of density for gases. One liter weighs .0899 gram. Calculations in which this value enters may be simplified by taking it as .09 gram, a value near enough to the actual one for most practical purposes. It is one of the most nearly permanent gases, having only within the last few years been condensed to a liquid. Its boiling point under atmospheric pressure is $-252.5°$ C.

Hydrogen is combustible; that is, when heated to about 500° C. it becomes capable of combining with the oxygen of the air; light and heat being evolved in the process. The flame of burning hydrogen is pale, and, under atmospheric pressure, is scarcely luminous, though it becomes bright if the pressure is increased. The heat evolved by it is very great, the burning of 1 gram producing heat enough to raise 34,462 grams of water from 0° to 1° C. It does not support combustion or respiration; a lighted candle placed in it is extinguished and animals die when confined in it.

Hydrogen is capable of being absorbed or occluded by many metals at a more or less elevated temperature. Of these metals, palladium is the most remarkable, being able at an ordinary temperature to take up over 900 times its own volume of hydrogen.

OXYGEN

Symbol O. Atomic weight 16. Valence II. Molecular weight 32.

40. Occurrence.—Oxygen occurs abundantly in nature, both in the free state and in combination with other elements. It occurs uncombined in the atmosphere, of which it constitutes about one-fifth. In the combined form, it constitutes eight-ninths, by weight, of water, and nearly one-half of the earth's solid crust. It also occurs in a more active state as **ozone,** having the formula O_3.

41. Properties.—Oxygen is an odorless, colorless, and tasteless gas. It is somewhat heavier than air, the ratio being about 1.105 for oxygen to 1 for air. It is only slightly soluble in water, 100 volumes of water dissolving about 3 volumes of oxygen. Oxygen can be condensed to a light blue liquid that boils at $-184°$ C. Oxygen is capable of entering into combination with nearly all the elements, but in the state in which it is usually obtained heat is necessary to accomplish this union.

Combustion, in the ordinary sense of the word, is the result of the union of oxygen with some other element,

the process of union being attended with light and heat. When, for instance, hydrogen, sulphur, carbon, phosphorus, sodium, or iron are brought in contact with oxygen at a suitable temperature, they burn, evolving heat and light, thus producing oxides of these substances. Oxygen, therefore, is an intensely active substance, in which the rapidity of ordinary combustion is vastly increased. It is respirable when pure, and causes a quickening of the circulation.

COMPOUNDS OF HYDROGEN AND OXYGEN

42. There are two compounds of hydrogen and oxygen, namely, *water*, H_2O, and *hydrogen peroxide*, H_2O_2. Water is one of the most important substances in nature, and while hydrogen peroxide is of considerable interest to the chemist and of some importance in the arts, it is not of sufficient importance to justify more than a reference to it in this Section.

43. Water.—Every one is familiar with the occurrence of **water** in its three physical forms: as a solid (ice), as a liquid (water), and as a gas (steam). Between the temperatures of 0° C. and 100° C., water is a tasteless, colorless, odorless, and limpid liquid. The most characteristic property of water is its great solvent power, there being comparatively few substances that it does not dissolve in large or small quantities. It has neither an acid nor an alkaline reaction and is a poor conductor of heat and electricity. When cooled to 0° C., it solidifies into ice; when heated to 100° C., it is changed to steam. It unites with most oxides, forming either acids or bases. It is the standard of specific gravity for liquids and solids; and in the metric system 1 cubic centimeter of water at 4° C. weighs 1 gram.

Chemically pure water is usually obtained by freezing natural water from the small quantity of foreign substance that it contains; and as most of these bodies are in the state of solution, the water is commonly purified by distillation.

By means of an electric current, water can be decomposed into the elements hydrogen and oxygen, the volume of the hydrogen produced being twice that of the oxygen, and

its weight one-eighth that of the oxygen. When hydrogen burns, it unites with the oxygen of the air, forming water. If 2 volumes of hydrogen are mixed with 1 volume of oxygen, and the mixture ignited (best by means of an electric spark), a violent explosion takes place; and if the original gases have been measured at a temperature above 100° C., and the resulting water, as steam, be measured at the same temperature, it will be found that the 3 volumes of mixed gases yield 2 volumes of steam.

44. Oxidation and Reduction.—It has been seen that oxygen is an exceedingly active element, combining with many other substances to form oxides. Such addition of oxygen is called **oxidation**. Copper, for example, if heated in oxygen, is converted into the black copper oxide, CuO. If this copper oxide is heated in hydrogen, the hydrogen will combine with the oxygen of the copper oxide and set free, or reduce, the metal. Thus, $CuO + H_2 = Cu + H_2O$. Any process by which oxygen is removed from a compound is called a **reduction** process, and the substance used to bring about this reduction is called a **reducing agent.**

The terms oxidation and reduction, however, mean more than the addition or removal of oxygen. Oxidation may be defined as any process by which oxygen is added to an element or compound, or by which the valence of an element is increased. The reverse of this is reduction, which may be defined as any process by which oxygen is removed from a compound, or by which the valence of an element is decreased.

It is evident that in any process of reduction there must also be oxidation, the reducing agent being oxidized. Thus, in the example of the reduction of copper oxide by the reducing agent hydrogen, the hydrogen itself is oxidized.

CHLORINE

Symbol Cl. Atomic weight 35.5. Valence I, III, V, and VII. Molecular weight 71.

45. Occurrence.—**Chlorine** is not known to exist free in nature, but, as chlorides, it is widely distributed in

combination with metals. The most important of these is *sodium chloride*, $NaCl$, or common salt, which not only occurs in enormous quantities dissolved in sea-water, and the water of salt springs, but also occurs in immense beds of rock salt.

46. Properties.—Chlorine is a yellowish-green gas with a suffocating odor and taste, causing coughing when present in only minute quantities, and if inhaled in larger quantities, causing serious inflammation of the lungs and air passages. It is nearly $2\frac{1}{2}$ times as heavy as air. At a temperature of $-40°$ C., or at a pressure of four atmospheres, it condenses to a dark yellow liquid, having a specific gravity of 1.38. It is quite soluble in water, 1 volume of water dissolving between 2 and 3 volumes of the gas, forming a solution that has essentially the same properties as the gas itself.

In its chemical properties, chlorine is an exceedingly active substance; even at an ordinary temperature, it combines with many elements and acts on many compounds. Many metals combine with chlorine with the evolution of light, and all metals are capable of combining with it, forming chlorides. Chlorine also unites readily with many nonmetals. The attraction of chlorine to hydrogen is specially strong, the two gases exploding violently when mixed together and exposed to sunlight. Owing to its slight attraction to oxygen, it does not burn in the air at any temperature. Chlorine is largely used as a bleaching and disinfecting agent.

47. Hydrochloric acid, HCl, also known as **muriatic acid,** is the only known compound of hydrogen and chlorine. It is a colorless gas with a sharp, pungent odor, fuming strongly in the air. It cannot be breathed and extinguishes flame. By cold and pressure, it may be condensed to a colorless, limpid liquid. Hydrochloric acid is remarkably soluble in water, 1 volume of which dissolves 450 volumes of the gas at 15° C., forming a strongly acid liquid. This solution of the gas in water is what is ordinarily called **hydrochloric acid.**

Hydrochloric acid is a strong acid, readily turning blue litmus red, neutralizing bases, and attacking many metals with the formation of the corresponding chloride and the liberation of hydrogen. With zinc the reaction is:

$$\underset{\text{zinc}}{Zn} + \underset{\text{hydrochloric acid}}{2HCl} = \underset{\text{zinc chloride}}{ZnCl_2} + \underset{\text{hydrogen}}{H_2}$$

THE HALOGEN GROUP

48. The three elements, *bromine*, *iodine*, and *fluorine*, together with *chlorine*, constitute the group known as the **halogens.** As most acids are oxyacids, their corresponding salts must contain oxygen, and consequently are ternaries. The halogens, however, possess the property of yielding salts by direct union with the metals, and these salts are, of course, binaries. All of this group form acids analagous to hydrochloric acid, and these acids consist of the element combined with hydrogen and contain no oxygen. In general, these elements resemble chlorine, bromine being a heavy, red, bad-smelling, and very volatile liquid. Iodine is a dark solid, but is easily converted, by heat, into a violet vapor. Fluorine is a gas. In chemical properties, these elements also resemble chlorine; but bromine is, in general, less active than chlorine, and iodine less than bromine. On the other hand, fluorine is more active than chlorine, its tendency to combine with other elements being so great that difficulty is encountered in preparing it in the free state. Its acid, *hydrofluoric acid*, *HF*, is of interest from the fact that it dissolves silica and glass.

SULPHUR

Symbol S. Atomic weight 32. Valence II, IV, and VI.

49. Occurrence.—Sulphur occurs free in nature, principally in the vicinity of active or extinct volcanoes. It is separated from the accompanying rock by fusion. Besides occurring in the free state, it is found in certain mineral springs in the form of hydrogen sulphide, and is otherwise

widely distributed in nature in combination with various metals, as sulphides and sulphates.

50. Properties.—Sulphur is capable of existing in three distinct *allotropic** forms or modifications. The most common form of sulphur, the one that is found in the natural state, and to which the others tend to change, is a lemon-yellow brittle solid, crystallizing in orthorhombic octahedrons, and having a specific gravity of 2.05; insoluble in water and soluble in carbon bisulphide. Another form is also crystalline; but the crystals are different from the first. Still a third variety is plastic, having a consistency like gum. Each variety melts at 115° C., becoming a pale-yellow limpid liquid; as the temperature is increased, it becomes dark in color and viscous, until, between 200° and 250° C., the vessel may be inverted without the sulphur running out of it. At about 350° C., it becomes a liquid again, which boils at 440° C. During cooling, these phenomena occur in reverse order.

The molecular weight of sulphur varies, being less at high temperatures than at low temperature. The molecular formula for sulphur vapor slightly above its boiling point is S_6, and, at temperatures above 860°, S_2. When heated to 260° C. in the air, sulphur takes fire and burns with a pale-blue flame and emits a suffocating odor. Sulphur readily combines with the metals, many of which take fire if thrown in its vapor. Sulphur combines with many of the non-metals.

In the arts, sulphur is extensively used in the manufacture of sulphuric acid and gunpowder, for bleaching vegetable and animal matter, and for other purposes.

51. Compounds.—Sulphur forms a very large number of compounds. **Hydrogen sulphide,** H_2S, is a colorless combustible gas having a very disagreeable odor, which resembles that of rotten eggs. **Sulphur dioxide,** SO_2, is a colorless incombustible gas, having the suffocating odor of

* Whenever an element occurs in two or more distinct forms, it is said to exhibit allotropy, and the less common varieties are said to be allotropic forms of that element. Oxygen exists in the allotropic form as ozone.

burning sulphur. **Sulphuric acid,** H_2SO_4, is a heavy, oily liquid, devoid of odor, and a very strong dibasic acid, that is, it contains two atoms of hydrogen capable of being replaced by a metal. It is an exceedingly important substance, being used extensively in the manufacture of other acids, alkalies, and other substances.

NITROGEN

Symbol N. Atomic weight 14. Valence III and V. Molecular weight 28.

52. Occurrence.—**Nitrogen** occurs in the free state in the air, of which it constitutes about four-fifths by bulk; it is found in native nitrates, combined with oxygen and metals, and is an essential constituent of many bodies of plant or animal origin.

53. Properties.—Nitrogen is a colorless, odorless, and tasteless gas. It differs remarkably in its properties from oxygen, with which it is associated in the air. While oxygen is specially characterized by its great chemical activity, nitrogen is one of the most inactive bodies, entering into direct combination with only a few elements. With oxygen itself, it only combines at exceedingly high temperatures. Nitrogen extinguishes burning bodies introduced into it. It is one of the gases most difficult to condense to a liquid, boiling at −194° C. It is very slightly soluble in water.

54. Air.—The **atmosphere** is mainly a mixture of nitrogen and oxygen, containing approximately 21 per cent. oxygen and 79 per cent. nitrogen by volume, or 23 per cent. oxygen and 77 per cent. nitrogen by weight. However, it does not consist wholly of these two elements, but always contains small amounts of carbon dioxide, CO_2, vapor of water, and argon. Exceedingly minute quantities of helium, neon, krypton, and xenon are also always present, and in particular localities, practically any other gas may be present. The presence of ammonia, hydrogen sulphide, sulphur dioxide, oxides of nitrogen, and ozone can generally be detected by careful examination.

55. Compounds of Nitrogen.—Notwithstanding the fact that the element nitrogen is exceedingly inactive chemically, some of its compounds are very active substances. *Ammonia*, NH_3, is a gas having a pungent irritating odor. It can be liquefied rather easily. Ammonia is very soluble in water, and it is this solution of the gas that is commercially well known as ammonia or *hartshorn*. The solution of ammonia acts like a base, the ammonia combining with the water to form *ammonium hydrate*.

$$\underset{\text{ammonia}}{NH_3} + \underset{\text{water}}{H_2O} = \underset{\text{ammonium hydrate}}{NH_4OH}$$

The group NH_4, or *ammonium*, is a compound radical, behaving as a metal.

Nitric acid, HNO_3, is a liquid with an acid odor; it is a powerful acid, and has a strong oxidizing action. It is sometimes known as *aqua fortis*.

PHOSPHORUS

Symbol P. Atomic weight 31. Valence III and V.

56. Occurrence.—Phosphorus never occurs free in nature. It is present in bones in the form of calcium phosphate, $Ca_3(PO_4)_2$. Several minerals also contain phosphates of different metals.

57. Properties.—Phosphorus exists in several allotropic modifications, the common form being a pale yellow, nearly transparent, wax-like solid that is very poisonous, easily inflammable, and that glows and emits a white smoke if exposed to the air. It burns in air if heated to 44° C., and therefore must be kept under water. Red phosphorus, another variety, is not a poison, and can be kept in the air. While the molecules of most elements consist of two atoms, the phosphorus molecule contains four atoms.

CARBON

Symbol C. Atomic weight 12. Valence II(?) and IV.

58. Occurrence.—**Carbon** occurs in nature in two crystalline modifications, known as *diamond* and *graphite*. More or less impure carbon is found in the various kinds of coal and, combined with hydrogen, in bitumen and petroleum. With oxygen and calcium, it forms limestone. In addition to these, carbon is an essential constituent of all compounds of organic origin. Few elements are capable of assuming so many aspects as carbon: the transparent, colorless, hard, crystalline diamond; the opaque, soft, metallic-looking graphite; the dull and porous wood charcoal—all these are carbon. It also occurs under other conditions as lampblack, anthracite, coke, etc.

59. Properties.—Carbon in the form of the *diamond* is crystalline, usually transparent, colorless, and has great brilliancy. Its specific gravity is about 3.5. It is a bad conductor of both heat and electricity, and is the hardest substance known.

The form of carbon known as *graphite* is a lead-colored solid with a metallic luster and a specific gravity of from 2 to 2.2; it crystallizes in a form different from the diamond. Next to the metals, it is one of the best conductors of heat and electricity, is soft, and is used in the manufacture of lead pencils; it is used also as a lubricant, as a facing material in foundry work, and for other purposes.

Various sorts of amorphous carbon are known as *charcoal*, *lampblack*, *coke*, etc. These are all devoid of crystalline structure, and are exceedingly useful substances. The power to conduct heat and electricity is generally greater the longer the carbon has been exposed to a high temperature.

All varieties of carbon are practically infusible and non-volatile, even at the highest temperatures, except that of an electric arc, in which the carbon seems to vaporize. The molecular weight of carbon is not known, but the molecules are probably composed of a large number of atoms. At

ordinary temperatures carbon is insoluble in all known solvents.

Carbon can unite directly with many elements; but in the case of all elements, except fluorine, it is necessary to bring the two together at high temperatures. The number of known compounds of carbon is far greater than of any other element. The elements most frequently entering into these compounds are hydrogen, oxygen, and nitrogen.

Though the temperature necessary to bring about the combination varies, being highest for graphite and diamond, when any of the three varieties of carbon burn in sufficient oxygen the product is *carbon dioxide*, CO_2, which is a colorless gas with a slightly pungent odor and taste. It is about $1\frac{1}{2}$ times as heavy as air, can be liquefied by cold and pressure, and neither burns nor supports combustion. Water dissolves about its own volume of carbon dioxide, and the resulting liquid contains *carbonic acid*, H_2CO_3. This acid, however, cannot be isolated in the pure state, as it readily breaks up into H_2O and CO_2; but its salts are of considerable importance. Since it contains two hydrogen atoms that can be replaced by metals, it forms normal carbonates as well as acid carbonates or *bicarbonates*.

Carbon monoxide, CO, is a colorless, odorless, poisonous gas. It is nearly as heavy as air, is hard to condense to a liquid, boiling at $-190°$ C., is very slightly soluble in water, and burns with a blue flame, forming carbon dioxide. It is a strong reducing agent. All ordinary forms of fuel consist of carbon alone, or of carbon with more or less hydrogen. On complete combustion, the products are carbon dioxide and (if hydrogen is in the fuel) water. When the combustion is incomplete, carbon monoxide may also be formed.

In consequence of its strong affinity for oxygen at high temperatures, carbon is an excellent reducing agent, and many metals are reduced from their oxides by means of one of the forms of carbon. In such operations, the carbon not only burns with the oxygen of the air to furnish the necessary heat, but part of it removes the oxygen from the oxide of the metal.

SILICON

Symbol Si. Atomic Weight 28. Valence IV.

60. Occurrence and Properties.—**Silicon** does not occur free in nature, but it is found most abundantly in combination with oxygen. In combination with oxygen, as well as with aluminum, potassium, calcium and other elements, it constitutes a large portion of all known rock formations.

There exists a considerable analogy between carbon and silicon. It is a solid occurring in two forms. *Crystalline silicon* forms black octahedral crystals, of a specific gravity 2.5, that will scratch glass; it is not oxidized on heating in the air, and it melts at about 1,500°. *Amorphous silicon* is a reddish-brown powder with a specific gravity of 2.35; it burns in the air, forming *silicon dioxide* SiO_2.

61. Silicon dioxide, better known under the name of **silica,** occurs widely distributed in nature. Its purest natural form is a transparent and colorless variety of quartz known as *rock crystal*, which is recognized by its great hardness, scratching glass almost as readily as the diamond. Sand, of which the white varieties are nearly pure silica, appears to have been formed by the disintegration of silicious matter, and has very often a red or yellow color, owing to the presence of iron oxide. Flint consists essentially of silica colored with various impurities. Silicon dioxide, in the form in which it is usually obtained, is a white amorphous powder. It has a specific gravity of 2.6, and is fusible only by means of the oxyhydrogen blowpipe.

THE METALS

62. As a rule, **metals** are distinguished physically by being opaque, by having a certain luster known as metallic luster, and by being good conductors of heat and electricity. Except mercury, which is a liquid at ordinary temperatures, all metals are solids. Chemically, the metals are capable of replacing the hydrogen of acids, forming salts.

Combinations of metals with each other are called **alloys**, while alloys of mercury are known as **amalgams**. Generally, alloys are not definite chemical compounds, but are merely mixtures of different metals; only occasionally is an alloy a true chemical compound. The properties of metals and alloys are sometimes modified to a remarkable extent by the presence of minute quantities of other elements.

There is a similarity between the compounds of the metals with the non-metals and the corresponding compounds of hydrogen with the same non-metals. In fact, most metallic compounds may be considered as being constructed on the model of a corresponding hydrogen compound. Thus, metallic oxides may be considered as water in which the two hydrogen atoms have been replaced by a metal; hydrates may be considered as water having one of its hydrogen atoms replaced by a metal. Metallic sulphides are derived from hydrogen sulphide, H_2S. From the definition of a salt, it is seen that salts must be considered as derived from the acids.

SODIUM

Symbol Na. Atomic weight 23. Valence I. Specific gravity .97.

63. Occurrence and Properties.—Sodium is abundantly and widely distributed in nature, but occurs only in combined forms. As sodium chloride, it is found not only in sea-water, but also in enormous deposits of rock salt; as sodium nitrate (Chile saltpeter) it occurs in South America. As sodium silicate, it is a constituent of many minerals and crystalline rocks. Traces of sodium salts are always found in the atmospheric dust.

Sodium is a shining, white, soft metal, melting at 95.6° C. and volatile at 742° C., forming a colorless vapor. It quickly oxidizes in the air, hence it is preserved in petroleum. It burns with a yellow flame.

Sodium forms a large number of compounds, the best known of which are: *sodium hydrate*, $NaOH$ (caustic soda),

sodium chloride, $NaCl$ (common salt), *sodium sulphate*, Na_2SO_4 (Glauber's salt), *sodium nitrate*, $NaNO_3$ (Chile saltpeter), *sodium carbonate*, Na_2CO_3, and *sodium bicarbonate*, $NaHCO_3$.

POTASSIUM

Symbol K. Atomic weight 39. Valence I. Specific gravity .86.

64. Occurrence and Properties.—Compounds of **potassium** occur in nature very extensively. Potassium exists principally in the silicates, especially feldspar and mica. The largest source of supply, however, is the double salts of potassium and magnesium, known as *carnallite*, $MgCl_2.KCl_2,6H_2O$, and *kainite*, $K_2SO_4.MgSO_4.MgCl_2,6H_2O$, found in the mines of Stassfurt, Germany.

Potassium has a silver-white luster, and is almost as soft as wax at ordinary temperatures. It melts at 62.5° C. and boils at about 720° C., turning then to a green vapor. It quickly oxidizes in moist air, its surface becoming covered with potassium hydrate. Like sodium, it has to be preserved under petroleum. It decomposes water, forming potassium hydrate and hydrogen; considerable heat being evolved, the hydrogen ignites and burns with a violet flame, due to the volatilized potassium.

CALCIUM

Symbol Ca. Atomic weight 40. Valence II. Specific gravity 1.58.

65. Occurrence, Properties, and Compounds.—**Calcium** does not occur free in nature. Calcium carbonate and calcium sulphate occur in very large deposits. Calcium silicate is a constituent of nearly all silicious minerals, and calcium phosphate occurs as apatite and phosphorite. Calcium, as a metal, in its free state, is of no commercial interest or value; but a number of its compounds are of great importance. *Calcium oxide*, CaO, is known as burnt lime and is used, on account of its high refractory power, in the construction of crucibles. *Calcium hydrate*, $Ca(OH)_2$,

or slaked lime, is used in the manufacture of cement and for building purposes. *Calcium carbonate*, $CaCO_3$, constitutes limestone and marble. It is insoluble in pure water, but water charged with carbon dioxide has a solvent action, the carbonic acid converting the calcium carbonate into calcium bicarbonate, $Ca(HCO_3)_2$. *Calcium sulphate*, $CaSO_4 + 2H_2O$, is soluble in 400 parts of water and loses part of its water on being heated to 120° C.; the product, $2CaSO_4 + H_2O$, is called burnt gypsum or plaster of Paris. If this is mixed into a paste with water, it develops heat and quickly hardens. If gypsum is heated to 160° C., it loses all its water and no longer combines with water, being then called dead burnt plaster.

MAGNESIUM

Symbol Mg. Atomic weight 24. Valence II. Specific gravity 1.75.

66. Occurrence and Properties.—Magnesium occurs as magnesium carbonate in magnetite and, mixed with calcium carbonate, in many impure limestones; it occurs also as a silicate in asbestos, talc, and meerschaum. The metal itself is white; it does not tarnish in dry air, but oxidizes readily in moist air. It burns with a dazzling white light, and on this account is of great value in flash-light photography. Like calcium carbonate, magnesium carbonate is insoluble in pure water, but is soluble in water charged with carbon dioxide. Magnesium sulphate is readily soluble in water. All soluble magnesium salts have a bitter taste.

ALUMINUM

Symbol Al. Atomic weight 27. Valence III. Specific gravity 2.583.

67. Occurrence.—Next to oxygen and silicon, **aluminum** is the most abundant element in nature. It never occurs in the free state, but exists as the oxide Al_2O_3 in

corundum and emery. Mica, clay, feldspar, and many other minerals contain aluminum silicate.

68. Properties.—Aluminum is a silvery white metal and is the lightest of the common metals; it is ductile and malleable, and fuses at about 657° C. It is permanent in the air, since it soon becomes coated with a thin layer of oxide which protects the metal from further attack. It is little affected by dilute nitric or sulphuric acids; but dilute hydrochloric acid dissolves it readily. It is also readily dissolved by strong alkalies, such as sodium hydrate.

Aluminum is a powerful reducing agent, reducing many oxides with a vigorous evolution of heat. It enters into some important alloys; for example, aluminum bronze, which is an alloy of copper and aluminum and is characterized by being very hard and of a golden color. *Magnalium* is an alloy with magnesium, and possesses properties similar to brass.

Aluminum hydrate, $Al(OH)_3$, is a white, jelly-like mass, formed by adding ammonium hydrate to a solution of an aluminum salt.

Aluminum hydrate and sulphuric acid yield *aluminum sulphate*, $Al_2(SO_4)_3$. It forms a characteristic class of double sulphates called **alums**, the general formula of which is $X_2SO_4 . Al_2(SO_4)_3, 24H_2O$, X being generally K or (NH_4).

IRON

Symbol Fe. Atomic weight 56. Valence II, III, IV, and VI. Specific gravity 7.8.

69. Occurrence.—**Iron** is the most important and one of the most abundant of metals. It is doubtful whether it occurs in the metallic state on the earth's surface, the native iron so found probably being almost entirely of meteoric origin. In combination, it is found in nearly all kinds of rock, clay, etc., its presence being generally indicated by the color of the rock, iron being the most common of natural mineral coloring ingredients. In most rocks, however, the amount of iron is very small, not enough to be

of any practical value. Iron ores are, however, rather abundant, the most important being the oxides Fe_2O_3 (hematite), Fe_3O_4 (magnetite), hydrated oxide $2Fe_2O_3,3H_2O$ (limonite), and a carbonate $FeCO_3$ (siderite). A sulphide of iron, FeS_2 (pyrite), contains a large percentage of iron; but it is not used as an ore on account of the injurious effects of the sulphur on the metal.

70. Properties.—Few metals have their properties modified to such an extent by very small quantities of other elements as iron. A large number of widely different kinds of iron, such as *cast iron*, *wrought iron*, and the various kinds of *steel*, are well known. While the causes of the wide variation in properties are not thoroughly understood, in the main these variations are due to the presence in the metal of varying quantities of other elements, the most important being carbon and silicon, though sulphur, phosphorus, manganese, tungsten, molybdenum, etc., are important, as well as some rarer elements not so universally present in commercial grades of iron or steel. The different varieties of commercial iron may be divided into three classes—wrought iron, cast iron, and steel. The first is nearly pure iron; the second contains generally from 2 to 6 per cent. of carbon, with varying quantities of silicon and other elements; in composition, the third lies between the other two.

Wrought iron has a white brilliant color; when melted in its pure state and then cooled, it crystallizes in cubes; but when forged or rolled out, its fracture becomes fibrous (depending, however, on the way it is broken), and the iron in this state is remarkably tough, though comparatively soft. It melts at about 1,600° C., but becomes a soft, wax-like mass at a considerably lower temperature, and is then capable of being welded. Cast iron melts at about 1,200° C., but does not soften before melting. Steel is more easily melted than wrought iron, but not as easily as cast iron. It can be hardened and tempered. A solid mass of iron does not tarnish in dry air, but if heated it oxidizes readily, with the production of black scales of oxide, Fe_3O_4; and when more strongly

heated, or when heated in pure oxygen, it burns, with the formation of the same scales. In pure water, iron does not lose its brilliancy, but if a trace of carbonic acid is present and air is permitted to reach it, the iron begins at once to oxidize or rust on its surface, forming a hydrated ferric oxide, Fe_2O_3,XH_2O. Iron decomposes steam at a red heat, liberating hydrogen and forming the same black oxide produced by its combustion in oxygen.

Among the important compounds of iron may be mentioned ferric oxide, Fe_2O_3, which is not only an important ore of iron, but is also of considerable importance as a mineral paint.

NICKEL

Symbol Ni. Atomic weight 58.5. Valence II and IV. Specific gravity 8.8.

71. Occurrence and Properties.—Nickel does not occur free in nature, except in small quantities in some meteorites, which are alloys of iron and nickel. Native compounds of nickel are also not very abundant. Among the important minerals containing nickel are *garnierite*, a complex silicate of nickel and magnesium, and *niccolite* (copper nickel, or kupfernickel), $NiAs$.

Nickel is almost as white as silver, is very tough, and has a high metallic luster. It dissolves sparingly in hydrochloric and sulphuric acids, but freely in nitric acid. It is permanent in the air, though slight tarnishing takes place. It is employed in nickel-plating metallic objects, and is a constituent of several alloys.

German silver contains about 50 per cent. copper, 25 per cent. zinc, and 25 per cent. nickel. Nickel coins contain 75 per cent. copper and 25 per cent. nickel. A small amount of nickel added to steel greatly increases its toughness, and much nickel is used for this purpose, especially in the manufacture of armor plate.

MANGANESE

Symbol Mn. Atomic weight 55. Valence II, III, IV, VI, and VII(?). Specific gravity 7.5.

72. Occurrence and Properties.—Manganese is found chiefly as *pyrolusite*, MnO_2, *braunite*, Mn_2O_3, and *rhodochrosite*, $MnCO_3$. Manganese sulphide, arsenide, and silicate are also known as minerals. The metal itself has not been applied to any very useful purpose in the arts, but forms some useful alloys.

Manganese is best prepared by reducing MnO_2 by means of aluminum powder. It is a grayish-white, hard, brittle metal. It is feebly magnetic, difficult to fuse (1,900° C.), and oxidizes readily in the air. It dissolves easily in dilute hydrochloric and sulphuric acids, Mn displacing H_2. It resembles iron in its tendency to combine with carbon at a high temperature to form a compound corresponding with cast iron; in this form the manganese is not oxidized by air.

Spiegeleisen and *ferromanganese* are alloys containing iron, manganese, and carbon, and are largely used in the production of steel.

73. Manganese dioxide, or peroxide, MnO_2, is the chief compound in which manganese is found in nature; it is also the source from which practically all manganese compounds are obtained. Its chief mineral form is pyrolusite, which forms steel-gray prismatic crystals of specific gravity 4.9; but it is also found amorphous, as *psilomelane*, and in the hydrated state as *wad*. In commerce, pyrolusite is known as black manganese and is largely imported, as well as mined, in the United States of America, for the use of the steelmaker, the manufacturer of bleaching powder, and the glassmaker. It is also used as a source of oxygen, which it evolves when heated to redness without fusing, leaving the red *oxide of manganese*, Mn_3O_4. Manganese dioxide is an indifferent oxide and does not readily combine with acids. Cold HCl, however, dissolves it, giving a brown

solution from which water precipitates a brown oxychloride. If the brown solution, which probably contains $MnCl_4$, be heated, it evolves chlorine, and becomes pink $MnCl_2$. Nitric acid is almost without action on MnO_2. Strong sulphuric acid evolves oxygen from it:

$$MnO_2 + H_2SO_4 = MnSO_4 + H_2O + O$$

Even dilute sulphuric acid produces the same change if some substance ready to combine with oxygen is added, as, for instance, ferrous sulphate or oxalic acid. Hence, a mixture of MnO_2 and H_2SO_4 is frequently used as an oxidizing agent.

When heated in hydrogen, the oxides of manganese are not reduced to the metals, like those of iron, but are converted into MnO. *Manganous oxide*, MnO, is a greenish powder, but it may also be obtained in transparent, emerald-green crystals. It easily absorbs oxygen from the air. It is a basic oxide, dissolving in acids to form the manganous salts, and has been found in nature in manganiferous dolomite.

CHROMIUM

Symbol Cr. Atomic weight 52. Valence II, III, and VI. Specific gravity 6.8.

74. Occurrence and Properties.—Chromium does not occur native, but is found largely as chrome iron ore, having the formula $Fe(CrO_2)_2$, and occasionally as lead chromate, $PbCrO_4$, in the mineral crocoisite. Chromium is a light-gray, or tin-white, lustrous, crystalline substance. It is very hard and difficult to fuse, and oxidizes very slowly when heated in the air, but heated in oxygen, or in the oxyhydrogen flame, it burns to chromic oxide, Cr_2O_3. It is insoluable in nitric acid, but dissolves in hydrochloric and hot sulphuric acids. Bivalent chromium forms cnromous compounds, trivalent chromium forms the chromic compounds, hexavalent chromium forms chromium trioxide and the chromates. Alloys of chromium with iron are used as chrome steel.

MOLYBDENUM

Symbol Mo. Atomic weight 96. Valence II, III, IV, V, and VI (?). Specific gravity 9.

75. Occurrence and Properties.—**Molybdenum** occurs principally as sulphide, or *molybdenite*, and as lead molybdate, or *wulfenite*. It is prepared by reducing the oxides or chlorides of molybdenum by a current of hydrogen at a high temperature. It is a white, very hard, brittle metal, and is almost infusible. It oxidizes, when heated in the air, to molybdenum trioxide. It forms four oxides: MoO, Mo_2O_3, MoO_2, and MoO_3.

TUNGSTEN

Symbol W. Atomic weight 184. Valence II, IV, V, and VI (?). Specific gravity 16.6.

76. Occurrence and Preparation.—**Tungsten** occurs in nature in a number of minerals. Its symbol, W, is derived from the mineral *wolframite*, which is tungstate of iron and manganese, and from which it was first obtained; it occurs in *scheelite*, which is calcium tungstate; in *stolzite*, which is lead tungstate; etc. Metallic tungsten is obtained by reducing tungsten trioxide, WO_3, with hydrogen or aluminum powder as an iron-gray metal of specific gravity 16.6 to 19.129. It is extremely hard, very infusible, and unaffected by either hydrochloric or sulphuric acid, though it is converted into tungstic acid by the action of nitric acid. When dissolved in about 10 times its own weight of fused steel, tungsten forms an extremely hard alloy, known as *tungsten steel*, that is used for lathe, planer, and other machine-shop tools. One of the most peculiar properties of tungsten is its ability to increase the magnetic power of the steel with which it is alloyed. A horseshoe magnet of ordinary steel, weighing 2 pounds, for instance, is generally considered of very good quality if it lifts 7 times its own weight, but a similar magnet made of an alloy of steel and tungsten is able to lift nearly 20 times its own weight.

VANADIUM

Symbol V. Atomic weight 51. Valence III and V. Specific gravity 5.5.

77. Occurrence and Preparation.—Vanadium occurs in nature in the form of vanadates, which are analogous to phosphates. It has been found in clay, coal, and some rare minerals. The method employed to obtain it depends on the composition of the compound treated. By treating lead vanadate with nitric acid, evaporating to expel the excess of acid, and dissolving out the lead nitrate with water, V_2O_5 remains. This may be purified by dissolving in ammonia, crystallizing the ammonium metavanadate formed, and decomposing this by heat, when vanadium pentoxide V_2O_5, is left as a reddish-yellow solid. This is sparingly soluble in water, but dissolves readily in hydrochloric acid. By heating the chloride in hydrogen, vanadium is obtained as a silver-white element with metallic luster, having a specific gravity of 5.5. It is not oxidized by the air at ordinary temperature, but burns when strongly heated in air. It is practically insoluble in hydrochloric acid, but is dissolved by nitric acid.

COPPER

Symbol Cu. Atomic weight 63. Valence II. Specific gravity 8.9.

78. Occurrence and Properties.—Copper has been used by man from the earliest times. It is sometimes found native, immense masses of the metal being found in the region around Lake Superior. Copper also occurs rather abundantly as carbonate and oxide; but its most important source is the sulphide, being in this form usually associated with sulphides of iron and other metals.

Copper is a lustrous metal, flesh red in color, and somewhat softer than iron. When rubbed, it exhales a peculiar odor. It is one of the best conductors of both heat and electricity, is malleable, ductile, and possesses considerable

tenacity. It melts at 1,065° C., and is slightly volatile at a white heat. In dry air, at ordinary temperatures, it is unaltered; but in the presence of moisture and carbon dioxide, it becomes covered with a green layer of basic carbonate of copper called *verdigris*. Heated to redness in the air, scales of oxide form on its surface. It dissolves readily in nitric acid. Weak acids, alkalies, and salt solutions act on it slowly; hence, as all its salts are poisonous, copper vessels should not be used in cooking.

Copper is an exceedingly useful metal. In the pure state, it is very largely used as a conductor of the electric current. Many of the most useful alloys contain copper, brass being an alloy of copper and zinc, and bronze an alloy of copper and tin.

MERCURY

Symbol Hg. Atomic weight 200. Valence I and II. Specific gravity 13.5.

79. Occurrence.—Mercury has been known from the earliest times; its Latin name *argentum vivum*, of which the English name *quicksilver* is a literal translation, refers to its fluidity as well as to its color. The name mercury, from the planet of that name, was given by the alchemist to all volatile substances, but only this one has retained it. The symbol of mercury, *Hg*, is derived from the Latin name *hydrargyrum*. It occurs native only sparingly, the chief ore being the sulphide, called *cinnabar*, which is found principally in Idria, Austria; Almaden, Spain; and New Almaden, California.

80. Properties.—Mercury is a brilliant, silver-white metal. It has a specific gravity of 13.59 at 0° C. and is the only metal that is liquid at the ordinary temperatures. Cooled to – 39.5° C., it solidifies to a malleable, tin-white mass, can easily be cut, and crystallizes in regular octahedrons. Even at ordinary temperatures mercury has a small vapor tension, and gives off minute quantities of its vapor into the atmosphere in contact with it. It boils at about 357° C.,

yielding a colorless vapor of specific gravity 6.976. When heated almost to its boiling point in air, it becomes coated with the red oxide HgO. It is unaltered in air; neither hydrochloric nor dilute sulphuric acid attacks it, but strong, boiling sulphuric acid, and even dilute nitric acid, dissolve it readily. Chlorine and sulphur unite directly with it.

Mercury is used in the arts for filling thermometers and barometers, and very extensively for extracting gold and silver from their ores. With most of the metals it forms alloys, known as *amalgams;* in some cases these amalgams possess a definite composition and crystalline form, for example, Hg_6Na, which is brittle and crystallizes in regular forms. Sodium amalgam is a convenient reducing agent, for when brought into contact with water or with solutions in water, hydrogen is evolved. Tin amalgam is sometimes used for producing the silver coating on glass for mirrors.

Mercury forms two series of compounds, the mercurous, in which this metal is univalent, and the mercuric, in which it is bivalent.

ZINC

Symbol Zn. Atomic weight 65. Valence II. Specific gravity 7.

81. Occurrence and Properties.—Zinc is not found native. The chief ore is the sulphide, ZnS, though considerable carbonate, oxide, and silicate is mined and reduced.

Zinc is a bluish-white metal of crystalline structure. It is hard and brittle at ordinary temperatures, but between 100° C. and 150° C. it becomes malleable and ductile, and may be rolled into sheets, which preserve to some extent their malleability on cooling. At 200° C., it is again hard and brittle. At 419° C., it melts; and at 1,040° C., it boils, or, if air is present, takes fire and burns with a luminous, greenish flame, forming zinc oxide. It oxidizes readily when exposed to moist air, and is easily attacked by acids.

Zinc is not only used as sheet zinc, but is used in some alloys, as, for example, brass. It is extensively used also in

making the so-called *galvanized iron*, which is iron covered with a coating of zinc by being dipped in melted zinc. Zinc oxide, ZnO, is used as a white paint, and zinc chloride, $ZnCl_2$, is used as a soldering flux.

LEAD

Symbol Pb. Atomic weight 207. Valence II and IV. Specific gravity 11.36.

82. Occurrence and Properties.—Lead was one of the metals known to the ancients. It does not occur native. The principal ore is the sulphide or *galena*, PbS; it also occurs as carbonate, as sulphate, and in other compounds.

Lead is a soft, bluish-white, brilliant metal; it is malleable and ductile, but possesses little tenacity. It melts at 335° C, and is slightly volatile at higher temperatures. A freshly-cut surface of lead tarnishes in ordinary air, but remains bright in perfectly dry air, and also in water entirely free from air. When melted in the air, it rapidly absorbs oxygen and becomes covered with a film of oxide, which, by the continuous action of heat and air, is transformed into a yellow powder, known as *litharge* or *massicot*. Ordinary water, in general, acts on lead, dissolving it and partly precipitating it as carbonate; some of the lead, however, remains in solution. This action is particularly noticeable in well water that contains nitrates or chlorides. Lead water pipes should therefore be used with great caution. Lead is only slightly attacked by sulphuric or hydrochloric acids at ordinary temperatures, but is readily attacked by nitric acid. In the presence of air and moisture, it is acted on by very feeble acids, such as acetic and carbonic acids.

Lead, when taken into the system, unites with certain tissues and is retained there, until finally sufficient of the metal accumulates to produce poisoning, of which acute colic and paralysis of the muscles (especially of the arms) are characteristics. Lead forms several oxides, one of which, red lead, Pb_3O_4, is used as a paint. A basic carbonate of lead constitutes *white lead*, an exceedingly useful paint.

TIN

Symbol Sn. Atomic weight 119. Valence II and IV. Specific gravity 7.3.

83. Occurrence and Properties.—Metallic tin is not found in nature. Tin ore occurs in only a few localities. The most important ore is the oxide SnO_2. Tin is a white metal, resembling silver; it is soft, malleable, and ductile, but possesses little tenacity. It crackles when a bar is bent, producing what is known as *the cry of tin.* It is easily fusible, melting at about 230° C. Tin does not lose its luster on being exposed to air at ordinary temperature; but if strongly heated it takes fire, forming a white powder of stannic oxide, SnO_2. It is attacked by acids. Tin is extensively used for plating iron. What is commonly known as tin is really sheet iron plated with that metal. It is a constituent of some important alloys, bronze being an alloy of tin and copper, and ordinary solder being an alloy of tin and lead.

ANTIMONY

Symbol Sb. Atomic weight 120. Valence III and V. Specific gravity 6.7.

84. Occurrence and Properties.—Antimony occurs native, but more commonly as the sulphide Sb_2S_3. It is a bluish-white, brittle body. It melts at 450° C., and at a white heat may be distilled. The brittleness of antimony renders it useless in its natural state; but it is a constitutent of a number of useful alloys. Type metal is an alloy of lead and antimony. Many alloys used for bearings in machinery contain antimony.

BISMUTH

Symbol Bi. Atomic weight 208. Valence III and V. Specific gravity 9.8.

85. Occurrence and Properties.—Bismuth is found in the metallic state and also as oxide, sulphide, carbonate,

etc. The metallic bismuth of commerce is seldom pure, but mostly contains arsenic, iron, and traces of other metals. Bismuth is a hard brittle metal. It is white, but has a slight reddish tinge. It melts at 264° C., and on solidifying expands about one thirty-second of its bulk. It remains unaltered in dry air, but tarnishes in moist air. At a red heat, it burns with a bluish-white flame, forming bismuth oxide. Bismuth is not used alone, but is a constitutent of some very easily fused alloys and of some bearing alloys.

HEAT

(PART 1)

MANIFESTATIONS AND MEASUREMENT OF HEAT

1. The Nature of Heat.—The sensations of warmth and cold are familiar to every one. If the hand is placed in water, a sensation is produced; and, according to the sensation, the water is pronounced cold, lukewarm, or hot. It is customary to ascribe the cause of the differences between these states of the water to something called **heat**; thus, when the water gives the sensation of warmth, it is said to have been heated, or to have had heat added to it; when it gives the opposite sensation, it is said to have been cooled, or to have had heat abstracted from it.

Regarding the nature of heat, there have been two theories. The older theory, now discarded, assumed that it was a substance, a sort of fluid without weight, which filled the spaces between the particles of a body, and that a body was hotter or colder according as it had more or less of the fluid stored in it.

The modern theory is that heat is a result of the rapid vibration of the molecules of a body. The application of heat to a body causes a more rapid vibration, while the withdrawal of heat causes a less rapid vibration; and it is to the rate of vibration that the sensation of hotness or coldness is due. As will be shown later, heat is a form of energy. For the present, it is sufficient to consider heat merely as something that can be recognized and measured.

2. Sensible Heat.—The heat that manifests itself to the senses is called **sensible heat,** because any change from a given state to a hotter or colder state is indicated at once by the sense of feeling or by the aid of an instrument called a **thermometer.** The more sensible heat a body possesses, the hotter it is; the more sensible heat that is taken away from it, the colder it is.

3. Temperature.—The term **temperature** is used to indicate how hot or cold a body is. When heat is added to a body and it becomes hotter, its temperature is said to rise; when it cools, its temperature is said to fall. According to the modern theory, the temperature of a body is a measure of the speed with which the molecules composing it are vibrating. A rise in temperature indicates an increase of molecular speed, and a fall in temperature indicates a decrease of molecular speed.

MEASUREMENT OF TEMPERATURE

4. Thermometers.—Owing to the imperfection of the senses, it is impossible to determine by their aid, with any degree of accuracy, the temperatures of different bodies; hence, for this purpose, the **thermometer** is used. In these instruments the effects of heat on bodies are made use of in obtaining the temperature, the most common method being to utilize the expansive effect of heat on liquids. Liquids are used for ordinary purposes instead of solids or gases, because in the first the expansion is too small, and in the second it is too great. Mercury and alcohol are the only liquids used—the former because it boils only at a very high temperature, and the latter because it does not solidify at ordinary temperatures.

5. Mercury Thermometers.—In Fig. 1 is shown a mercurial thermometer with two sets of graduations on it. The one on the left, marked *F*, is the **Fahrenheit graduation,** named after its orignator, and is the one commonly used in the United States and England; the one on the right

marked *C*, is the **centigrade graduation**, generally used by scientists throughout the world because its graduations are better adapted for calculations. As will be seen, the instrument consists of a closed glass tube terminating in a bulb at the lower end. Before closing the upper end, the tube is partially filled with mercury, and the air above it is driven out by heating the mercury to near its boiling point. When the tube above the mercury is filled with mercurial vapor, it is sealed; on cooling, the vapor condenses and a vacuum results. The expansion or contraction of the mercury on the application or withdrawal of heat from the body with which the bulb is in contact causes the highest point of the mercury column to rise and fall, and since for equal changes of temperature the mercury rises or falls equal distances, this instrument, when properly made and graduated, indicates any change in temperature with great accuracy.

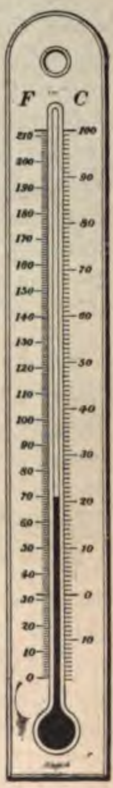


Fig. 1

6. Graduation of Thermometers.—The inside diameter of a good thermometer tube should be the same throughout its length. The graduation of the thermometer is accomplished as follows: It is first placed in melting ice and the point to which the mercury column falls is marked **freezing**. It is then placed in the steam rising from water boiling in an open vessel, and the point to which the mercury column rises is marked **boiling**.

There are now two fixed points—the freezing point and the boiling point. If it is desired to make a Fahrenheit thermometer, the distance between these two fixed points is divided into 180 equal parts, called degrees. The freezing point is marked 32°, and the boiling point 212°. Thirty-two parts are marked off from the freezing point downwards, and the last one is marked 0°, or zero. The graduations are carried above the boiling point and below the zero point as far as desired.

This thermometer was invented in 1714, and was the first to come into general use.

7. In graduating a centigrade thermometer, the freezing point is marked 0°, or zero, and the boiling point 100°; the distance between the freezing and boiling points is divided into 100 equal parts; these equal divisions are carried as far below the freezing point and above the boiling point as desired. The reason that Fahrenheit placed the zero point on his thermometer 32° below freezing was because that temperature was the lowest he could obtain, and he supposed that it was impossible to obtain a lower one. Where there can be any doubt as to the thermometer used, the first letter of the name is placed after the degree of temperature. For example, 183° F. means 183° above zero on the Fahrenheit instrument; 183° C. means 183° above zero on the centigrade instrument.

8. In Russia and a few other countries, another instrument is used, called the **Réaumur** thermometer; the freezing point is marked 0°, or zero, and the boiling point 80°, the space between these two points being divided into 80 equal parts; 183° R. would mean 183° on the Réaumur thermometer.

9. Of these three thermometers, the centigrade is used the most; but, since the Fahrenheit instrument is the one in general use in the United States, all temperatures given here will be understood to be in Fahrenheit degrees, unless otherwise stated. In order to distinguish the temperatures below the zero point from those above, the sign of subtraction is placed before the figures indicating the number of degrees below zero. Thus, −18° C. means a temperature of 18° below the zero point on the centigrade thermometer; −25.4° F. means 25.4° below zero on the Fahrenheit thermometer.

10. Absolute Temperature.—As was stated in *Pneumatics*, absolute zero, or −460° F., is the temperature at which all vibratory motion of the molecules ceases. It is

supposed that, at this temperature, and under a heavy pressure, so that the molecules would be brought close enough together, even hydrogen would solidify. The absolute zero on the centigrade scale is $-273\frac{1}{3}°$ C.

The **absolute temperature** is the temperature measured above the point of absolute zero. Hence, on the Fahrenheit scale, the absolute temperature T is $460° + t°$ when $t =$ the ordinary temperature, and is above zero. If $t°$ is below zero, its value is negative, and the absolute temperature T is $460° + (-t°) = 460° - t°$.

Throughout the following pages, where temperatures are mentioned, t will denote the ordinary temperature indicated by the thermometer, and T the absolute temperature.

EXAMPLE.—What are the absolute temperatures corresponding to 212°, 32°, and −39.2°?

SOLUTION.—Since no scale is specified, the Fahrenheit is the one intended to be used.

$$T = 460° + 212° = 672°. \text{ Ans.}$$
$$T = 460° + 32° = 492°. \text{ Ans.}$$
$$T = 460° - 39.2° = 420.8°. \text{ Ans.}$$

The absolute temperature on the centigrade scale is $T = 273\frac{1}{3}° + t°$ when $t°$ is above zero, or $T = 273\frac{1}{3} - t°$ when $t°$ is below zero.

EXAMPLE.—What are the absolute temperatures corresponding to 100°, 4°, and −40° C.?

SOLUTION.— $T = 273\frac{1}{3}° + 100° = 373\frac{1}{3}°$ C. Ans.
$T = 273\frac{1}{3}° + 4° = 277\frac{1}{3}°$ C. Ans.
$T = 273\frac{1}{3}° - 40° = 233\frac{1}{3}°$ C. Ans.

11. Conversion of Centigrade and Fahrenheit Temperatures.—It is frequently necessary to change from the centigrade scale to the Fahrenheit scale, or the reverse. Since the number of degrees between the freezing point and boiling point on the centigrade scale is 100 and on the Fahrenheit 180, it is evident that if F equals the number of degrees Fahrenheit, and C equals the number of degrees centigrade,

$$F : C = 180 : 100, \text{ or } F = \tfrac{180}{100} C = \tfrac{9}{5} C$$

And,

$$C = \tfrac{100}{180} F = \tfrac{5}{9} F$$

That is, the number of Fahrenheit degrees above freezing is $\frac{9}{5}$ of the number of centigrade degrees above the same point. But since, on the Fahrenheit scale, the freezing point is 32° above zero, 32 must be added to the number of degrees above the freezing point.

Let t_c = centigrade temperature;

and t_f = Fahrenheit temperature.

Then, $$t_f = \tfrac{9}{5}\,t_c + 32 \qquad (1)$$

and $$t_c = \tfrac{5}{9}\,(t_f - 32) \qquad (2)$$

Formulas **1** and **2** may be expressed as follows:

Multiply the temperature centigrade by $\frac{9}{5}$, and add 32°; the result will be the temperature Fahrenheit.

Subtract 32° from the temperature Fahrenheit, and multiply by $\frac{5}{9}$; the result will be the temperature centigrade.

EXAMPLE 1.—Change: (*a*) 100° C., (*b*) 4° C., and (*c*) −40° C. into Fahrenheit temperatures.

SOLUTION.—

(*a*) $t_f = \frac{9}{5}t_c + 32 = \frac{9}{5} \times 100 + 32 = 212°$ F. Ans.

(*b*) $t_f = \frac{9}{5} \times 4 + 32 = 39.2°$ F. Ans.

(*c*) $t_f = \frac{9}{5} \times -40 + 32 = -40°$ F. Ans.

EXAMPLE 2.—Change: (*a*) 60° F., (*b*) 32° F., and (*c*) −20° F. into their corresponding centigrade temperatures.

SOLUTION.—

(*a*) $t_c = \frac{5}{9}(t_f - 32) = \frac{5}{9}(60 - 32) = 15\frac{5}{9}°$ C. Ans.

(*b*) $t_c = \frac{5}{9}(32 - 32) = 0°$ C. Ans.

(*c*) $t_c = \frac{5}{9}(-20 - 32) = -28\frac{8}{9}°$ C. Ans.

12. Alcohol Thermometers.—Since mercury freezes at −39° F., which corresponds to about −39.5° C., it cannot be used to obtain temperatures below this point. For this purpose alcohol is used instead of mercury. This liquid has not been frozen until very recently, and then only at the extremely low temperature of −202.9° F. Since alcohol vaporizes at 173° F., the boiling point of water cannot be marked on the alcohol thermometer by heating it to that point. The freezing point is determined as for mercury. An alcohol and a mercurial thermometer are placed in a vessel containing hot water or other liquid, and the point to which the alcohol column rises is marked. Suppose that the

point to which the mercury column rises is marked 132°, then the distance on the alcohol thermometer between the point marked and the freezing point would be divided into $132 - 32 = 100$ equal parts, and each one of these parts would correspond to one degree on the mercurial thermometer. These equal divisions are then carried below the zero point as far as desired.

There are many other kinds of thermometers, some of which depend on the expansion and contraction of different metals and gases when heated and cooled. For temperatures above 675° F., the point at which mercury vaporizes, other means are employed to obtain the temperatures.

EXPANSION OF BODIES BY HEAT

13. The volume of any body—solid, liquid, or gaseous—is always changed if the temperature is changed, other conditions remaining the same; nearly all bodies expand when heated and contract when cooled. In solids, expansion may be considered in three ways, according to the conditions: (1) Expansion in one direction, as the elongation of an iron bar; this is called **linear expansion**; (2) **surface expansion**, which refers to an increase in area; (3) **cubic expansion**, which refers to a general increase in the whole volume.

14. In Fig. 2 is shown an apparatus for exhibiting the linear expansion of a solid body. A metal rod *a* is fixed at one end by a screw *b*, the other end passing freely through the eye *c*, held in the post, and pressing against the short arm of the indicator *f*. The rod is heated in the way shown, and its elongation causes the indicator to move along the arc *de*.

An illustration of surface expansion is afforded in machine shops, particularly in locomotive shops, where piston rods, crankpins, etc. are shrunk in and tires are shrunk on their centers. In shrinking on a tire, it is bored a little smaller than the wheel center and is then heated until it is expanded

enough to go over the wheel center. It is then cooled with cold water, when it contracts, tending to regain its original size, but is prevented by the wheel center, which is a trifle larger. The tire is thus caused to bind around the center

FIG. 2

with great force, and the excessive friction between the tire and the center prevents them from separating.

Cubic expansion may be illustrated by means of a *Gravesandes' ring*. This consists of a brass ball *a*, Fig. 3, which at ordinary temperatures passes freely through the ring *m*, of very nearly the same diameter. When the ball is heated, it expands so much that it will no longer pass through the ring.

FIG. 3

The bars of a furnace must not be fitted tightly at their extremities, but must be free at one end; otherwise, in expanding, they will crack the masonry. In laying the rails on railways, a small space is left between the successive rails; for, if they touched, the force developed by the

expansion would cause them to curve. Long straight lines of steam piping must be fitted with expansion joints. If a glass vessel is heated or cooled too rapidly, it cracks, especially if it is thick; the reason for this is that, since glass is a poor conductor of heat, the sides become unequally heated and, consequently, unequally expanded. The expansion of liquids is shown in the mercurial and alcohol thermometers; the expansion of gases was discussed to some extent in *Pneumatics*.

15. Coefficient of Expansion.—Suppose that the temperature of the metal rod, shown in Fig. 2, was 32° F. before heating, and that the rod was exactly 10 feet long; that after

TABLE I

COEFFICIENTS OF EXPANSION

Name of Substance	Linear Expansion C_1	Surface Expansion C_2	Cubic Expansion C_3
Cast iron	.00000617	.00001234	.00001850
Copper	.00000955	.00001910	.00002864
Brass	.00001037	.00002074	.00003112
Silver	.00001060	.00002120	.00003180
Wrought iron	.00000686	.00001372	.00002058
Steel (untempered) . . .	.00000599	.00001198	.00001798
Steel (tempered)	.00000702	.00001404	.00002106
Zinc	.00001634	.00003268	.00004903
Tin	.00001230	.00002460	.00003690
Mercury	.00003334	.00006668	.00010010
Alcohol	.00019259	.00038518	.00057778
Gases			.00203252

the temperature had been raised 1°, or to 33°, the bar was 10 feet + $\frac{1}{1200}$ inch long. The linear expansion then was (10 feet + $\frac{1}{1200}$ inch) − 10 feet = $\frac{1}{1200}$ inch, and the ratio of this expansion to the original length of the bar was $\frac{1}{1200} : 10 \times 12 = \frac{1}{1200 \times 120} : 1 = .000006944$.

For every increase of temperature of 1°, the rod became longer by .000006944 of its length. This number .000006944,

which is equal to the expansion of the rod for 1° rise of temperature divided by the original length, is called the **coefficient of linear expansion.** If the temperature of the rod were increased 100° instead of 1°, the amount of elongation would be $.000006944 \times 100 = .0006944$ of its length, or $.0006944 \times 120 = .083328$ inch, or $\frac{1}{12}$ inch. Table I contains the coefficients of expansion, per degree Fahrenheit, for a number of solids, mercury, and alcohol, and the average cubic expansion of gases. No liquids are given, except mercury and alcohol, for the reason that the coefficient of expansion of a liquid is different at different temperatures.

16. Formulas for Expansion.—

Let L = length of any body;

l = amount of expansion or contraction due to heating or cooling the body;

A = area of any section of the body;

a = increase or decrease of area of the same section after heating or cooling the body;

V = volume of the body;

v = increase or decrease in volume due to heating or cooling the body;

C_1, C_2, and C_3 = coefficients taken from Table I;

t = difference of temperature between original temperature and temperature of body after it has been heated or cooled.

Then,

$$l = L C_1 t \qquad (1)$$

$$a = A C_2 t \qquad (2)$$

$$v = V C_3 t \qquad (3)$$

EXAMPLE 1.—How much will a bar of untempered steel, 14 feet long, expand if its temperature is raised 80°?

SOLUTION.—Since only one dimension is given, that of length, linear expansion only can be considered. From Table I the coefficient of linear expansion per unit of length for a rise in temperature of 1° is found to be .00000599 for untempered steel. Hence, using formula **1**, $l = L C_1 t = 14 \times .00000599 \times 80 = .0067088$ ft., or $.0067088 \times 12 = .0805056$ in. Ans.

This seems a very small amount, but in engineering constructions, where long pieces are rigidly connected, it must be taken into account. If the cross-section of the bar were 2 inches square, and the bar were fitted tightly between two supports, an expansion of the above amount would exert a pressure against the supports of about 58,000 pounds.

EXAMPLE 2.—An iron rod $1\frac{1}{2}$ inches in diameter and 100 feet long is used as a tie-rod in constructing a bridge. It was put in place and securely fastened to two rigid supports during a warm day in summer when the temperature in the sunlight was, say 110°. On a cold day in winter, when the thermometer registers zero, how much will the bar tend to shorten, owing to this change in temperature?

SOLUTION.—From formula **1**,

$$l = .00000686 \times 100 \times 110 = .07546 \text{ ft.} = .90552 \text{ in. Ans.}$$

If this rod were rigidly secured, so that it could neither stretch nor shorten, it would exert a pull on the supports estimated at about 33,400 lb.

EXAMPLE 3.—The wheel center of a locomotive driver is turned to exactly 50 inches in diameter. If the steel tire is bored 49.94 inches in diameter, to what temperature must the tire be raised in order that it may be easily placed over the center? Assume that the diameter of the tire is increased until it is $\frac{1}{1000}$ inch larger than the center, and that the original temperature is 60°.

SOLUTION.—For this case, formula **2** may be used. The original diameter of the tire is 49.94 in., and it is to be increased to 50.001 in. The area of a circle 49.94 in. in diameter is 1,958.79 sq. in.; the area of a circle 50.001 in. in diameter is 1,963.58 sq. in. The difference between the two areas is 1,963.58 − 1,958.79 = 4.79 sq. in. = a in formula **2**. Hence, since C_2 = .00001198, and A = 1,958.79, substitute these values in $a = A\,C_2\,t$, and $4.79 = 1{,}958.79 \times .00001198 \times t = .023466\,t$. Therefore,

$$t = \frac{4.79}{.023466} = 204.13°, \text{ and } 204.13° + 60° = 264.13°. \quad \text{Ans.}$$

NOTE.—Owing to the form of the equation here denoted by formula **2**, and to the manner in which the coefficients C_2 were determined, this example may be more easily solved by means of formula **1**. Thus, regard the diameter as a linear dimension and apply formula **1**. Increase in diameter is $l = 50.001 - 49.94 = .061$ in. $L = 49.94$ and $C_1 = .00000599$. Substituting in $l = L\,C_1\,t$, $.061 = 49.94 \times .00000599 \times t$, or $t = \frac{.061}{49.94 \times .00000599} = 203.92°$, and $203.92° + 60° = 263.92°$. Ans. The slight difference in the two results is immaterial, and was to have been expected.

EXAMPLE 4.—What is the decrease in volume of a copper cylinder 30 inches long and 22 inches in diameter if cooled from 212° to 0°?

SOLUTION.—The volume is $V = 22^2 \times .7854 \times 30 = 11{,}404$ cu. in. Apply formula **3**, $v = V\,C_3\,t$. By substituting,

$$v = 11{,}404 \times .00002864 \times 212 = 69.24 \text{ cu. in.} \quad \text{Ans.}$$

17. It will be found, on trial, that the three preceding formulas for calculating expansion will not work backwards; that is, if the length of a bar, after it has been heated, be found by formula **1**, Art. **16**, and an attempt be made to reduce the bar to its original length by again applying the same formula and substituting for t the same value as in the first case, the value obtained for l will be slightly different in the two cases. The difference, however, is so slight that it is neglected in practice. If, however, it is desired to obtain exactly the same result in both cases, the following more cumbersome formula must be used, in which t_1, t_2, l_1, and l_2 are, respectively, the original and final temperatures and the original and final lengths, and C_1 has the same value as in formula **1**, Art. **16**:

$$l_2 = \left[\frac{1 + C_1\,(t_2 - 32)}{1 + C_1\,(t_1 - 32)}\right] l_1$$

18. Expansion of Water.—Although, as stated before, the expansion of solids and liquids is nearly uniform throughout all ranges of temperature, water is a marked exception to the general rule. If water is cooled down from its boiling point, it continually contracts until it reaches 39.1° F., when it begins to expand, until it freezes at 32° F. On the other hand, if water at 32° F. is heated, it contracts until it reaches 39.1° F., when it commences to expand. Therefore, the density of water is greatest where this change occurs. The importance of this exception is seen in the fact that ice forms on the surface of water, since it is lighter than the warmer body of water lying at varying depths below it. Were it not for this fact all the large bodies of water would freeze solid, and the climate of the earth would thereby be seriously affected. The coefficient of expansion of water is such a changeable quantity (varying with the temperature) that a special table of coefficients is required.

EXAMPLES FOR PRACTICE

1. What are the absolute temperatures corresponding to: (*a*) 120° C., and (*b*) 120° F?

Ans. $\begin{cases} (a)\ 393\frac{1}{3}° \text{ C.} \\ (b)\ 580° \text{ F.} \end{cases}$

2. Change $-10°$ C. to the corresponding Fahrenheit reading.
Ans. 14° F.

3. (*a*) How much will an iron tie-rod 60 feet long expand when the temperature is raised from 40° to 110°? (*b*) Calculate the expansion by the formula in Art. **17** also. (*c*) What is the difference of the two results?

Ans. $\begin{cases} (a) & .345744 \text{ in.} \\ (b) & .345725 \text{ in.} \\ (c) & .000019 \text{ in.} \end{cases}$

4. To what temperature must a steel tire of 59.93 inches internal diameter be raised in order that its diameter may be 60.0015 inches? Original temperature is 71°. Ans. 270°

HEAT PROPAGATION

19. Heat is propagated through matter and space in three ways—by *conduction*, by *convection*, and by *radiation*.

20. Conduction.—The progress of heat from places of higher to places of lower temperature in the same body is called **conduction.** The rate at which heat is conducted

TABLE II

HEAT CONDUCTIVITY OF METALS

Metal	Conductivity	Metal	Conductivity
Silver	100.0	Iron	11.9
Copper	73.6	Steel	11.6
Gold	53.2	Lead	8.5
Aluminum . . .	31.3	Platinum . . .	8.4
Zinc	28.1	Bismuth . . .	1.8
Tin	15.2	Mercury . . .	1.3

varies greatly with different substances, the good conductors being those in which conduction is most rapid, and the bad conductors being those in which it is very slow. A non-conductor is a substance that will not conduct heat. No perfectly non-conducting substances are known, but a number of materials are such poor conductors of heat that they are ordinarily called non-conductors. The metals furnish the best conductors, and of these, silver stands first, and copper second. Fluids, both liquid and gaseous, are very

poor conductors of heat. Water, for example, can be made to boil at the top of a vessel while a cake of ice is suspended in the water within a few inches of the surface. If thermometers are placed at different depths, while water boils at the top, it is found that the conduction of heat downwards is very slight.

Representing the conductivity of silver by 100, Table II shows the conducting power of a number of the metals.

Organic substances conduct heat poorly. It is because of this fact that trees withstand great and sudden changes in the atmosphere without injury. The bark is a poorer conductor than the wood beneath it. Cotton, wool, straw, bran, etc. are all poor conductors. Rocks and earth are poorer conductors the less dense and homogeneous is the mass; hence the length of time required for the heat of the sun's rays to penetrate the earth. In Central Europe, the air near the ground has the highest temperature in the month of July, but at a depth of from 25 to 28 feet in the earth the time of highest temperature is in the month of December.

21. Convection.—The transfer of heat by the motion of the heated matter itself is called **convection.** It can take place only in liquids and gases. For example, as heat is applied to the bottom and sides of a vessel of water, the heaviness of the water nearest the source of heat is decreased; it rises, and the colder and heavier water above descends and takes its place. There is thus a constant circulation going on, and this tends to equalize the temperature of the whole mass by bringing the hotter parts of the water in contact with the colder.

22. Radiation.—The communication of heat from a hot body to a colder one across an intervening space is called **radiation.** The best example of radiated heat is that received from the sun, the distance intervening in this case being 93,000,000 miles. A person standing in front of a fire, but at some distance from it, feels a sensation of warmth that is not due to the temperature of the air, for, if a screen be interposed between him and the fire, the sensation

immediately ceases, which would not be the case if the surrounding air had a high temperature. Hence, bodies can send out rays that excite heat and penetrate the air without heating it. This is **radiant heat**, and it manifests itself in all directions around the body.

The intensity of heat radiation from a given source:

1. *Varies as the temperature of the source;*
2. *Varies inversely as the square of the distance from the source;*
3. *Grows less as the inclination of the rays to the surface grows less.*

The truth of all these laws has been established by careful experiment.

Radiant heat is transmitted in a vacuum as well as in air. This is demonstrated by the following experiment: In the top of a glass flask, a thermometer t is fixed in such a manner that its bulb occupies the center of the flask, Fig. 4. The neck of the flask is next carefully narrowed by means of a blowpipe; the flask is then attached to an air pump, and a vacuum is produced in the interior. This being accomplished, the tube is sealed at the narrow part. On immersing in hot water, or on bringing the flask near some hot charcoal, the mercury is seen to rise at once. It can rise only by reason of the radiation through the vacuum in the interior, for glass is such a poor conductor that the heat could not travel with sufficient rapidity through the sides of the flask and the stem of the thermometer to cause this almost instantaneous rise.

FIG. 4

23. The **radiating power of heated surfaces** also depends very greatly on the form, shape, and material of which they are composed. If a cubical vessel, filled with hot water, has one of its vertical sides coated with polished

silver, another with tarnished lead, a third with mica, and the fourth with lampblack, experiment has shown that the radiating powers will be represented relatively by the numbers 25, 45, 80, 100; hence, bright surfaces radiate less heat than dark ones having the same temperature.

In the same way, it is found that the heat-absorbing power of bodies varies in a similar manner. Lampblack reflects few of the heat rays that impinge on it; nearly all are absorbed, while, on the other hand, polished silver reflects the greater percentage of the rays and absorbs only about $2\frac{1}{2}$ per cent.

Some substances neither absorb nor reflect the heat rays to any extent, but transmit nearly all of them just as glass transmits light. For example, rock salt reflects less than 8 per cent. of the radiation it receives, absorbs almost none, and transmits 92 per cent.

It is apparent that there is a sort of exchange going on between heated bodies at all times, which tends toward an equalization of temperature. The hot bodies are always cooling, and the cold bodies are always tending toward a rise in temperature, so that heat is created only to be diffused and apparently lost. That it is not lost, however, will be shown in the subsequent pages.

24. Dynamical Theory of Heat.—The view now generally taken as to the mode in which heat is propagated is thus stated in Ganot's Physics: "A hot body is one whose molecules are in a state of vibration. The higher the temperature of a body, the more rapid are these vibrations, and a diminution in temperature is but a diminished rapidity of the vibrations of the molecules. The propagation of heat through a bar is due to a gradual communication of this vibratory motion from the heated part to the rest of the bar. A good conductor is one which readily takes up and transmits the vibratory motion from molecule to molecule, while a poor conductor is one which takes up and transmits the motion with difficulty. But even through the best of the conductors the propagation of this motion is comparatively slow. How then can be explained the instantaneous

perception of heat when a screen is removed from a fire, or when a cloud drifts from the face of the sun? In this case, the heat passes from one body to another without affecting the temperature of the medium which transmits it. In order to explain these phenomena, it is imagined that all space, the space between the planets and the stars, as well as the interstices in the hardest crystal and the heaviest metal—in short, matter of any kind—is permeated by a medium having the properties of matter of infinite tenuity, called **ether.** The molecules of a heated body, being in a state of intensely rapid vibration, communicate their motion to the ether around them, throwing it into a system of waves which travel through space and pass from one body to another with the velocity of light. When the undulations of the ether reach a given body, the motion is given up to the molecules of that body, which in their turn begin to vibrate; that is, the body becomes heated. This process of this motion through the ether is termed radiation, and what is called a ray of heat is merely one series of waves moving in a given direction."

MEASUREMENT OF HEAT

HEAT UNITS

25. The British Thermal Unit.—To measure heat, some standard unit is required. The unit commonly employed in English-speaking countries *is the amount of heat required to raise the temperature of 1 pound of water from 62° to 63° F.* This unit is called the **British thermal unit,** usually written B. T. U.

For accurate work, it is necessary to specify the particular degree on the thermometric scale, for it is found by experiment that the heat required to raise the temperature of a pound of water 1° is not the same for all parts of the scale. For ordinary calculations, however, this is not required.

26. The Calorie.—The amount of heat necessary to raise the temperature of 1 kilogram of water 1° C. is called

a **calorie.** One kilogram equals 2.2046 pounds and 1° C. $= \frac{9}{5} \times 1°$ F.; hence, a calorie is $2.2046 \times \frac{9}{5} = 3.9683$ B. T. U. The calorie is used in France, and in other countries in which the metric system of weights and measures has been adopted.

SPECIFIC HEAT

27. Definition of Specific Heat.—If equal weights of two substances are heated under exactly similar conditions to a certain temperature, it will take longer to heat one than to heat the other. If, at this higher temperature, they are plunged into water, in vessels containing equal quantities of water at the same temperature, the temperature of the water will be raised more by the substance that required the longer time to heat. For example, it requires more time to raise the temperature of 1 pound of iron from 70° to 300° than to raise the temperature of 1 pound of lead to the same point under the same conditions. If each is then cooled in a separate vessel containing, say, 5 pounds of water at 70°, the water in which the iron cools will be heated to a higher temperature than that in which the lead cools. This indicates that it takes more heat to raise the temperature of 1 pound of iron a certain number of degrees than it does to raise the temperature of 1 pound of lead the same number of degrees, and that the iron, at the same temperature as the lead, held more heat than did the lead.

The **specific heat** of a substance is the ratio between the amount of heat required to raise the temperature of the substance 1° and the amount of heat required to raise the temperature of the same weight of water 1°. Thus, if the specific heat of lead is .0314, the amount of heat required to raise a certain weight of lead 1° will raise the same weight of water only .0314 of 1°, or, what is the same thing, .0314 B. T. U. will raise the temperature of 1 pound of lead 1° F.

EXAMPLE.—The specific heat of copper is .0951; how many B. T. U. will it take to raise the temperature of 75 pounds 180°?

SOLUTION.—Since it takes .0951 B. T. U. to raise 1 lb. of copper 1°, it will take $75 \times 180 \times .0951$ B. T. U. to raise 75 lb. 180°. Hence, the heat required is $.0951 \times 75 \times 180 = 1,283.85$ B. T. U. Ans.

28. Heat Required for a Given Rise of Temperature.—The following formula gives the number of B. T. U. required to raise the temperature of a substance a given number of degrees, or the number of B. T. U. given up by a body in cooling a given number of degrees:

Let G = weight of body, in pounds;
s = specific heat of substance composing the body;
t_1 = original temperature of body;
t_2 = final temperature of body;
Q = number of B. T. U. required, or given up, in changing temperature of body from $t_1°$ to $t_2°$.

Then, $$Q = Gs(t_2 - t_1)$$

EXAMPLE.—A piece of wrought iron weighing 31.3 pounds and having a temperature of 900°, is cooled to a temperature of 60°; how many units of heat did it give up? The specific heat of wrought iron is .1138.

SOLUTION.—Using the above formula, $Q = Gs(t_2 - t_1)$,
$31.3 \times .1138 \times (60 - 900) = -2{,}992$ B. T. U. Ans.

If a body is cooled from a temperature t_1 down to a temperature t_2, the value of Q as given by the above formula will be negative, the minus sign indicating that the heat is withdrawn.

29. In Table III are given the specific heats of a number of substances under constant pressure.

The reason that there are two values for the specific heat of gases is that less heat is required to raise the temperature of a gas when the volume is constant than when the pressure is constant and the volume varies. This point will be more fully discussed later.

30. Mixing Bodies of Unequal Temperatures.—If a certain quantity of water having a temperature of 40° is mixed with a like quantity having a temperature of 100°, the temperature after mixing will be $\frac{40 + 100}{2} = 70°$. But, if 5 pounds of copper having a temperature of 100° is immersed in 5 pounds of water having a temperature of 40°, the resulting temperature will not be 70°.

TABLE III
SPECIFIC HEATS
Solids

Substance	Specific Heat	Substance	Specific Heat
Copper	.0951	Cast iron	.1298
Gold	.0324	Lead	.0314
Wrought iron	.1138	Platinum	.0324
Steel (soft)	.1165	Silver	.0570
Steel (hard)	.1175	Tin	.0562
Zinc	.0956	Ice	.5040
Brass	.0939	Sulphur	.2026
Glass	.1937	Charcoal	.2410
Aluminum	.2143	Nickel	.1089

Liquids

Water	1.0000	Lead (melted)	.0402
Alcohol	.6200	Sulphur (melted)	.2340
Mercury	.0333	Tin (melted)	.0637
Benzine	.4500	Sulphuric acid	.3350
Glycerine	.5550	Oil of turpentine	.4260

Gases

	Specific Heat	
	Constant Pressure	Constant Volume
Air	.23751	.16902
Oxygen	.21751	.15507
Nitrogen	.24380	.17273
Hydrogen	3.40900	2.41226
Superheated steam	.48050	.34600
Carbon monoxide	.24790	.17580
Carbon dioxide	.21700	.15350

When different substances having different specific heats and different temperatures are mixed or are brought into close contact, the resulting temperature may be found by the following formula, provided that there is no change of state in any substance, as when ice melts, etc.:

$$t = \frac{G_1 s_1 t_1 + G_2 s_2 t_2 + G_3 s_3 t_3 + etc.}{G_1 s_1 + G_2 s_2 + G_3 s_3 + etc.}$$

in which t = final temperature of mixture;

G_1, s_1, and t_1 = weight, specific heat, and temperature, respectively, of one body;

G_2, s_2, and t_2 = same for second body;

G_3, s_3, and t_3 = same for a third body, etc.

Apply this formula to the case of the copper immersed in water. The specific heat of water is 1 and the specific heat of copper from Table III is .0951; then the final temperature is found from the equation

$$t = \frac{5 \times 1 \times 40 + 5 \times .0951 \times 100}{5 \times 1 + 5 \times .0951} = 45.21°, \text{ nearly}$$

EXAMPLE 1.—If 21 pounds of water at a temperature of 52° is mixed with 40 pounds of water at a temperature of 160°, what is the temperature of the mixture?

SOLUTION.—Since the specific heat of water is 1, it may be left out in applying the formula, and the temperature is found to be

$$t = \frac{21 \times 52 + 40 \times 160}{21 + 40} = 122.82°. \text{ Ans.}$$

EXAMPLE 2.—A copper vessel weighing 2 pounds is partly filled with water having a temperature of 80° and weighing 7.8 pounds. A piece of wrought iron weighing $3\frac{1}{4}$ pounds and having a temperature of 780° is dropped into this water. What is the final temperature?

SOLUTION.—Substituting the values given in the formula, and remembering that the original temperatures of the copper vessel and the water that it contains are the same, then

$$t = \frac{2 \times .0951 \times 80 + 7.8 \times 80 + 3.25 \times .1138 \times 780}{2 \times .0951 + 7.8 + 3.25 \times .1138} = 110.97°, \text{ nearly.}$$

Ans.

EXAMPLE 3.—A wrought-iron ball weighing 1 pound is placed in a furnace; when it has attained the temperature of the furnace, it is taken out and placed in a copper vessel weighing $\frac{1}{2}$ pound and containing exactly 2 pounds of water at a temperature of 75°. Assuming that no water escapes as steam, and that the temperature of the ball, water, and vessel after mixing is 156°, what is the temperature of the furnace?

SOLUTION.—Substituting the values given in preceding formula,

$$156 = \frac{1 \times .1138 \times t_1 + 2 \times 75 + .5 \times .0951 \times 75}{1 \times .1138 + 2 + .5 \times .0951}$$

or,
$$156 = \frac{.1138\, t_1 + 153.566}{2.16135}$$

and clearing of fractions, $156 \times 2.16135 = .1138\, t_1 + 153.566$; hence, $.1138\, t_1 = 183.604$,

or,
$$t_1 = \frac{183.604}{.1138} = 1{,}613.4°. \quad \text{Ans.}$$

31. Calculating Specific Heat.—By means of the formula in Art. **30**, the specific heat of a substance may be obtained. Thus, in the formula,

$$t = \frac{G_1 s_1 t_1 + G_2 s_2 t_2 + G_3 s_3 t_3 + etc.}{G_1 s_1 + G_2 s_2 + G_3 s_3,\ etc.}$$

suppose that the specific heat s_3 is required, and that all the other quantities, including t, are known. Solving the above equation for s_3, $t(G_1 s_1 + G_2 s_2 + etc.) + t\, G_3 s_3 = G_1 s_1 t_1 + G_2 s_2 t_2 + G_3 s_3 t_3 + etc.$, or $t\, G_3 s_3 - t_3 G_3 s_3 = G_1 s_1 t_1 - G_1 s_1 t + G_2 s_2 t_2 - G_2 s_2 t + etc.$,

or,
$$s_3 = \frac{G_1 s_1 (t_1 - t) + G_2 s_2 (t_2 - t) + etc.}{G_3 (t - t_3)}$$

EXAMPLE.—A silver vessel weighing 13 ounces is suspended by a string; 1 pound 4 ounces of water having a temperature of 120° is poured into it, and in this is placed a piece of metal weighing 14 ounces and having a temperature of 100°. If the temperature of the vessel is 72°, and the final temperature is 117°, what is the specific heat of the piece of metal?

SOLUTION.—Using the formula, and letting G_1, s_1, and t_1 represent, respectively, the weight, specific heat, and temperature of the silver vessel G_2, s_2, and t_2 the same quantities for the water, and G_3, s_3, and t_3 those for the piece of metal,

$$s_3 = \frac{G_1 s_1 (t_1 - t) + G_2 s_2 (t_2 - t)}{G_3 (t - t_3)}$$

$$= \frac{13 \times .057\,(72 - 117) + 20 \times 1\,(120 - 117)}{14\,(117 - 100)} = \frac{-33.345 + 60}{238} = .112$$

Ans.

All weights must be reduced to either pounds or ounces before substituting.

EXAMPLES FOR PRACTICE

1. How many units of heat are required to raise the temperature of 10 ounces of platinum from 80° to 2,000°? Ans. 38.88 B. T. U.

2. In order to determine the specific heat of a certain alloy, a piece weighing 12½ ounces was heated to a temperature of 320°, and was then immersed in 2 pounds 6 ounces of water contained in a lead vessel weighing 4 pounds 7 ounces. The temperature of the water and of the vessel being 70°, what was the specific heat of the alloy if the final temperature was 79°? Ans. .1202

3. In order to determine the temperature of a chimney, a silver bar weighing 20 ounces is placed in it until it has attained the same temperature. It is then immersed in 1 pound of water contained in a brass vessel weighing 10 ounces. The temperature of the vessel and water being 65°, and the final temperature 98½°, what is the temperature of the chimney? Ans. 596°

4. An iron casting weighing 3 tons is cooled from 2,100° to 100°; how many units of heat does it give up? Ans. 1,557,600 B. T. U.

LATENT HEAT

32. Doctor Black's Experiment.—Heretofore, the phenomena relating to sensible heat only have been considered; heat that is not sensible, that is, heat the existence of which is not revealed by the senses or by a thermometer, will now be considered. If a quantity of pounded ice at a temperature of 32° be put in a vessel and held over the flame of a spirit lamp, heat passes rapidly into the ice and melts it; but a thermometer resting in this mixture of ice and water shows no tendency to rise; it will remain at 32° until all the ice has been melted. Where has the heat gone that was supplied to the ice? This question was first investigated by Doctor Black, of Edinburgh, in 1760, and is easily explained by the modern dynamical theory of heat.

Doctor Black took 1 pound of water and 1 pound of ice, both having a temperature of 32°, and placed them in two vessels suspended in a chamber that was kept at as nearly a uniform temperature as possible. At the end of ½ hour the temperature of the water was 39.2°, but the ice did not reach that temperature until 10½ hours had passed, being melted,

of course, in the meantime. Doctor Black reasonably assumed that the ice received the same quantity of heat that the water did in each $\frac{1}{2}$ hour, because it was placed in exactly the same position with respect to the surrounding air; that is to say, it received $39.2 - 32 = 7.2$ units of heat every $\frac{1}{2}$ hour, or 14.4 units every hour, and $14.4 \times 10\frac{1}{2} = 151.2$ units in $10\frac{1}{2}$ hours. Hence, it took $151.2 - 7.2 = 144$ units of heat to change the 1 pound of ice at 32° into water at 32°. This value will be used hereafter whenever the occasion arises for using it.

If 1 pound of water having a temperature of 212° be mixed with 1 pound of water having a temperature of 32°, the temperature of the mixture will be $\frac{212 + 32}{2} = 122°$, the boiling water giving up 90° and the cold water receiving 90°, thus bringing both to a common temperature. If 1 pound of ice at a temperature of 32° be mixed with 1 pound of water at a temperature of 212°, the temperature of the mixture will be only 50° instead of 122, as in the previous case. Here, the water has given up $212 - 50 = 162$ units of heat in order to bring both bodies to a common temperature. Since the temperature of the ice was raised from 32° to 50°, it follows that $50 - 32 = 18$ units of heat were used to raise the temperature of the ice after it had been melted into water, and that $162 - 18 = 144$ units of heat were necessary to convert the ice at 32° into water of the same temperature.

33. Latent Heat of Fusion.—The extra amount of heat that is necessary to convert a solid into a liquid of the same temperature without raising the temperature of the solid is called the **latent heat of fusion**, and the temperature at which this change of state in the body takes place is called the **melting point**, or **temperature of fusion**. All solids probably have a latent heat of fusion, the word *probably* being used because some solids have never been melted, except at such high temperatures that accurate measurements are not possible.

The value of the latent heat varies greatly for different substances, being 144 units for ice, while for frozen mercury its value is only 5.09; that is, to change 1 pound of frozen mercury at its temperature of fusion (– 39° F.) into liquid mercury of the same temperature requires only 5.09 units of heat.

It is reasonable to suppose that if 144 units of heat are required to convert 1 pound of ice at 32° into water at 32°, then the same number of heat units will be given up when water at 32° is changed into ice at 32°. Experiment has verified this supposition.

34. Latent Heat of Vaporization.—If water be heated to its boiling point of 212° under a constant pressure of 14.696 pounds per square inch, it has been found, by experiment, that about 965.8 units of heat are required to change 1 pound into steam at 212°. This extra number of units of heat necessary to convert a liquid into vapor of the same temperature and pressure is called the **latent heat of vaporization,** and the temperature at which this change of state takes place is called the **temperature of vaporization.**

35. Nature of Latent Heat.—According to the modern theory of heat, the extra quantity of heat necessary for a change of state of a body is used in forcing the molecules of a body farther apart, and in overcoming the force of cohesion. This latent heat is not lost, but performs work in giving additional energy to the molecules of a body, and it always reappears when the body resumes its former state. Thus, for instance, 1 pound of steam under a pressure of one atmosphere contains about $965.8 + 180 = 1{,}145.8$ units of heat more than does 1 pound of water at 32°. Hence, if 1 pound of steam at 212° be mixed with $\frac{965.8}{180} = 5.37$ pounds of water at 32°, the temperature of the mixture will be exactly 212°, or the boiling point of water; in other words, the steam raised 5.37 pounds of water from the freezing point to the boiling point without lowering its own temperature by merely changing from steam into water. If 1 pound

of water at a temperature of 32° be changed into ice of the same temperature, 144 units of heat will be given up during this change of state.

36. In Table IV are given the temperatures of fusion and of vaporization, and the latent heats of fusion and vaporization for the cases in which they have been determined with sufficient accuracy:

TABLE IV

HEATS OF FUSION AND VAPORIZATION

Substance	Temperature of Fusion Degrees F.	Temperature of Vaporization Degrees F.	Latent Heat of Fusion B. T. U.	Latent Heat of Vaporization B. T. U.
Water	32	212	144.00	965.8
Mercury . . .	− 39	675	5.09	
Sulphur	237.2	831	16.86	
Tin	446		25.65	
Lead	626		9.67	
Zinc	786	1,900	50.63	
Alcohol	− 202.9	173		372
Oil of turpentine	14	313		124
Linseed oil . .		600		
Aluminum . .	1,220		51.4	
Copper	2,000			
Cast iron . .	2,192			
Wrought iron .	2,912			
Steel	2,520			
Platinum . . .	3,200			

EXAMPLE 1.—How many units of heat will be required to change 12 pounds of ice at a temperature of −20° C. into steam of 212° F.?

SOLUTION.—By formula **1**, Art. **11**, $t_f = (\frac{9}{5} \times -20) + 32 = -4°$. This is equivalent to 32° + 4° = 36° F. below the freezing point. Table III, the specific heat of ice was given as .504; hence, it will take 12 × 36 × .504 = 217.73 B. T. U. to raise the temperature of 12 lb. of ice from −4° to 32°. To convert this ice into water of 32° will require 144 × 12 = 1,728 B. T. U. To raise this water from 32° to a tempera-

of 212° will require 12 × 180 = 2,160 B. T. U. To convert it into steam of 212° will require 965.8 × 12 = 11,589 B. T. U. The total number of units of heat required is therefore

217.73 + 1,728 + 2,160 + 11,589 = 15,695 B. T. U. Ans.

EXAMPLE 2.—How many units of heat will it take to evaporate 25 pounds of alcohol from a temperature of 70°?

SOLUTION.—The temperature of vaporization of alcohol is 173°, and the specific heat is .62; the increase in temperature from 70° will be 173° − 70° = 103°. The number of units of heat required will be 25 × 103 × .62 = 1,596.5 heat units. The latent heat of vaporization is 372; hence, 1,596.5 + 25 × 372 = 10,896.5 B. T. U. will be required. Ans.

37. Freezing Mixtures.—A solid may be changed into a liquid, not only by melting it, but also by dissolving it, as salt or sugar is dissolved in water. Since the particles of the solid body must be torn asunder, in opposition to the forces that hold them together, it is reasonable to suppose that a certain amount of heat will be required to do this. That such is a fact may be easily proved by any one having a thermometer. Put a thermometer in a vessel of water and leave it there until it indicates the temperature of the water, then put in some salt or sugar, and stir so as to make it dissolve more quickly. It will be found that the mercury has fallen several degrees. In fact, if any solid is dissolved in a liquid that does not act chemically on it, the temperature of the mixture will be lower than if the solid did not dissolve. It is this principle that is taken advantage of in the so-called freezing mixtures. A mixture of one part of nitrate of ammonia and one part of water will reduce the temperature from 50° to 4°, a fall of 46°. The effects are still more striking when both bodies are solids, one of which is already at the freezing point. Thus, in a mixture of two parts of snow, or finely pounded ice, and one part of common salt, the temperature is lowered from 32° to −5°, a fall of 37°, while in a mixture of four parts of potash and three parts of snow or pounded ice the temperature is lowered from 32° to − 51°, a fall of 83°.

38. Latent Heat in Nature.—Latent heat plays an important part in the formation and melting of ice and snow

in nature. It takes a long time and severe cold to freeze the water of a river to any depth, even though the thermometer goes far below the freezing point. This is because 144 units of heat must be given up by every pound of water, after being brought to the freezing point, before the ice can form. If it were not for this fact, the rivers, lakes, and other bodies of water would be frozen solid as soon as the water reached the freezing point, and would be melted as soon as the temperature rose above that point. In the spring all the snow on the hills would be melted during a warm day, and great floods would be the consequence. As it is, 144 units of heat must be supplied to every pound of snow at 32° to convert it into water at 32°, and considerable time must elapse before the whole of this large quantity of heat can be supplied.

EXAMPLES FOR PRACTICE

1. If 1 pound of steam at 212° and 7 pounds of ice at 32° are mixed, what will be the resulting temperature? Ans. 49.23°

2. How many units of heat are required to melt 10 pounds of mercury at −39° and raise it to a temperature of 0°? Ans. 63.9 B. T. U.

3. How many pounds of oil of turpentine at 60° can be vaporized by the heat from 1 pound of coal, if the coal gives out 13,400 B. T. U. during combustion? Ans. 57.8 lb.

4. How many pounds of water at 32° can be vaporized by the heat from the combustion of 1 pound of coal of the quality used in example 3? Ans. 11.7 lb.

5. How many pounds of coal of the same quality as in example 3 are required to raise 100 pounds of wrought iron from 85° to its melting point? Ans. 2.4 lb.

SOURCES OF HEAT

39. Heat is derived from the following sources: *Physical sources*, which are, the radiation of heat from the sun, terrestrial heat, change of state in bodies, and electricity; *chemical sources*, or molecular combinations, more especially combustion; *mechanical sources*, comprising friction, percussion, and pressure.

40. Physical Sources.—The greatest of all the sources of heat is the sun. Most scientists are of the opinion that all the heat received or given up by the earth has, or has had, its source in the sun; but it is not required to discuss this theory fully here. It is the heat radiated from the sun and received by the earth that causes the changes of seasons, and that causes the water in the rivers, lakes, and seas to evaporate and form the clouds, to be again precipitated as rain or snow. Without this heat, no living thing—animal or vegetable—could exist.

The earth possesses a heat peculiar to itself, called *terrestrial heat.* When a descent is made below the surface, the temperature is found to gradually increase. This is not caused by the heat radiated from the sun, for the material comprising the earth is such a poor conductor that the heat of the sun's rays penetrates only a very short distance below the surface. The explanation usually given for this phenomenon is that the interior of the earth is in a molten condition. The terrestrial heat exerts but a slight effect, not raising the temperature of the surface more than $\frac{1}{20}°$.

If a liquid be poured on a finely divided solid, as a sponge, flour, starch, roots, etc., the temperature will be increased from 1° to 10° according to conditions. This phenomenon may be called *heat produced by capillarity.*

The heat produced by a change of state has already been described; it is the heat given off when a body is converted from a gas or liquid to a liquid or solid.

Extremely high temperatures may be produced by the electric current. By means of it, quicklime, firebrick, osmium, porcelain, and several other substances, which until very recently have resisted every attempt to melt them, may be made to run like water.

41. Chemical Sources.—Whenever substances that act chemically on one another are brought together and allowed to combine, heat is evolved. When heat is produced by oxygen uniting with carbon or some other substance, and is accompanied by light, it is called *combustion.*

42. Mechanical Sources.—The friction between any two bodies rubbed together produces heat. Rubbing one hand briskly against the other will soon make the hands too warm for comfort. The friction between a journal and its bearing causes heat; the heat causes the journal and bearing to expand, the journal expanding more rapidly because it is smaller and is heated more quickly; the expansion causes a greater pressure on the bearing, thus producing more friction and heat. If the bearing is not properly oiled, the heat will become so intense in a short time that the soft metal in the bearing will melt.

Fig. 5

When meteors or so-called shooting stars strike the earth's atmosphere their velocity is so great (sometimes as high as 150 miles a second) that the friction of the atmosphere causes them to take fire almost instantly. Friction is always accompanied by the production of heat.

Heat is also generated by percussion. The repeated blows of a hammer on a piece of iron, lead, or other metal, will soon make it quite hot.

The generation of heat by compression was mentioned in connection with gases. It is a matter of common experience that when a gas is compressed its temperature rises. The cylinder of an air compressor is too warm to rest the hand on, and a bicycle pump can be made quite warm by vigorous use. The same thing is also true of solids and liquids, but the results are not so marked.

The production of heat by the compression of gases is easily shown by means of the pneumatic syringe shown in Fig. 5, which consists of a glass tube with thick sides, hermetically closed with a leather piston. At the bottom is a small cavity in which a piece of cotton, moistened with ether or carbon disulphide, is placed. The tube is filled with air

and the piston is suddenly plunged downwards. The compression of the air generates so much heat that the cotton is ignited and can be seen to burn when the piston is suddenly withdrawn. The ignition of the cotton in this experiment indicates a temperature of at least 570°, since it will not ignite at a lower temperature under these conditions.

THERMODYNAMICS

FUNDAMENTAL RELATIONS OF HEAT AND WORK

43. Production of Heat by Work.—In Art. 42, it was shown that heat may be produced mechanically by friction, percussion, or the compression of gases. In every case in which heat is thus produced mechanically, there is work done. In the case of the journal and bearing, the frictional resistance at the contact surface is overcome and work is done against this resistance; in compressing a gas, there is work done in moving the piston; when a bar of iron is heated by hammering, work is done on the bar by the impact of the hammer.

Furthermore, the quantity of heat produced depends on the amount of work done. If a journal is rough and unlubricated, the friction between journal and bearing is greater than when it is smooth and lubricated; as a consequence, more work is done in overcoming friction in a given time and it is a matter of experience that more heat is produced; that is, the journal heats in a shorter time. In compressing a gas, the further the compression is carried the more work there is done and the more the gas is heated. Likewise, the more work expended in hammering a bar of iron, the hotter it becomes; that is, the more heat there is generated. The statements of this paragraph lead therefore to the following important principle:

The performance of mechanical work results in the production of heat, and the quantity of heat thus produced depends, in some way, on the amount of work done.

44. Performance of Mechanical Work by Heat. Heat may be produced by the expenditure of work, and, conversely, work may be produced by the expenditure of heat. As an example, let the metal rod *a*, Fig. 6, be heated; it will lengthen and raise the mass *b* a distance equal to the increase of length; here heat has been expended, and as a result work has been done in raising the block. The expansion of gases furnishes instances of the production of work by heat. In Fig. 7, air is confined in the cylinder *a* by the piston *b*. Let the confined air be heated; then, if its pressure remains the same, which will be the case if the piston is free to move, the volume of the confined air must be increased, for the temperature is increased and the volume must increase with the absolute temperature when the pressure remains constant. But if the air expands, the piston must rise, and consequently work is done. The more heat expended, the more the air expands and the more work there is done.

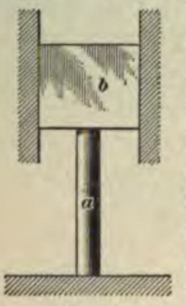


Fig. 6

Perhaps the most conclusive proof that heat produces work is furnished by Hirn's experiments on the steam engine. Hirn measured the heat taken into the engine cylinder per stroke, from the boiler, and the heat rejected per stroke to condenser. In every case, the heat given up to the condenser was less by a considerable amount than that received from the boiler, proving that some of the heat received was expended in the performance of the work required to drive the engine.

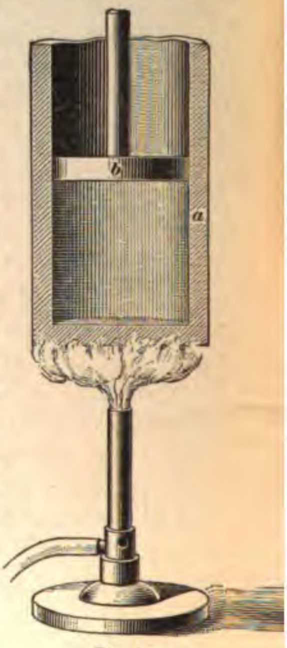


Fig. 7

45. Mechanical Theory of Heat.—The fact that heat is produced by the expenditure of work, and vice versa,

leads to the conclusion that heat is simply a form of energy. When work is done and reappears as heat, it has simply been transformed into energy of another form. To illustrate this statement, take the case of work done on a gas in compressing it. If the piston, Fig. 7, is pushed downwards, the air below is heated and its temperature rises. Work has been done on the air, and as a result heat has been generated. Before the piston was moved, the confined air had a definite temperature; the molecules were vibrating to and fro with an average speed that may be denoted by v_1, and it is the continual striking of the molecules against the piston that gives rise to what is called the pressure on the containing walls and piston.

Let the piston move downwards and compress the air. The molecules, being now closer together, strike the containing walls with greater frequency, thus causing the increased pressure. It is found that the temperature is also increased; this results from the fact that the molecules are now moving with an average speed v_2, which is greater than the original speed v_1. According to the principles of mechanics, a body of weight G having a speed v possesses the kinetic energy $\frac{Gv^2}{2g}$. Considering the molecules of the gas as small bodies, the kinetic energy of a molecule moving with the speed v_1 is $\frac{Gv_1^2}{2g}$ when G denotes the weight of the molecule. The sum of the kinetic energies of all the molecules of the confined air is the kinetic energy of the weight of air. After compression, the molecules have the velocity v_2, and therefore the kinetic energy is $\frac{Gv_2^2}{2g}$, which is greater than the original kinetic energy $\frac{Gv_1^2}{2g}$, hence, the total kinetic energy of the contained air has been increased. Also, there may be a change in the potential energy of the air, but this point will be considered later.

When work is done on a body, that work is stored up in the body as kinetic energy and the increase of energy is

equal to the work done. For example, the net work done in starting a railway train from rest and getting it up to speed is stored in the train and is given back when the train comes to rest. In the same way, the work done in compressing the air is expended in increasing the total energy of the air; that is, in making the molecules move at a higher rate of speed.

Consider next the reverse operation. Suppose the air under the piston to be heated by a flame while the piston is held stationary. The temperature of the air rises and the pressure is correspondingly increased. Now, remove the flame and let the piston be released so that it is free to move. The pressure of the confined air being greater than the pressure of the atmosphere above the piston, there is a net upward force that causes the piston to rise. As the piston rises, the confined air expands and its temperature falls. The fall of temperature indicates that the molecules of the air move with less speed, and this in turn indicates that the air as it expands is losing kinetic energy. According to the law of conservation of energy, the energy thus lost by the air must reappear somewhere; it cannot be destroyed. Where it reappears in this case is easily seen; the energy lost by the air is precisely that required to do the work of raising the piston.

From the foregoing, it is apparent that *the heat in a body is simply the stock of energy the body possesses;* this energy may be kinetic energy, due to the motions of the molecules composing the body, or potential energy, due to the relative positions of the molecules and their distances from each other; or it may be partly kinetic and partly potential. To heat a body is to increase its stock of energy; and, conversely, to cool a body is to decrease its stock of energy.

46. The Mechanical Equivalent of Heat.—As already explained, heat is measurable, and quantities of heat are expressed in terms of a unit called the British thermal unit. Work and energy are expressed in foot-pounds. Since heat is merely a form of energy, there should be some numerical

relation between the two units, that is, a B. T. U. should be equal to some definite number of foot-pounds, or the reverse.

This relation has been determined by many experimenters. Doctor Joule, of Manchester, England, found as the result of many experiments that the heat required to raise the temperature of 1 pound of water 1° F. could, if expended in work, raise a weight of 772 pounds a distance of 1 foot; that is, 1 B. T. U. = 772 foot-pounds. Later experiments of Professor Rowland, of Baltimore, Maryland, show that Joule's figure is too low, and that 778 is the correct value. This number, 778, is called the *mechanical equivalent of heat*, and sometimes *Joule's equivalent;* it is denoted by the letter J.

47. The First Law of Thermodynamics.—The formal statement of the relation between heat and work constitutes the first law of thermodynamics.

Law.—*Heat and mechanical work are mutually convertible. A unit of heat requires for its production, or produces by its disappearance, J units of work.*

Let Q = heat produced or given up, in B. T. U.;
W = work, in foot-pounds, produced by Q or required to produce Q;
J = Joule's equivalent.

Then the first law of thermodynamics is expressed by the formula,

$$JQ = W, \text{ or } Q = \frac{W}{J}$$

EXAMPLE 1.—The combustion of 1 pound of coal results in the generation of 13,700 B. T. U.; if all this heat could be transformed into work, what horsepower could be obtained by the burning of 300 pounds of coal per hour?

SOLUTION.—The B. T U. liberated per minute is $\frac{300}{60} \times 13{,}700 = 68{,}500$. Since 1 B. T. U. = 778 ft.-lb., the work per minute is $68{,}500 \times 778 = 53{,}293{,}000$ ft.-lb. Hence, since 1 H. P. is equal to 33,000 ft.-lb. per min., the horsepower is $53{,}293{,}000 \div 33{,}000 = 1{,}614.9$ H. P. Ans.

EXAMPLE 2.—A journal 4 inches in diameter bears a load of 10,000 pounds and makes 80 revolutions per minute. The coefficient of friction is .02. How much heat is produced per hour?

SOLUTION.—The frictional resistance is $10{,}000 \times .02 = 200$ lb. A point on the circumference travels $3.1416 \times \frac{4}{12} \times 80 \times 60 = 5{,}026.56$ ft. per hr. The work done against friction is, therefore, $W = 5{,}026.56 \times 200 = 1{,}005{,}312$ ft.-lb., and from the above formula the heat produced is

$$Q = \frac{W}{J} = \frac{1{,}005{,}312}{778} = 1{,}292.2 \text{ B. T. U.} \quad \text{Ans.}$$

48. Three Effects of Heat.—When a quantity of heat is imparted to a body, it, in general, performs three kinds of work:

1. The temperature of the body is raised—that is, the sensible heat is increased; in consequence, the molecules are caused to move at greater speeds and the kinetic energy is increased. The work required to raise the temperature, or what is the same thing, to increase the kinetic energy, is called the **vibration work.**

2. Usually, the heated body expands and, on the whole, the molecules are farther apart than they were before the body was heated. Since the molecules attract each other, work must be expended in moving them farther from each other. After being thus separated, the molecules, when they again approach each other, possess a certain capacity for doing work; hence, the expanded body has a certain potential energy that is due merely to the separation of the molecules. The work required thus to increase the potential energy is called the **disgregation work.**

3. The body, in expanding, must overcome an external pressure through some definite distance and work is thus done against the external pressure. For example, the expansion of the air in the cylinder shown in Fig. 7 causes the piston to rise. Above the piston is the pressure of the atmosphere, and the piston, in rising, does work against this pressure. To the work expended in overcoming external pressure, the name **external work** is given.

49. Equation of Energy.—According to the first law of thermodynamics, the heat absorbed must be precisely equal to the total work done; hence, the heat, in B. T. U. $\times$ 778 = vibration work + disgregation work + external work.

Let K = vibration work, in foot-pounds;
D = disgregation work, in foot-pounds;
W = external work, in foot-pounds;
Q = heat imparted, in B. T. U.

Then,
$$JQ = K + D + W \qquad (1)$$

K is the increase of the kinetic energy of the body and D is the increase of the potential energy; hence, the sum $K + D$ is the total change of energy. If E_1 denotes the energy possessed by the body originally, and E_2 the energy after the heat is imparted, both expressed in foot-pounds, then $K + D = E_2 - E_1$, and, substituting in formula **1**,

$$JQ = E_2 - E_1 + W \qquad (2)$$

that is, the heat imparted to a body is equal to the increase in the energy of the body plus the external work.

50. Formula **1**, Art. **49**, may now be applied under various circumstances.

1. *Heating a Solid Body.*—A solid body expands but little; the disgregation and external works are therefore small and may be neglected when the vibration work is being considered. When heating a solid, as a piece of iron, it is assumed, therefore, that all the heat is used in raising the temperature.

2. *Melting a Solid.*—During the melting of a solid body, there is no change of temperature; hence, the vibration work K becomes zero. Usually, there is little change in the volume during the melting, and the external work is therefore small. Nearly all the heat is expended in disgregation work, which in this case consists in changing from the molecular state of a solid to that of a liquid; that is, in tearing the molecules from their fixed positions relative to each other in the solid and giving them the freedom that molecules have in the liquid state. If the very small external work be neglected, the disgregation work is the equivalent of the latent heat of fusion (see Art. **33**).

3. *Heating a Liquid.*—In heating a liquid, the conditions are the same as in the heating of a solid.

4. *Vaporization of a Liquid.*—There is no change of temperature during vaporization; therefore, $K = 0$, and $JQ = D + W$. Since the volume of the vapor is many times greater than that of the liquid, there is considerable external work, though the larger part of the heat Q is expended in disgregation work.

5. *Heating a Gas.*—In the case of a gas, the molecules are so far apart, considering their size, that their attraction for each other is almost inappreciable, and practically no work is required to separate them farther; hence, the disgregation work is taken as zero, and formula **1**, Art. **49**, becomes

$$JQ = K + W$$

The relative magnitudes of K and W depend on the conditions under which the gas is heated. Suppose that the air in the cylinder, Fig. 7, is being heated. The piston may be held in the original position, in which event the external work W is zero and all the heat is expended in raising the temperature. Or the piston may be raised at such a rate that the decrease of temperature due to the expansion of the air just offsets the increase due to the heating; in this case, the temperature remains constant, $K = 0$, and all the heat is expended in doing external work. The work W may even be negative; thus, imagine the piston to be pushed down while the air is being heated so that work is done by the external pressure instead of against that pressure. As work done by the air against the external pressure has been considered as positive, the work done on the air by the external pressure must be considered as negative. In this case, therefore, $JQ = K - W$, or $JQ + W = K$. The work W assists the heat Q in doing the vibration work K.

51. Abstraction of Heat.—When heat is abstracted or taken from a body, the three works K, D, and W are, in general, negative and the equation of energy takes the form

$$-JQ = (-K) + (-D) + (-W)$$

or

$$-JQ = -K - D - W$$

The negative signs indicate, respectively, that the heat is abstracted instead of added, that the temperature falls, that

the molecules approach each other, and that the body thus gives up potential energy, and that work is done by the external pressure on the body as the latter contracts.

Under special conditions, one of the quantities may be positive. Take, for example, the case of freezing water; the temperature remains constant; hence, $K = 0$, and, as is well known, water in freezing expands and the work W is therefore positive. The energy equation for this case is therefore

$$-JQ = -D + W$$

As another example, suppose that the piston in Fig. 7 is pushed downwards, so as to compress the air confined below it, and suppose further that the lower part of the cylinder is surrounded by a stream of cold water, which abstracts heat from the air. If the air is compressed very slowly, its temperature will fall and K will be negative; but if the air is compressed quickly, the temperature will rise, notwithstanding the abstraction of heat by the cold water, and K will therefore be positive. In the latter case,

$$-JQ = K - W$$

A thorough understanding of formula **1**, Art. **49**, the energy equation, is very necessary as a preparation for the work that is to follow. Arts. **49**, **50**, and **51** should be thoroughly studied, and the nature of the processes described completely understood.

HEAT

(PART 2)

THERMODYNAMICS OF GASES

FUNDAMENTAL RELATIONS OF GASES

General Equation of Gases.—As explained in *matics*, the relation between the pressure, volume, and erature of 1 pound of a gas is expressed by the equation

$$pV = RT$$

For a weight of G pounds,

$$pV = GRT$$

hich p = absolute pressure, in pounds per square inch;
V = volume, in cubic feet;
R = a constant;
T = absolute temperature;
G = weight, in pounds.

the formulas given in the following pages, it is conent to express pressures in pounds per square foot, er than in pounds per square inch. Therefore, let P equal ssure in pounds per square foot. Then $P = 144p$, and

$$PV = 144pV = 144RT$$

or air, R was given, in *Pneumatics*, as .37; hence, $144R$ $144 \times .37 = 53.28$, and

$$PV = 53.28T$$

The symbol R may also be used to denote 53.28, with the lerstanding that the pressure is to be taken in pounds per are foot.

The other form of the general equation, given in *Pneumatics*, $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$, will frequently be used in this and succeeding Sections.

2. Specific Heats of a Gas.—From the definition in *Heat*, Part 1, the specific heat of a gas is numerically equal to the quotient obtained by dividing the British thermal units (B. T. U.) imparted to a pound of gas by the rise in temperature; that is, if Q is the B. T. U. imparted; s, specific heat; and t_1 and t_2, the initial and final temperatures, respectively; then

$$s = \frac{Q}{t_2 - t_1}$$

For gases, $JQ = K + W$ (see *Heat*, Part 1); hence,

$$Q = \frac{K+W}{J}, \text{ and } s = \frac{K+W}{J(t_2 - t_1)}$$

in which Q = heat in B. T. U.;
K = vibration work, in foot-pounds;
W = external work, in foot-pounds;
t_1 and t_2 = initial and final temperatures, in degrees Fahrenheit;
J = mechanical equivalent of heat, or 778 foot-pounds per B. T. U.

The specific heat of a gas may have any value, depending on the conditions under which the heat is imparted; for, as has been shown, in *Heat*, Part 1, K and W may be varied at pleasure; either may be made negative; and for a given rise in temperature the sum $K + W$ may be made small or large.

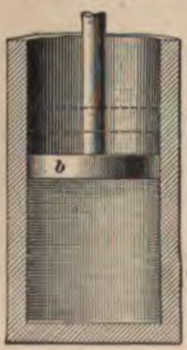


Fig. 1

There are two specific heats of special importance: that at *constant pressure* and that at *constant volume*. Suppose that the cylinder shown at a, Fig. 1, contains air, and that the piston b is free to move. The pressure of the confined air is due only to the weight of the piston and to the pressure of the atmosphere on the piston, and evidently this pressure is the same for all

positions of the piston. As heat is added to the air, the expansion causes the piston to rise to some higher position, as that shown by the dotted lines, and external work is done. The amount of this external work may now be calculated. Suppose the weight of the confined air to be 1 pound, and let V_1 denote the volume before heating and V_2 the volume after heating. The constant pressure is P pounds per square foot. If A is the area in square feet, PA is the total pressure of the piston on the air; and if h is the distance the piston rises, in feet, the external work done is

$$W = P A h \text{ foot-pounds}$$

But $A h$ is the volume swept through by the piston and is, therefore, the increase in the volume of the air; hence,

$$A h = V_2 - V_1$$

and

$$W = P(V_2 - V_1) \qquad (1)$$

From the general equation, then,

$$P V_1 = R T_1$$

and

$$P V_2 = R T_2$$

where T_1 and T_2 denote, respectively, the initial and final absolute temperatures. Subtracting the first from the second, $P V_2 - P V_1 = R T_2 - R T_1$, or $P(V_2 - V_1) = R(T_2 - T_1)$ which is also equal to W; that is, the work done by 1 pound of air when heated at constant pressure is R times the rise in temperature.

Let c_p denote the specific heat at constant pressure; then,

$$c_p = \frac{K}{J(t_2 - t_1)} + \frac{R}{J} \qquad (2)$$

$t_2 - t_1$ being the rise in temperature.

For the derivation of formula **2**, see Appendix I at the end of this Section. Values of c_p for various gases are given in *Heat*, Part 1.

Suppose that the piston in Fig. 1 is held in one position, so that the air cannot expand, but must retain the same volume, and that heat is added. Under this condition, there is no external work done, that is, $W = 0$. Let the specific heat at constant volume be denoted by c_v; then,

$$c_v = \frac{K + W}{J(t_2 - t_1)} = \frac{K}{J(t_2 - t_1)} \qquad (3)$$

3. Relation Between Specific Heats at Constant Pressure and Constant Volume.—Comparing the expressions for c_p and c_v in Art. **2**, it appears that c_p is the larger. That this must be the case is evident, for in heating at constant pressure part of the heat is used in doing external work, and therefore more heat is required for the same rise in temperature.

For a given rise in temperature $(t_2 - t_1)$, the change of energy K, which depends only on the change of temperature, has a definite value; hence, $\frac{K}{J(t_2 - t_1)}$, in formula **3**, Art. **2**, is the same as in formula **2**, and subtracting formula **3** from formula **2**,

$$c_p - c_v = \left[\frac{K}{J(t_2 - t_1)} + \frac{R}{J}\right] - \frac{K}{J(t_2 - t_1)} = \frac{R}{J}$$

For air, Regnault found the value of c_p to be .23751; hence, $.23751 - c_v = \frac{53.28}{778}$, and

$$c_v = .23751 - \frac{53.28}{778} = \frac{.23751 \times 778 - 53.28}{778} = .16902$$

The ratio $\frac{c_p}{c_v}$ is frequently used, and is denoted by k.

For air, $k = \frac{.23751}{.16902} = 1.4052$, say 1.405. The constant k has been determined experimentally. The value thus found agrees closely with the value given above.

4. Change of Energy of a Gas.—From *Heat*, Part 1, the change of energy in the gas, sometimes called the change of **intrinsic energy**, is, in general,

$$E_2 - E_1 = K + D$$

in which E_1 and E_2 = initial and final energies of the gas, in foot-pounds;
K = vibration work, in foot-pounds;
D = disgregation work, in foot-pounds.

For a gas, however, the disgregation work is practically zero, and

$$E_2 - E_1 = K$$

But, from formula **3**, Art. **2**, $K = Jc_v(t_2 - t_1)$ for 1 pound of gas; hence, for G pounds,

$$E_2 - E_1 = Jc_v G(t_2 - t_1) = Jc_v G(T_2 - T_1) \qquad (1)$$

Experiments have shown that when a gas expands without doing external work, the temperature remains practically unchanged. To be exact, there is a very slight change of temperature, owing to the small amount of heat required to do the disgregation work; but for a gas, as already stated, the disgregation work may be neglected.

EXAMPLE 1.—Six pounds of air is heated at constant volume from 60° F. to 83° F.; what is the increase of energy?

SOLUTION.— $E_2 - E_1 = 778 \times .16902 \times 6 \times (83 - 60) = 18{,}147$ ft.-lb., nearly. Ans.

A second formula for the change of energy is

$$E_2 - E_1 = \frac{P_2 V_2 - P_1 V_1}{k - 1} \qquad (2)$$

If the pressures are expressed in pounds per square inch,

$$E_2 - E_1 = \frac{144(p_2 V_2 - p_1 V_1)}{k - 1} \qquad (3)$$

For derivation of formula **2**, see Appendix II. In most cases, formula **2** is more convenient than formula **1**, as the weight of the air need not be known.

EXAMPLE 2.—Air confined in a cylinder has a volume of 15 cubic feet and a pressure of 60 pounds per square inch, absolute; the air is heated and expands to a volume of 22 cubic feet, and the pressure is 55 pounds per square inch. What is the change of energy of the air?

SOLUTION.— $E_2 - E_1 = \dfrac{144(55 \times 22 - 60 \times 15)}{1.405 - 1} = 110{,}222$ ft.-lb. Ans.

EXPANSION OF GASES

5. A gas may pass from one state to another in a number of ways; in practice, however, the expansion of a gas takes place according to one of a few well-defined laws. The most common forms of expansion are the following:

1. *Expansion at constant pressure.*
2. *Isothermal expansion*, during which the temperature remains constant.

3. *Adiabatic expansion*, in which the gas expands without receiving heat from or giving up heat to any external body.

4. *Expansion according to the law* $P V^n = c =$ a constant, in which the gas expands in such a way that the pressure always varies inversely as the nth power of the volume.

In connection with the expansion of a gas from some initial state to a second state, according to any given law, it is desired to know the following: (1) The external work done; (2) the change of energy in the gas; (3) the heat added to or abstracted from the gas; (4) the relations between pressure and volume, pressure and temperature, and volume and temperature, respectively.

It is desirable to take up, in order, the four expansions just noted and derive expressions for external work, heat added, etc. The derivation of some of the formulas cannot be accomplished except by the use of higher mathematics; in these cases the formulas must be taken for granted.

EXPANSION AT CONSTANT PRESSURE

6. The expansion of a gas when the pressure remains constant may be represented, graphically, in the way shown in Fig. 2. From any point O, the line OP is drawn vertically and the line OV horizontally. From O, a length OA_1 is laid off to represent, to some scale, the initial volume of the gas, and from A_1 the line A_1A is drawn parallel to OP, and of such a length as to represent, to some scale, the initial pressure of the gas. The point A is said to represent the initial state of the gas, as regards pressure and volume; its distance from OP represents the volume and its distance from OV represents the pressure. As the gas changes its state, the point representing the state must move; thus, if the volume increases, it must move away

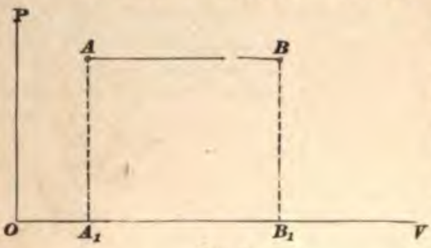


FIG. 2

from OP so that its perpendicular distance from OP continually represents the volume; while, if the pressure increases, it must move away from OV, and vice versa.

In the case under consideration, the volume increases but the pressure remains the same; hence, the point must move away from OP, but it must remain at the same distance from OV; that is, it must move along AB parallel to OV. The point B is located by making OB_1 equal to the final volume, and drawing B_1B perpendicular to OV.

As shown by formula **1**, Art. **2**, the external work is the product of the pressure and the increase in volume; that is,

$$W = P(V_2 - V_1) \qquad (1)$$

in which V_1 = initial volume;

V_2 = final volume.

In Fig. 2, the width A_1B_1 represents the increase of volume, $V_2 - V_1$, and the height $A_1A = B_1B$, the constant pressure P; hence the area of the rectangle A_1ABB_1 represents the external work.

The change of energy is given by either formula **1** or formula **2**, Art. **4**. These formulas are of general application and hold good for all expansions or changes of state of a perfect gas.

The heat added per pound of gas during the expansion is evidently the product of the specific heat and the rise in temperature, that is, $c_p(T_2 - T_1)$. For G pounds,

$$Q = Gc_p(T_2 - T_1) \qquad (2)$$

Another expression for the heat added is as follows:

$$JQ = E_2 - E_1 + W = \frac{k}{k-1}(PV_2 - PV_1) \qquad (3)$$

This equation has the advantage that the weight G need not be known; for its derivation, see Appendix III.

To obtain the relation between the volume and the temperature, the general equation $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$, from *Pneumatics*, is used. Since $P_1 = P_2$ (the pressure being constant), $\frac{V_1}{T_1} = \frac{V_2}{T_2}$; that is, the volume varies directly as the absolute temperature.

EXAMPLE.—Air having a volume of 5 cubic feet, a pressure of 60 pounds per square inch, absolute, and a temperature of 40° F., expands at constant pressure until the volume is 8 cubic feet. Compute: (*a*) the final temperature; (*b*) the external work; (*c*) the change in energy; (*d*) the heat imparted during the expansion.

SOLUTION.—(*a*) $T_1 = 460 + 40 = 500$, $V_1 = 5$, and $V_2 = 8$.

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$, hence $\frac{5}{500} = \frac{8}{T_2}$, or $T_2 = \frac{8 \times 500}{5} = 800$, and $t_2 = 800 - 460 = 340°$ F. Ans.

(*b*) $P = 144p = 144 \times 60 = 8{,}640$ lb. per sq. ft.

$W = P(V_2 - V_1) = 8{,}640 \times (8 - 5) = 25{,}920$ ft.-lb. Ans.

(*c*) By formula **2**, Art. **4**, and remembering that $P_2 = P_1$, $E_2 - E_1 = \frac{P(V_2 - V_1)}{k - 1} = \frac{8{,}640 \times (8 - 5)}{1.405 - 1} = 64{,}000$ ft.-lb., nearly. Ans.

(*d*) By formula **3**, $JQ = E_2 - E_1 + W = 64{,}000 + 25{,}920 = 89{,}920$ ft.-lb., and $Q = \frac{89{,}920}{778} = 115.58$ B. T. U. Ans.

ISOTHERMAL EXPANSION

7. In **isothermal expansion**, gas expands at constant temperature; then, in the general formula $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$, $T_1 = T_2$, and in consequence $P_1 V_1 = P_2 V_2$; that is, the gas in expanding follows Boyle's law (see *Pneumatics*). The graphic representation of this expansion is shown in Fig. 3. As the expansion progresses, the volume of the gas increases and the pressure falls; hence, starting from the point A, which represents the initial pressure and

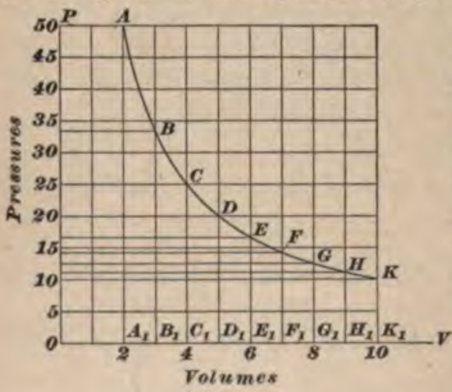


FIG. 3

volume, as in Fig. 2, the moving point recedes from OP as the volume increases, and approaches OV as the pressure decreases. As explained in *Pneumatics*, the curve traversed must be such that the product of the perpendicular distances

of any point from OP and OV is the same as the corresponding product for any other point. Hence, as the volume and pressure are known for any state, the pressure may be found for any assumed volume or the volume for any pressure. This enables one to plot the curve by finding points that indicate the different pressures and volumes.

8. Equilateral Hyperbola.—The curve shown in Fig. 3 is called the **isothermal expansion curve,** or the **expansion curve of constant temperature.** It is known in mathematics as the **equilateral hyperbola,** and, hence, when used on indicator diagrams, is sometimes called the **hyperbolic expansion curve.** If the initial volume, pressure, and final volume are known, the curve may be constructed graphically without calculating the different points, as was done in Fig. 3. Thus, in Fig. 4, let OY and OX be two lines at right angles to each other. These lines are

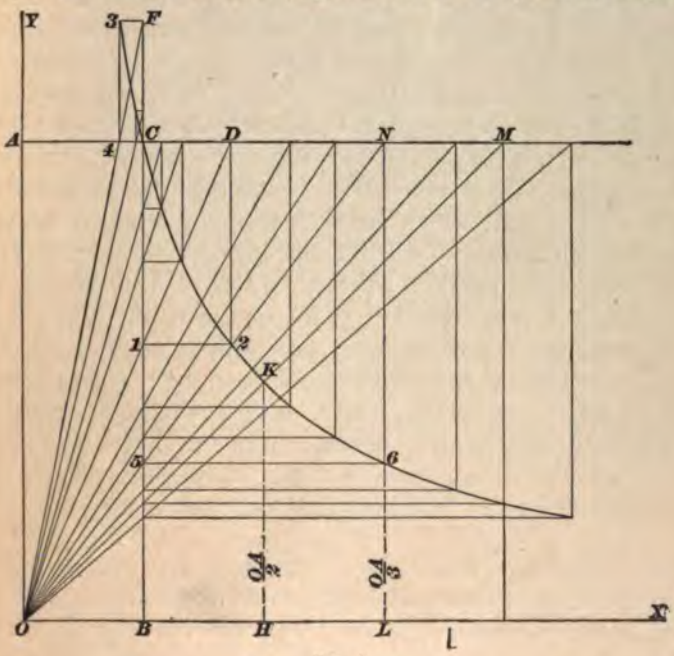


FIG. 4

known in mathematics as the **coordinate axes,** the line OY being called the **axis of ordinates,** or **axis of Y,** and the line OX, the **axis of abscissas,** or **axis of X.** Let OA represent the absolute initial pressure and OB the initial volume. Through A draw the indefinite straight line AM parallel to the axis OX, and through B draw the indefinite straight line BF parallel to the axis OY. The point C, where these two lines meet, is the point where the expansion is to begin; consequently, it is one point on the curve. Through the point O, called the **origin,** which is the point of no volume and no pressure, draw a number of lines, OF, OD, ON, OM, etc., cutting BF at F, *1*, *5*, etc., and AM at *4*, D, N, etc. Through the points F, *1*, *5*, etc. draw lines parallel to the axis OX, and through *4*, D, N, etc. draw lines parallel to the axis OY. These lines intersect in the points *3*, *2*, *6*, etc., which are points on the required isothermal expansion line. To prove this, lay off BH equal to OB, and draw HK parallel to the axis OY, intersecting the curve in K. Now, if K is a point on the isothermal expansion line, HK must be equal in length to one-half of OA, since, when the volume is twice as great, the pressure is only half as great. Similarly, if $HL = BH = OB$, $L\,6$ must be one-third as long as OA. By measurement, this will be found to be the case. This curve and this method of constructing it are much used in "working up" indicator diagrams.

9. External Work.—The **external work** done during the isothermal expansion is represented by the area AA_1K_1K, Fig. 3, under the curve AK. The following formula, which is developed by the use of higher mathematics, represents the work done during this isothermal expansion:

$$W = P_1 V_1 \log_e \frac{V_2}{V_1} \qquad (1)$$

in which V_1 and V_2 are the initial and final volumes, P_1 is the initial pressure in pounds per square foot, and $\log_e$ denotes the *hyperbolic logarithm*. The hyperbolic logarithm differs from the common logarithm, in that the base 2.71828 is used instead of 10. It is apparent, therefore, that th

logarithms of the same number in these two systems must have a definite relation to each other. This relation is expressed by the factor 2.3026, by which the common logarithm, explained in *Logarithms*, must be multiplied to obtain the hyperbolic logarithm. For work not requiring great accuracy, the factor 2.3 is often used. In formulas containing a logarithm derived by means of the higher mathematics, the hyperbolic logarithm is generally used. When it is desired to use the common logarithm, therefore, *2.3026 log* must be substituted for log_e. Formula **1** is then written

$$W = 2.3026\, P_1 V_1 \log \frac{V_2}{V_1} \qquad \textbf{(2)}$$

In the following formulas, the common logarithm will be used. When the hyperbolic logarithm is used, it is always indicated by the subscript e, as shown in formula **1**.

EXAMPLE 1.—Find, by the exact method, formula **2**, the external work done in the expansion represented in Fig. 3.

SOLUTION.— $P_1 = 144\, p_1 = 144 \times 50 = 7{,}200$; $V_1 = 2$, $V_2 = 10$, and $\frac{V_2}{V_1} = 5$.

$$W = 2.3026 \times 7{,}200 \times 2 \times \log 5 = 2.3026 \times 7{,}200 \times 2 \times .69897$$
$$= 23{,}176 \text{ ft.-lb. Ans.}$$

Since $P_1 V_1 = P_2 V_2$, $\frac{V_2}{V_1} = \frac{P_1}{P_2}$, and formula **2** may be written in the form

$$W = 2.3026\, P_1 V_1 \log \frac{P_1}{P_2} \qquad \textbf{(3)}$$

EXAMPLE 2.—Air having an absolute pressure of 44 pounds per square inch and a volume of 3.75 cubic feet expands isothermally until the pressure reaches that of the atmosphere, 14.7 pounds per square inch; what is the external work?

SOLUTION.— $W = 2.3026 \times 44 \times 144 \times 3.75 \times \log \frac{44}{14.7}$. In the quotient $\frac{P_1}{P_2}$, pressures in pounds per square inch may be used for $\frac{P_1}{P_2} = \frac{144\, p_1}{144\, p_2} = \frac{p_1}{p_2}$. To obtain $\log \frac{44}{14.7}$, it is most convenient to take the logarithms of the numerator and denominator separately; thus,

$$\log \frac{44}{14.7} = \log 44 - \log 14.7 = 1.64345 - 1.16732 = .47613,$$

and $W = 2.3026 \times 44 \times 144 \times 3.75 \times .47613 = 26{,}049$ ft.-lb. Ans.

10. Change of Energy and Heat Imparted.—During isothermal expansion, there is no change in the energy of the gas; for, from formula **1**, Art. **4**, $E_2 - E_1 = Jc_v G(t_2 - t_1)$; and as the temperature remains the same, $t_2 - t_1 = 0$, and $E_2 - E_1 = 0$.

The heat added during the expansion is obtained by the general formula in *Heat*, Part 1; or, $JQ = (E_2 - E_1) + W = 0 + W = W$; that is to say, *the heat imparted to the gas is equivalent to the external work.* That this must be true is also evident from the general principles stated in *Heat*, Part 1. Of all the heat supplied to the gas, there is none needed for vibration work, because there is no rise in temperature; none is needed for disgregation work, because the substance is a gas; therefore, the whole must be expended in the performance of external work.

EXAMPLE.—In example 2 of Art. **9**, what amount of heat is imparted to the gas during the expansion?

SOLUTION.— $JQ = W = 26{,}049$ ft.-lb.

$$Q = \frac{W}{J} = \frac{26{,}049}{778} = 33.48 \text{ B. T. U.} \quad \text{Ans.}$$

ADIABATIC EXPANSION

11. Suppose a quantity of gas to be confined in the cylinder, Fig. 1, and that its pressure is greater than the atmospheric pressure on the upper side of the piston; and suppose further that the cylinder and piston are made of some non-conducting material, so that no heat can be imparted to or can escape from the contained gas. Because the upward pressure against the piston is greater than the downward pressure, the piston will rise, the gas will expand, and in so doing will perform external work. An expansion of this kind, in which the gas neither receives heat from nor gives up heat to an external body, but does external work, is called an **adiabatic expansion.**

12. Change of Energy and Work Performed.—Let the general equation of energy, $JQ = E_2 - E_1 + W$, be applied to the case of an adiabatic expansion. As no heat

is imparted to or abstracted from the gas, $Q = 0$; hence,

$$0 = E_2 - E_1 + W, \text{ or } W = -(E_2 - E_1) = E_1 - E_2$$

This expression shows that during an adiabatic expansion the energy decreases from an initial value E_1 to a final value E_2, and that the external work done is equal to this decrease of energy.

Using the expressions for $E_2 - E_1$ given in formulas **1** and **2**, Art. **4**,

$$W = -J c_v G (T_2 - T_1) = J c_v G (T_1 - T_2) \qquad \textbf{(1)}$$

and
$$W = -\frac{P_2 V_2 - P_1 V_1}{k - 1} = \frac{P_1 V_1 - P_2 V_2}{k - 1} \qquad \textbf{(2)}$$

Since the gas gives up energy while expanding, it follows that its temperature falls as the expansion progresses and that T_1 is greater than T_2.

EXAMPLE.—A mass of confined air weighing .84 pound and having a temperature of 100° F. expands adiabatically until the temperature drops to 30° F. (*a*) What is the external work? (*b*) What is the loss of energy?

SOLUTION.— $T_1 - T_2 = t_1 - t_2 = 100 - 30 = 70$. Using formula **1**,

(*a*) $W = 778 \times .16902 \times .84 \times 70 = 7{,}732.1$ ft.-lb. Ans.

(*b*) $E_1 - E_2 = W = 7{,}732.1$ ft.-lb. Ans.

13. Relation Between Pressure and Volume.—Suppose that there are two cylinders, each containing 1 pound of air under the same conditions as regards pressure, volume, and temperature. The state of the gas as regards pressure and volume is indicated by the point A, Fig. 5; $O A_1$ represents the initial volume V_1, and $A_1 A$ the initial pressure.

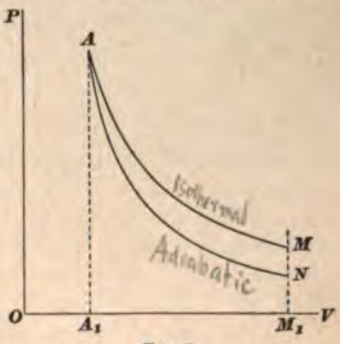


FIG. 5

Let the gas in one of the cylinders expand isothermally. The expansion then follows the law $P_1 V_1 = P_2 V_2 = P_3 V_3$, etc., and the final pressure $M_1 M$ is obtained from the general equation $P_2 V_2 = R T_2 = R T_1$, since $T_2 = T_1$; hence, $P_2 = \frac{R T_1}{V_2}$. Let the gas in the other cylinder expand adiabatically from the initial volume V_1 to the final volume V_2.

The final temperature T_2 is less than the initial T_1; therefore, the final pressure $M_1 N_1$ which is $P_2 = \frac{R\,T_2}{V_2}$, must be less than the final pressure, $P_2 = \frac{R\,T_1}{V_2}$, attained in the isothermal expansion. Starting from the same point A, the curve representing adiabatic expansion therefore lies wholly below that representing isothermal expansion.

For the isothermal expansion, the pressure varies inversely as the volume. For the adiabatic expansion, a different law is followed; the pressure varies inversely as some power of the volume. It can be shown by higher mathematics that the power is $k = \frac{c_p}{c_v}$, which, by Art. **3**, is equal to 1.405 for a perfect gas.

Then, $$P_1 : P_2 = V_2^k : V_1^k$$

and $$P_1 V_1^k = P_2 V_2^k \qquad (1)$$

or $$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^k \qquad (2)$$

As $\frac{P_1}{P_2} = \frac{p_1}{p_2}$, formula **2** may be written $\left(\frac{V_2}{V_1}\right)^k = \frac{p_1}{p_2}$, whence

$$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{k}} \qquad (3)$$

$$V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{k}} \qquad (4)$$

These formulas may be used to compute the final pressure or volume, as illustrated by the following examples:

EXAMPLE 1.—A mass of confined air having a volume of 20 cubic feet and a pressure of 80 pounds per square inch, absolute, expands adiabatically until the pressure has fallen to 38 pounds per square inch, absolute. (*a*) What is the final volume? (*b*) Calculate the external work done.

SOLUTION.—(*a*) From formula **4**, $V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{k}} = V_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.405}}$. The calculation evidently must be made with the aid of logarithms.

Then, $$\log V_2 = \log\left[V_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.405}}\right] = \log V_1 + \log \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.405}}$$

$$= \log V_1 + \frac{\log p_1 - \log p_2}{1.405} = \log 20 + \frac{\log 80 - \log 38}{1.405}$$

$$= 1.30103 + \frac{1.90309 - 1.57978}{1.405} = 1.53114$$

The number whose logarithm is 1.53114 is 33.97; therefore, $V_2 = 33.97$ cu. ft. Ans.

(*b*) Using formula **2**, Art. **12**,

$$W = \frac{144(80 \times 20 - 38 \times 33.97)}{1.405 - 1} = 109{,}916 \text{ ft.-lb. Ans.}$$

EXAMPLE. 2—If the air in example 1 expands until the final volume is 60 cubic feet, what is the final pressure?

SOLUTION.—By formula **2**, $\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^k$, from which $p_2 = p_1\left(\frac{V_1}{V_2}\right)^k = 80\left(\frac{20}{60}\right)^{1.405}$. Using logarithms,

$$\log p_2 = \log 80 + 1.405 (\log 20 - \log 60)$$
$$= 1.90309 + 1.405 (1.30103 - 1.77815) = 1.23274.$$

Hence, $p_2 = 17.1$ lb. per sq. in., nearly. Ans.

14. Relation Between Volume and Temperature. The general equation, $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$ (see *Pneumatics*), holds good for all expansions and changes of state. Clearing it of fractions,

$$P_1 V_1 T_2 = P_2 V_2 T_1 \qquad (1)$$

But

$$P_1 V_1^k = P_2 V_2^k \qquad (2)$$

in the case of adiabatic expansion. Dividing the second equation by the first, $\frac{V_1^k}{V_1 T_2} = \frac{V_2^k}{V_2 T_1}$, or $\frac{V_1^{k-1}}{T_2} = \frac{V_2^{k-1}}{T_1}$

whence,

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{k-1} = \left(\frac{V_2}{V_1}\right)^{.405} \qquad (3)$$

EXAMPLE.—Air at a temperature of 120° F. expands adiabatically from a volume of 20 cubic feet to a volume of 30 cubic feet; what is the temperature at the end of the expansion?

SOLUTION.—From formula **3**, $T_2 = T_1\left(\frac{V_1}{V_2}\right)^{.405}$. $T_1 = 460 + 120 = 580$, and $\left(\frac{V_1}{V_2}\right)^{.405} = \left(\frac{20}{30}\right)^{.405}$. Then, $T_2 = 580\left(\frac{20}{30}\right)^{.405}$. Using logarithms,

$$\log T_2 = \log 580 + .405(\log 20 - \log 30)$$
$$= 2.76343 + .405(1.30103 - 1.47712) = 2.69211.$$
$$T_2 = 492.17° \text{ and } t_2 = 492.17 - 460 = 32.17° \text{ F. Ans.}$$

15. Relation Between Pressure and Temperature. As in Art. **14**,

$$P_1 V_1 T_2 = P_2 V_2 T_1 \qquad (1)$$

and for adiabatic expansion,

$$P_1 V_1^k = P_2 V_2^k \qquad (2)$$

Taking the kth root of both members of formula **2**,

$$(P_1)^{\frac{1}{k}} V_1 = (P_2)^{\frac{1}{k}} V_2 \qquad (3)$$

Dividing formula **1** by formula **3**, member by member,

$$\frac{P_1 T_2}{(P_1)^{\frac{1}{k}}} = \frac{P_2 T_1}{(P_2)^{\frac{1}{k}}}, \text{ or } (P_1)^{1-\frac{1}{k}} T_2 = (P_2)^{1-\frac{1}{k}} T_1$$

and from this

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{1-\frac{1}{k}} = \left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}} \qquad (4)$$

Substituting for k its value, 1.405, formula **4** becomes,

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{.405}{1.405}} = \left(\frac{P_1}{P_2}\right)^{.2883} \qquad (5)$$

or,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{.2883} \qquad (6)$$

Formulas **4**, **5**, and **6** are homogeneous, and the pressures may, therefore, be taken either in pounds per square foot or in pounds per square inch.

EXAMPLE.—A mass of confined air at a pressure of 60 pounds per square inch, absolute, and a temperature of 140° F. expands adiabatically until the pressure falls to 20 pounds per square inch; calculate the final temperature.

SOLUTION.—By formula **6**, $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{.2883}$, and $T_2 = T_1\left(\frac{P_2}{P_1}\right)^{.2883}$. $T_1 = 460 + 140 = 600$. Therefore, $T_2 = 600\left(\frac{20}{60}\right)^{.2883}$. Log T_2 $= \log 600 + .2883(\log 20 - \log 60) = 2.64060$. $T_2 = 437.12$, and $t_2 = 437.12 - 460 = -22.88°$ F. Ans.

EXPANSION ACCORDING TO THE LAW PV^n = A CONSTANT

16. The isothermal and adiabatic expansions are special or limiting cases of the more general expansion, in which the relation between pressure and volume as the expansion progresses follows the law PV^n = a constant, or $P_1 V_1^n = P_2 V_2^n$ $= P_3 V_3^n$, etc.

For $n = 1$, the formula gives the isothermal case, in which sufficient heat is supplied to the gas to do the external work and there is no change in the energy. For $n = k = 1.405$, it gives the adiabatic case, in which there is no heat supplied

to the gas and the external work is done wholly at the expense of the energy of the gas.

Between these extremes, there are any number of expansions depending on the amount of heat supplied to the gas. Thus, if the heat imparted is enough to do one half the external work, the gas must give up enough energy to do the other half of the work; and, while the temperature falls, it does not fall as much as in the adiabatic case, in which the gas must give up enough energy to do all the external work. For this case, therefore, the curve representing the expansion would lie between the adiabatic and isothermal, Fig. 5, and the exponent n would lie between 1 and 1.405.

It is possible to imagine cases in which n does not lie between 1 and 1.405, but these rarely occur in engineering practice. Thus, if the heat imparted is more than sufficient to do the external work, n will be less than **1**, and the curve of the expansion will rise above the isothermal curve; on the other hand, if the gas gives up more energy than enough to do the external work, heat is abstracted from it, n is greater than 1.405, and the curve lies below the adiabatic.

17. External Work.—The following general formula for the external work done when the expansion follows the law $P_1 V_1^n = P_2 V_2^n$ is derived by the use of higher mathematics:

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

This formula is true for any expansion in which the value of n is greater than 1. It will be seen that it differs from the work formula for adiabatic expansion, formula **2**, Art. **12**, only in that n is substituted for k.

18. Heat Added to a Gas.—The change of energy is expressed by the formulas

$$JQ = (P_1 V_1 - P_2 V_2)\left(\frac{1}{n-1} - \frac{1}{k-1}\right) \qquad (1)$$

and
$$Q = G c_v \frac{n-k}{n-1}(T_2 - T_1) \qquad (2)$$

For the derivation of formulas **1** and **2**, see Appendix IV.

$P = \frac{a}{V^n}$ $W = \int P\,dv = \int \frac{a}{V^n}\,dv = a\int V^{-n}\,dv = a \left| \frac{V^{-n+1}}{-n+1} \right|$

Formulas **1** and **2** must, of course, give the same result. In cases arising in practice, it is usually more convenient to apply formula **1**, since the pressures and volumes are usually known rather than the temperatures and the weight of the gas.

In *Heat*, Part 1, the heat imparted to any substance is shown to be

$$Q = G s (T_2 - T_1)$$

in which T_1 and T_2 = the initial and final absolute temperatures;

s = specific heat;

G = weight.

Comparing this formula with formula **2**, it is seen that

$$s = c_v \frac{n-k}{n-1} \qquad (3)$$

Formula **3**, therefore, gives the specific heat of a gas expanding according to the law $p v^n$ = a constant.

For values of n lying between 1 and k, that is, between 1 and 1.405, s is negative. Thus, suppose $n = 1.2$; $s = c_v \frac{1.2 - 1.405}{1.2 - 1} = -1.025\, c_v$. For air, $c_v = .16902$ and $s = -1.025 \times .16902 = -.17325$. The negative specific heat simply signifies that the temperature falls rather than rises as heat is added.

19. Ratio of Change of Energy to External Work. For a given value of n, the change of energy of the gas is some definite percentage of the external work. Since there is usually a decrease of energy during expansion, let the decrease equal C times the external work, that is

$$E_1 - E_2 = C W \qquad (1)$$

Inserting the values of $E_1 - E_2$ and W from formula **2**, Art. **4**, and the formula of Art. **17**, respectively,

$$\frac{P_1 V_1 - P_2 V_2}{k-1} = C \frac{P_1 V_1 - P_2 V_2}{n-1}$$

whence $$C = \frac{n-1}{k-1} \qquad (2)$$

and $$n = C(k-1) + 1 \qquad (3)$$

EXAMPLE 1.—Air expanding according to the law $PV^{1.25}$ = a constant, performs 13,500 foot-pounds of external work. (*a*) What amount of energy is given up by the air? (*b*) How much heat is imparted to the air during the expansion?

SOLUTION.— $C = \frac{n-1}{k-1} = \frac{1.25-1}{1.405-1} = .6173.$

(*a*) $E_1 - E_2 = CW = .6173 \times 13{,}500 = 8{,}333.6$ ft.-lb. Ans.

(*b*) The energy imparted is equal to the difference between the work done and the energy given up by the air, or, $13{,}500 - 8{,}333.6 = 5{,}166.4$ ft.-lb. Then,

$$JQ = 5{,}166.4, \text{ and } Q = \frac{5{,}166.4}{778} = 6.64 \text{ B. T. U. Ans.}$$

EXAMPLE 2.—Air expands in such a way that two-thirds of the external work is done by the energy given up, and one-third by the heat imparted to the gas; what is the law of the expansion?

SOLUTION.—Here $C = \frac{2}{3}$. Using formula **3**,

$$n = C(k-1) + 1 = \tfrac{2}{3} \times .405 + 1 = 1.27$$

Hence, the air expands according to the law $PV^{1.27}$ = a constant, or $P_1 V_1^{1.27} = P_2 V_2^{1.27}$. Ans.

Referring to formula **2**, it is seen that when $n = 1$, the isothermal case, $C = 0$, showing that there is no change in energy; when $n = k$, the adiabatic case, $C = 1$, showing that all the work is furnished by the decrease of energy.

20. Relations Between Volume and Temperature, and Pressure and Temperature.—The formulas giving the ratio of temperatures for a given ratio of volumes or of pressures have the same form as formula **3** of Art. **14**, and formula **4** of Art. **15**; the only change is the substitution of n for k. Thus

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1} = \left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}}$$

21. Illustrative Example.—The solution of the following example illustrates the principles developed in the preceding articles:

EXAMPLE.—A mass of confined air has a pressure of 60 pounds per square inch, absolute, a volume of 8 cubic feet, and a temperature of 72° F.; it expands according to the law $PV^{1.2}$ = a constant, until the final pressure reaches 35 pounds per square inch. Find: (*a*) the final volume; (*b*) the final temperature; (*c*) the external work done; (*d*) the change of energy; (*e*) the heat imparted during the expansion.

SOLUTION.—(*a*) From the law of the expansion $P_1 V_1^{1.2} = P_2 V_2^{1.2}$, $V_2^{1.2} = V_1^{1.2}\left(\frac{P_1}{P_2}\right)$, or $V_2 = V_1\left(\frac{P_1}{P_2}\right)^{\frac{1}{1.2}} = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{1.2}}$, and

$$\log V_2 = \log V_1 + \frac{1}{1.2}(\log p_1 - \log p_2) = \log 8 + \frac{\log 60 - \log 35}{1.2} = 1.09816;$$

hence, $V_2 = 12.536$ cu. ft. Ans.

(*b*) From the formula in Art. **20**, $T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$. $T_1 = 460 + 72 = 532$, and $n = 1.2$; hence, $T_2 = 532\left(\frac{35}{60}\right)^{\frac{.2}{1.2}}$. Log $T_2 = \log 532 + \frac{.2}{1.2}(\log 35 - \log 60) = 2.68690$. $T_2 = 486.3°$ and $t_2 = 486.3 - 460 = 26.3°$. Ans.

(*c*) To find the work, use the formula in Art. **17**,

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1} = \frac{144(60 \times 8 - 35 \times 12.536)}{1.2 - 1} = 29{,}693 \text{ ft.-lb.}$$

Ans.

(*d*) From formula **2** of Art. **19**, $C = \frac{n-1}{k-1} = \frac{1.2 - 1}{1.405 - 1} = \frac{.2}{.405}$; and by formula **1** of Art. **19**,

$$E_1 - E_2 = C\,W = \frac{.2}{.405} \times 29{,}693 = 14{,}663 \text{ ft.-lb.} \quad \text{Ans.}$$

(*e*) From the general formula,

$$J\,Q = E_2 - E_1 + W = W - (E_1 - E_2) = 29{,}693 - 14{,}663 = 15{,}030 \text{ ft.-lb.};$$

hence, $Q = 15{,}030 \div 778 = 19.32$ B. T. U. Ans.

COMPRESSION OF GASES

22. The behavior of a gas when compressed according to some law is precisely the reverse of its behavior when expanding according to the same law. Let a given weight of gas expand from some initial state represented by the point A, Fig. 6, to some second state represented by the point B. The form of the curve $A\,B$ is determined by the law of the expansion, as expressed by the formula $P_1 V_1^n = P_2 V_2^n$, in which n lies between 1 and k. The characteristics of an expansion following this law are here given:

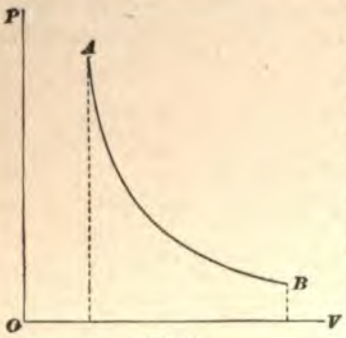


FIG. 6

The pressure falls as the volume increases.

The temperature falls.

External work is done by the gas.

The gas loses part of its energy.

Heat is imparted to the gas.

If the gas is compressed from the state *B* back to the state *A* along the same curve, that is, according to the same law, the following take place:

The volume decreases and the pressure rises.

The temperature rises.

External work is done on the gas.

The energy of the gas is increased.

Heat is abstracted from the gas.

In the case of the expansion, the initial state is represented by the point *A*; the initial conditions are distinguished by the subscript *1*, thus: P_1, V_1, and T_1. The final state is represented by *B*, and the pressure, volume, and temperature at this state are distinguished by the subscript *2*, thus: P_2, V_2, T_2.

In compressing the gas, *B* is taken as the initial state and *A* as the final state; the initial state *B* is distinguished by the subscript *1* and the final state *A* by the subscript *2*. Then all the formulas derived for the expansion of the gas hold good also for the compression, and the results will have proper signs. For example, the formula for external work, in Art. **17**, is

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1};$$

for expansion, P_1 and V_1 refer to the state *A*, and P_2 and V_2 to the state *B*, and if n is greater than 1, the product for *A* is the larger; that is, $P_1 V_1$ is greater than $P_2 V_2$, and the work W is positive. For compression, P_1 and V_1 refer to the state *B*, and P_2 and V_2 to the state *A*; hence, $P_2 V_2$ is greater than $P_1 V_1$, $P_1 V_1 - P_2 V_2$ is negative, and the result obtained for W is therefore negative. This is as it should be, for if the work done by the air during expansion is considered positive, the work done on the air during compression must be considered negative.

23. Compression of Air.—Probably the most important application of the principles of the thermodynamics of gases is found in the compression of air. The action of the ordinary air compressor is described in *Pneumatics*, which description should now be reviewed. In Fig. 7 is shown the compression cylinder with the piston at one end of the stroke. The cylinder is filled with free air, that is, air at atmospheric pressure. The volume of the air, which is equal to the volume swept through by the piston, is denoted by V_1, and the initial pressure, in pounds per square foot, by P_1. The state of the gas as regards pressure and volume is represented, with reference to the axes OP and OV, by the point A. OF is the length of the stroke of the piston, and may be conveniently taken to represent the volume V_1, and FA represents to some scale the pressure P_1.

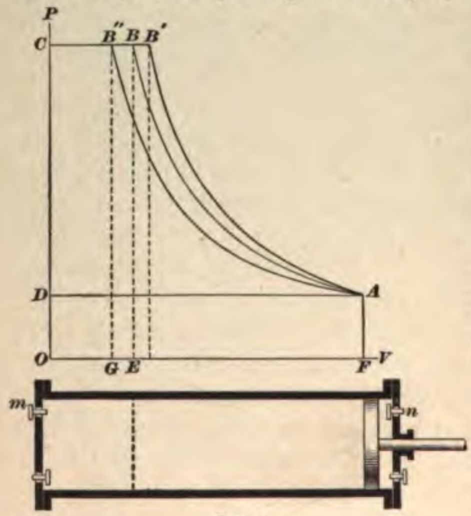


FIG. 7

As the piston moves to the left, the air is compressed and the pressure rises. If corresponding pressures and volumes are laid off from OV and OP, the points representing the successive states of the gas during the compression will lie on a curve AB. When the pressure EB is reached, the outlet valve m is forced open and the air is discharged into a receiver. The receiver is so large that the pressure is not raised appreciably by the addition of this air; hence, the pressure remains practically the same while the air is being forced from the cylinder. The line BC represents this operation. At B, the air has the volume CB and the pressure EB; since this pressure remains the same, the point representing the state of the gas must

move from B parallel to OV and approach OP. The volume CB is denoted by V_2, and the pressure EB by P_2.

The work of compressing the air from the state A to the state B is represented by the area $ABEF$ under the curve AB; and the work of pushing the air into the receiver after it has reached the state B is represented by the area of the rectangle $BCOE$ under BC. But as soon as the piston begins to move, air enters the cylinder through the valve n and exerts a pressure on the right-hand side of the piston and does work on the piston. The pressure of the air is P_1, represented by AF, and the work done on the piston is the product of this pressure by the volume swept through by the piston; hence, the work is represented by the area $ADOF$. The total work done is the sum of these three parts, each taken with its proper sign. According to Art. **22**, work done by the air on the piston is considered positive and that done by the piston on the air is negative.

The compression represented by the curve AB follows the law PV^n = a constant. From the formula of Art. **17**, the work of compression from A to B is

$$W_{AB} = \frac{P_1 V_1 - P_2 V_2}{n - 1} \qquad (1)$$

As explained in the preceding article, the work given by this formula will always have the proper sign. The work of expelling the air is

$$W_{BC} = EB \times BC = P_2 V_2 \qquad (2)$$

As this work is done by the piston on the air, it must be given the negative sign. The work done on the piston by the air in the right end of the cylinder is

$$W_{AD} = FA \times OF = P_1 V_1 \qquad (3)$$

This work is given the positive sign. The total work per stroke is therefore

$$W = W_{AB} - W_{BC} + W_{AD} \qquad (4)$$

which equals

$$\frac{P_1 V_1 - P_2 V_2}{n - 1} - P_2 V_2 + P_1 V_1 = (P_1 V_1 - P_2 V_2)\left(\frac{1}{n - 1} + 1\right)$$

or

$$W = \frac{n}{n - 1}(P_1 V_1 - P_2 V_2) \qquad (5)$$

EXAMPLE.—The initial volume of the free air in the cylinder is 16 cubic feet, and the air is compressed from atmospheric pressure to a pressure of 54 pounds per square inch gauge—that is, above the pressure of the atmosphere—according to the law $P V^{1.3}$ = a constant. (*a*) What is the work done per stroke? (*b*) If the compressor makes eighty working strokes per minute, what is the net horsepower required to drive it? (*c*) How much heat is abstracted from the air during the compression from V_1 to V_2?

SOLUTION.—First, the volume V_2 must be found. Since $p_1 V_1^n = p_2 V_2^n$, $\left(\frac{V_2}{V_1}\right)^n = \frac{p_1}{p_2}$, or $V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$. $V_1 = 16$ cu. ft.; $p_1 = 14.7$ lb.; $p_2 = 54$ lb.; gauge = 68.7 lb., absolute. Then, $V_2 = 16 \left(\frac{14.7}{68.7}\right)^{\frac{1}{1.3}}$. Log $V_2 = \log 16 + \frac{1}{1.3}(\log 14.7 - \log 68.7) = .68901$. Hence, $V_2 = 4.887$ cu. ft.

(*a*) From formula **5**,

$$W = \frac{n}{n-1}(P_1 V_1 - P_2 V_2) = \frac{1.3}{.3} \times 144 \times (14.7 \times 16 - 68.7 \times 4.887)$$
$$= -62{,}735 \text{ ft.-lb. Ans.}$$

(*b*) There are eighty working strokes per minute, and since the work found in (*a*) is the work of one stroke,

$$\text{H. P.} = \frac{80 \times 62{,}735}{33{,}000} = 152. \text{ Ans.}$$

(*c*) To determine the heat abstracted during the compression, formula **1** of Art. **18** may be used. Then,

$$JQ = 144\,(14.7 \times 16 - 68.7 \times 4.887)\left(\frac{1}{1.3 - 1} - \frac{1}{1.405 - 1}\right)$$
$$= -12{,}511 \text{ ft.-lb.}$$
$$Q = \frac{-12{,}511}{778} = -16.08 \text{ B. T. U. Ans.}$$

The result is negative, as it should be; for heat imparted to the gas has been considered as positive, and heat abstracted should therefore be negative.

24. Adiabatic and Isothermal Compression.—The fundamental formula, $JQ = E_2 - E_1 + W$, may now be considered in connection with the air-compression process. The work W being done on the gas is negative and, in practice, JQ is also negative; that is, heat is abstracted from the gas. The change of energy, however, is positive; that is, it increases, for during compression the temperature rises. Writing the formula with these signs,

$$-JQ = (E_2 - E_1) - W, \text{ or } W = E_2 - E_1 + JQ$$

It is to be noted that the W in this formula is the work of compression merely, that is, the work represented by the area $ABEF$, Fig. 7, and not the total work per stroke.

For adiabatic compression, $JQ = 0$ and $W = E_2 - E_1$. The entire work of compressing the air from A to B has been expended in increasing the energy of the air. If the compression is isothermal, there is no change of energy, $E_2 - E_1 = 0$ and $W = JQ$. The work of compression is taken away, as fast as it is performed, by the water-jacket. If the air were used as soon as it is compressed, the energy stored in it by adiabatic compression would be utilized and adiabatic compression would be as efficient as isothermal compression. As a matter of fact, the air is usually carried in mains perhaps several miles long, and in transmission cools to the temperature of the outside air and thus loses the energy due to the rise in temperature during compression.

Referring now to Fig. 7, suppose the compression to be adiabatic; then the compression curve is AB'. If it were isothermal, the compression curve would be AB'', lying below and to the left of AB'. That the adiabatic must lie to the right of the isothermal is evident; for at the final pressure, P_2, the final temperature is higher in the adiabatic case, and, therefore, the final volume CB' is greater than the final volume CB'' in the isothermal case. As has been seen, the net work per stroke is represented by the area $AB'CDA$. If the compressor is provided with a water-jacket effective enough to prevent the temperature from rising during compression, the work per stroke would be $AB''CDA$ and the work $B'AB''$ would be saved. In practice, the water-jacket is not so effective, and the actual compression curve AB lies between the adiabatic AB' and the isothermal AB''. The work saved is represented by the area $B'AB$. The following conclusions should now be evident:

The work that may be eventually derived from air at a given pressure is the same, whether it is compressed adiabatically or isothermally. The extra energy imparted to the air by raising its temperature in adiabatic compression is lost by radiation. The work of the compressor piston per

stroke is smaller the lower the final temperature is kept by the action of the water-jacket. Hence, the compression should be as nearly isothermal as possible.

25. Formula **5**, Art. **23**, holds good for any value of n except 1; that is, for any case but the isothermal. In that case, the work per stroke is represented by the area $A\,B''\,C\,D\,A$, Fig. 7, which is made up of the areas $A\,B''\,G\,F$, $B''\,C\,O\,G$, and $A\,D\,O\,F$, as described in Art. **23**. By formula **2**, Art. **9**, the work represented by the area $A\,B''\,G\,F$ is $2.3026\,P_1\,V_1 \log \frac{V_2}{V_1}$, and the remaining areas are represented by $P_1\,V_1$ and $P_2\,V_2$, as in Art. **23**. Then the total work $W = 2.3026\,P_1\,V_1 \log \frac{V_2}{V_1} + P_1\,V_1 - P_2\,V_2$. But in isothermal compression or expansion $P_1\,V_1 = P_2\,V_2$; hence,

$$W = 2.3026\,P_1\,V_1 \log \frac{V_2}{V_1} = 2.3026\,P_1\,V_1 \log \frac{P_1}{P_2}.$$

Hence, the total work, represented by the area $A\,B''\,C\,D\,A$, is the same as the work of compressing the air from A to B'', that is, from V_1 to V_2, which is represented by the area $A\,B''\,G\,F$.

EXAMPLE.—In the example of Art. **23**, what would be the work per stroke if the compression were isothermal, and what would be the percentage saved?

SOLUTION.—Here $P_1 = 144 \times 14.7$, $V_1 = 16$, and $P_2 = 144 \times (54 + 14.7)$; hence,

$$W = 2.3026 \times 144 \times 14.7 \times 16 \times \log \frac{14.7}{68.7} = -52{,}223 \text{ ft.-lb., nearly.}$$

Ans.

In the previous case, the work was 62,735 ft.-lb. Hence, the percentage saved is

$$100 \times \frac{62{,}735 - 52{,}223}{62{,}735} = 16.76 \text{ per cent. Ans.}$$

EXAMPLES FOR PRACTICE

1. If 5.68 cubic feet of air having a temperature of 50° F. is compressed adiabatically to a volume of 1.3 cubic feet, what is the final temperature? Ans. 466.7° F.

2. In example 1, if the initial pressure is 14.7 pounds per square inch, absolute, what is the final pressure?

Ans. 116.7 lb. per sq. in., absolute

3. With the same data as in examples 1 and 2, calculate the work required to compress the air when the compression is adiabatic.

Ans. 24,254 ft.-lb.

4. With the conditions the same as in example 3, calculate the work required when the compression is isothermal?

Ans. 17,730 ft.-lb.

5. Confined air having a volume of .8 cubic foot at a temperature of 120° and a pressure of 45 pounds per square inch, absolute, expands adiabatically to the pressure of the atmosphere. What is: (*a*) the final volume? (*b*) the final temperature? (*c*) the work done during expansion?

Ans. { (*a*) 1.774 cu. ft. (*b*) – 39.9° F. (*c*) 3,527.9 ft.-lb.

THERMODYNAMICS OF CLOSED CYCLES

DEFINITIONS AND PRINCIPLES

26. Cycle of Changes of State.—Thus far only those changes of state that follow some one law have been considered. **Cycles of changes of state** will now be taken up, in which there is a series of processes and in which the substance changes its state according to a succession of different laws.

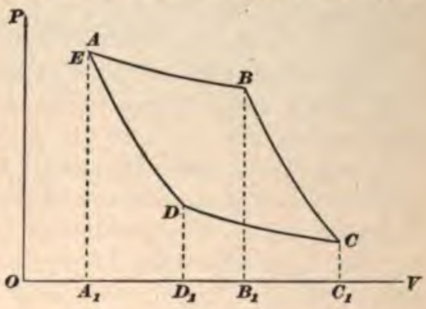


FIG. 8

A cycle of operations in which a substance, after passing through the several changes of state, is brought back to its initial state is called a **closed cycle**; otherwise it would be an **open cycle**. A closed cycle is represented graphically by a series of lines enclosing an area, as in Fig. 8. Thus, suppose the substance to start from the initial state represented by the point *A*, and to be subjected, in turn, to four changes of state represented in the pressure and volume

coordinates OP and OV, commonly called P–V coordinates, by the curves AB, BC, CD, and DE, respectively; as the substance is brought back to the initial state, the end E of the last curve must coincide with A, and the four curves must therefore form a closed figure.

The cycle of the fluid used in a heat engine consists usually of four changes of state and is represented by four curves. If the alternate curves are of the same kind, it is said to be a **simple cycle.**

27. Relation Between Heat Used and External Work.—Let W_{AB} denote the external work during the change from A to B, Fig. 8, and Q_{AB} the heat imparted during the change; and similarly for the other changes of state; also, let E_A, E_B, etc. denote the intrinsic energy of the substance in the states A, B, etc. Applying the general energy equation to each change of state, then

$$\text{for } AB,\ JQ_{AB} = E_B - E_A + W_{AB}$$
$$\text{for } BC,\ JQ_{BC} = E_C - E_B + W_{BC}$$
$$\text{for } CD,\ JQ_{CD} = E_D - E_C + W_{CD}$$
$$\text{for } DA,\ JQ_{DA} = E_A - E_D + W_{DA}$$

Adding the members of the four equations,

$$J(Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}) = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

That is, *the total external work is the equivalent of the total heat imparted.*

In any closed cycle, work must be done by the fluid during part of the cycle, and on the fluid during the remainder; that is, part of the work is positive and part negative. Referring to Fig. 8, assume that the cycle is made in the direction A–B, B–C, etc.; then from A to B and from B to C work is done by the substance, while from C to D and from D to A work is done on the substance. Therefore,

$$W_{AB} = + \text{area } A_1ABB_1$$
$$W_{BC} = + \text{area } B_1BCC_1$$
$$W_{CD} = - \text{area } C_1CDD_1$$
$$W_{DA} = - \text{area } D_1DAA_1$$

Adding, $W_{AB} + W_{BC} + W_{CD} + W_{DA} = A_1ABB_1 + B_1BCC_1 - C_1CDD_1 - D_1DAA_1 = ABCD$. That is, *the net work*

done by the substance during the cycle is represented by the area enclosed by the curves representing the successive changes of state.

EXAMPLE.—A given weight of air goes through a closed cycle of changes and in so doing has imparted to it 148 B. T. U. and gives up 123 B. T. U.; what is the net external work done?

SOLUTION.—The net heat imparted is $148 - 123 = 25$ B. T. U.
Then, $W = JQ = 778 \times 25 = 19{,}450$ B. T. U. Ans.

28. Heat Engine.—A **heat engine** is a machine or motor by which heat is transformed into work. The engine is supplied with some substance, called the **working fluid,** that is capable of receiving heat freely and of giving it up freely. This fluid receives heat from some external body, called the *source* or *hot body*, and gives up a smaller quantity of heat to another body, called the *refrigerator* or *cold body*. The heat not delivered to the refrigerator is transformed into work. In any heat engine, the working fluid goes through continuous cycles in which the original conditions are periodically repeated. In the ordinary reciprocating engine, there is a cycle for every revolution.

CARNOT'S CYCLE

29. Carnot's Heat Engine.—In 1824, Sadi Carnot, a French engineer, described an ideal engine having a simple cycle composed of isothermal and adiabatic changes of state. The conditions required by this engine cannot be complied with in practice and the engine cannot be actually constructed. Notwithstanding this fact, the study of the Carnot engine is of the first importance because it represents the limit of engine economy and is a standard by which engines may be compared with each other. The efficiency of an actual engine is always less than that of the ideal Carnot engine working between the same source of heat and the same refrigerator.

In Fig. 9, c represents the cylinder of a Carnot engine. Its walls are supposed to be perfectly non-conducting and its head a perfect conductor. The piston is also supposed to be a non-conductor of heat and to move in the cylinder

without friction. There is a source of heat s that is maintained at a constant temperature T_1, absolute, and a refrigerator r maintained at a constant temperature T_2, absolute; there is also a stand f that is a perfect non-conductor.

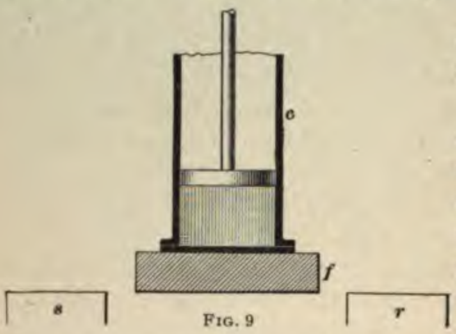


FIG. 9

The action of the engine is described as follows:

1. Let the fluid in the cylinder have the temperature T_1 of the source of heat, and let the cylinder be placed on the stand f. The piston is permitted to rise and the fluid expands adiabatically, as no heat can pass through the non-conducting walls. This expansion proceeds until the temperature drops to the temperature of the refrigerator r. Let A, Fig. 10, represent the initial state of the fluid as regards pressure and volume; then the adiabatic expansion is represented by the curve AB.

2. Next, the cylinder is placed on the refrigerator r and the fluid is compressed slowly. Heat passes through the conducting head into the refrigerator and if the compression is sufficiently slow, the passage from cylinder to refrigerator may be accomplished without any rise of temperature; that is, the compression is isothermal. This compression is represented by the curve BC.

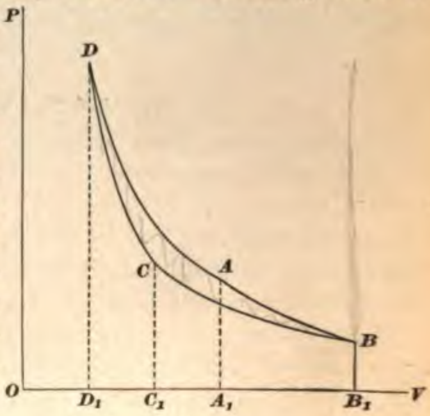


FIG. 10

3. The cylinder is now placed on the non-conducting stand f and the fluid is still further compressed; since no

heat can escape through the non-conducting walls, this compression is adiabatic and the temperature of the fluid rises. Let the adiabatic compression be continued until the temperature of the fluid is T_1, the same as the temperature of the source s. The curve CD represents this third operation.

4. For the next operation, the cylinder is placed on the source s and the fluid is permitted to expand. The source s supplies heat during the expansion and keeps the temperature of the fluid constantly at T_1; hence, the expansion is isothermal. Let the expansion proceed until the fluid attains the initial state A.

The fluid has now passed through a closed cycle of operations consisting of four changes of state, namely, adiabatic expansion, isothermal compression, adiabatic compression, and isothermal expansion. This cycle is called **Carnot's cycle.**

30. Efficiency of Carnot's Engine.—The efficiency of a heat engine is the ratio of the heat transformed into work to the whole heat supplied from the source. Thus, if an engine receives 1,000 B. T. U. from the source, transforms 150 B. T. U. into external work, and gives up the remaining 850 B. T. U. to the refrigerator, its efficiency is $\frac{150}{1000} = .15$, or 15 per cent.

Let Q_1 = heat absorbed by working fluid from source;
Q_2 = heat given up by working fluid to refrigerator;
e = efficiency.

The heat transformed into work is evidently $Q_1 - Q_2$; hence,

$$e = \frac{Q_1 - Q_2}{Q_1} \qquad (1)$$

Since in a perfect gas $Q = s\,T$, in which s is the specific heat of the gas and T the absolute temperature, $\frac{Q_1 - Q_2}{Q_1} = \frac{s\,T_1 - s\,T_2}{s\,T_1} = \frac{T_1 - T_2}{T_1}$, and

$$e = \frac{T_1 - T_2}{T_1} \qquad (2)$$

That is, *the efficiency of a Carnot engine working with a source at a temperature T_1 and a refrigerator at a temperature T_2*

is the ratio of the temperature range $T_1 - T_2$ to the temperature T_1 of the source.

31. The Reversed Heat Engine.—Carnot's heat engine is reversible; that is, it may run as described in Art. **29**, or it may traverse the cycle in the reverse order. The action of the reversed engine is as follows: Starting with the state D, Fig. 10, the cylinder is placed on the non-conducting stand and the fluid expands adiabatically to the state C. The cylinder is then placed on the refrigerator and the fluid expands isothermally as shown by the curve CB, at the same time receiving heat Q_2 from the refrigerator. Next, the cylinder is placed on the non-conducting stand and the fluid is compressed adiabatically to the state A. Finally, the cylinder is placed on the hot body and the fluid is compressed isothermally, as shown by AD. During this compression, the quantity of heat Q_1 is given up to the hot body by the fluid.

In the reversed engine, it will be observed that the total work done on the fluid during the compressions BA and AD is greater than the work done by the fluid during the expansions DC and CB; hence, the enclosed area $ADCB$ represents the net work done on the fluid by the piston. This external work is the equivalent of the difference $Q_1 - Q_2$ between the heat given to the hot body and that received from the cold body. The reversed engine, therefore, draws a certain quantity of heat Q_2 from the refrigerator, transforms a certain quantity of work W into heat, and delivers the sum $Q_2 + \frac{W}{J} = Q_1$ to the hot body.

32. Carnot's Principle.—*Of engines working between the same source and the same refrigerator, no engine can have a greater efficiency than Carnot's ideal reversible engine.* For, if another engine A should have a higher efficiency, it will do a larger amount of work than the Carnot engine for an equal expenditure of heat, both working between the same source of heat and the same refrigerator. It has been seen that when the Carnot engine is reversed, it

will require the same amount of work to drive it that it develops when running forwards. Suppose, now, that the two engines are connected together, so as to work in opposition to each other. Since engine A is capable of doing more work, the Carnot engine must run backwards. It, therefore, takes heat from the refrigerator and restores it to the hot body, as described in Art. **31**, while engine A takes heat from the hot body and gives up heat to the cold body. Assuming that there is no friction, engine A does just enough work to drive the Carnot engine, and the work of the one is equal to that of the other. But since A is more efficient, it takes less heat from the hot body than the Carnot engine would under the same conditions, and as the Carnot engine, when reversed, restores to the hot body the same amount of heat that it would take when running forwards, it follows that it must return to the hot body more heat than is drawn off by engine A. Heat would thus be delivered to a hot body from a cold body without outside aid.

Experience shows that this result cannot be attained. Heat of itself never passes from a body to a hotter body unless work is expended from without; and the supposition that a motor may run at the expense of the refrigerator leads to the conclusion that all the heat may be abstracted from the refrigerator, a result clearly impossible. Therefore, no engine can be more efficient than an ideal reversible Carnot engine for a given source and refrigerator.

33. The Second Law of Thermodynamics.—The formal statement of Carnot's principle constitutes the second fundamental law of thermodynamics. This law is stated in various ways, but each statement involves the same principle.

1. *Heat cannot, unaided by external agency, pass from a colder to a hotter body.*

2. *It is impossible to obtain work by cooling any portion of matter below the temperature of the coldest of surrounding objects.*

3. *The efficiency of the Carnot engine depends only on the temperatures of the source and refrigerator, and not on the nature*

of the working fluid; hence, all Carnot engines working between the same source and refrigerator have the same efficiency.

Carnot's principle, as shown in Art. **32,** is a direct consequence of either the first or the second statement of the law. The second statement is particularly suggestive. The atmosphere possesses an almost unlimited store of heat energy, and if some means could be found for utilizing this energy, motors could be driven without fuel. To thus utilize the heat energy of the atmosphere has been the dream of many inventors, but according to the second law it is not possible to do it. The temperature of the atmosphere may be regarded as a sort of sea level of temperature—the temperature to which all bodies either hotter or colder will ultimately attain if left to themselves. To obtain work from a hydraulic motor or waterwheel, a head of water is required; that is, there must be a fall from an altitude above the sea level. Similarly, to obtain work from a heat motor, there must be a fall of temperature, and this requires a source at a temperature higher than the temperature of the atmosphere, or, using the atmosphere as a source, a refrigerator at a temperature lower than that of the atmosphere. The last alternative is impossible, for there cannot be found in nature a portion of matter permanently colder than the atmosphere that can be used as a refrigerator.

34. Consequences of the Second Law.—According to the first law of thermodynamics, heat and work are mutually convertible; work may be transformed into heat, and vice versa. The questions now arise: With a given quantity of work, can the whole of the work or only a fraction of it be transformed into heat? and, conversely, with a given quantity of heat, can the whole or only a fraction of the heat be transformed into work? The entire quantity of work can be, and in fact usually is, transformed into heat; for example, the entire work done by an engine running with a friction brake is expended in overcoming friction and is converted into heat. On the other hand, only a fraction of the heat available can be converted into work. A quantity Q_1 is

taken from a source of heat, a smaller quantity Q_2 must be given up to a refrigerator of lower temperature, and only the difference $Q_1 - Q_2$ is transformed into work. To convert the whole of Q_1 into work, a refrigerator with a temperature at absolute zero must be found.

The next question is, what is the maximum value of the fraction $\frac{Q_1 - Q_2}{Q_1}$ of the heat Q_1 that can be converted into work? A Carnot engine converts the fraction $\frac{T_1 - T_2}{T_1}$ of the heat Q_1 into work, and according to the second law no other device can do more; hence, $\frac{T_1 - T_2}{T_1}$ is the maximum value sought. Evidently, the value of this fraction may be increased by lowering the temperature T_2 of the refrigerator. The value of the fraction $\frac{T_1 - T_2}{T_1}$ may also be increased by increasing the higher temperature T_1.

EXAMPLE.—A good ordinary steam engine requires the consumption of $2\frac{1}{2}$ pounds of coal per hour for each horsepower; taking the heating value of a pound of coal as 13,700 B. T. U., what fraction of the total heat liberated by the combustion is converted into work?

SOLUTION.—The heat resulting from the combustion is $13,700 \times 2\frac{1}{2} = 34,250$ B. T. U.; 1 H. P. is the performance of 33,000 ft.-lb. of work per min., or $33,000 \times 60 = 1,980,000$ ft.-lb. in 1 hr. The heat equivalent of this work is $Q = \frac{W}{J} = \frac{1,980,000}{778} = 2,545$ B. T. U. Of the 34,250 B. T. U. supplied, only 2,545 B. T. U. is ultimately converted into work. The efficiency is therefore $\frac{2545}{34250} = .0743$; that is, 7.43 per cent. of the total heat appears as work. Ans.

35. It must not be supposed that a heat engine is a poor and inefficient contrivance because it utilizes but a small part of the total heat supplied to it. A large part of that heat is absolutely unavailable for conversion into work, and the efficiency of the engine should be based on the ratio of the heat utilized to the available heat, rather than to the total heat. A simple hydraulic analogy will illustrate this point. Suppose that a supply of water in the Rocky Mountains is at an elevation of 12,000 feet above the sea level,

and that the water is carried to a level 600 feet lower, and is there used to drive an impulse waterwheel. If the wheel uses 90 per cent. of the energy of the fall of 600 feet, its efficiency is said to be 90 per cent. But if the water could fall to the sea level it would have energy due to a head of 12,000 feet; hence, even a perfect wheel 600 feet below the upper level can utilize but $\frac{600}{12000} = .05$, or 5 per cent., of the total energy referred to sea level. It would be manifestly unfair to credit the wheel with only 5 per cent. efficiency when it is using 90 per cent. of the energy available at that location. Similarly, it is unfair to credit a heat engine with an efficiency of only 5 to 25 per cent. based on the total heat supplied when the engine utilizes perhaps 80 per cent. of the available heat; that is, the heat that could possibly be utilized under the same conditions by a perfect Carnot engine. It would be quite as reasonable to expect the waterwheel to utilize the total fall to sea level as to expect the heat engine to utilize the total fall from the temperature of the hot body to absolute zero.

HEAT
(PART 2)

APPENDIX I

Derivation of formula **2**, Art. **2**

Let c_p denote the specific heat at constant pressure; then, since $\frac{K+W}{J}$ represents the heat imparted to the air, and $t_2 - t_1$ the rise in temperature,

$$c_p = \frac{K+W}{J(t_2-t_1)} = \frac{K+R(T_2-T_1)}{J(t_2-t_1)}$$

The difference $T_2 - T_1$ is equal to the difference $t_2 - t_1$, the temperatures above zero; hence,

$$c_p = \frac{K}{J(t_2-t_1)} + \frac{R(T_2-T_1)}{J(T_2-T_1)} = \frac{K}{J(t_2-t_1)} + \frac{R}{J} \qquad \textbf{(2)}$$

APPENDIX II

Derivation of formula **2**, Art. **4**

Multiply and divide the second member of formula **1**, Art. **4**, by R; then,

$$E_2 - E_1 = \frac{Jc_vG}{R}(RT_2 - RT_1) = \frac{Jc_v}{R}(GRT_2 - GRT_1)$$

From the general equation, $PV = GRT$; hence,

$$GRT_2 = P_2V_2 \text{ and } GRT_1 = P_1V_1$$

Substituting these values

$$E_2 - E_1 = \frac{Jc_v}{R}(P_2V_2 - P_1V_1)$$

Now, from Art. **3**, $\frac{R}{J} = c_p - c_v$, and, $\frac{J}{R} = \frac{1}{c_p - c_v}$; hence,

$$E_2 - E_1 = \frac{c_v}{c_p - c_v}(P_2V_2 - P_1V_1)$$

Dividing numerator and denominator of the fraction by c_v,

$$E_2 - E_1 = \frac{1}{\frac{c_p}{c_v} - 1}(P_2V_2 - P_1V_1), \text{ or finally,}$$

$$E_2 - E_1 = \frac{1}{k-1}(P_2V_2 - P_1V_1) = \frac{P_2V_2 - P_1V_1}{k-1} \qquad \textbf{(2)}$$

APPENDIX III

Derivation of formula **3**, Art. **6**

By formula **2**, Art. **4**, the increase of energy is

$$E_2 - E_1 = \frac{P V_2 - P V_1}{k - 1}$$

and the external work done is

$$W = P V_2 - P V_1$$

From the energy equation, *Heat*, Part 1,

$$JQ = E_2 - E_1 + W = \frac{P V_2 - P V_1}{k - 1} + P V_2 - P V_1$$

$$= \frac{P V_2 - P V_1}{k - 1} + \frac{(k - 1)\ (P V_2 - P V_1)}{k - 1}$$

$$= \frac{k}{k - 1}(P V_2 - P V_1)$$

APPENDIX IV

Derivation of formulas **1** and **2**, Art. **18**

The change of energy as given by formula **2**, Art. **4**, is

$$E_2 - E_1 = \frac{P_2 V_2 - P_1 V_1}{k - 1}$$

and from the formula in Art. **17**,

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

Then, applying the general equation,

$$JQ = E_2 - E_1 + W = \frac{P_2 V_2 - P_1 V_1}{k - 1} + \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

or $$JQ = (P_1 V_1 - P_2 V_2)\left(\frac{1}{n - 1} - \frac{1}{k - 1}\right) \qquad (1)$$

Since $P_1 V_1 = G R T_1$ and $P_2 V_2 = G R T_2$,

$$JQ = G R (T_1 - T_2)\left(\frac{1}{n - 1} - \frac{1}{k - 1}\right)$$

From the formula of Art. **3**,

$$R = J(c_p - c_v) = J c_v \left(\frac{c_p}{c_v} - 1\right) = J c_v (k - 1)$$

Substituting this value for R,

$$JQ = G J c_v (k - 1)\ (T_1 - T_2)\left(\frac{1}{n - 1} - \frac{1}{k - 1}\right)$$

whence, $$Q = G c_v (T_1 - T_2)\left(\frac{k - 1}{n - 1} - 1\right)$$

$$= G c_v \frac{k - n}{n - 1}(T_1 - T_2)$$

or, changing signs,

$$Q = G c_v \frac{n - k}{n - 1}(T_2 - T_1) \qquad (2)$$

ENTROPY AND STEAM

ENTROPY

presentation of Heat by an Area.—When a changes in volume, pressure, and temperature, it ther receives heat from some external source, or up heat. The quantity of heat transferred may nted by an area as work nted by an area in the ated in *Heat*, Part 2, in ordinates, or vertical s, were represented by and the abscissas, or horiensions, by volumes.

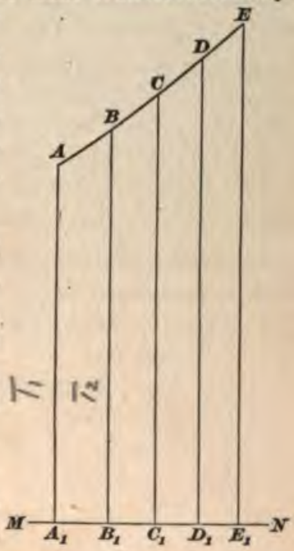


Fig. 1

y substance, say copper, at tial temperature t_1. Then, rizontal line MN, Fig. 1, line, erect a vertical line e $A_1 A$, and let its length to some scale, the initial mperature $T_1 = t_1 + 460°$, e temperature above zero. g the copper, the heat Q_1 nd the temperature rises T_2. Erect a second ordito represent the temperature T_2 to the same scale represents T_1 and place it at such a distance to f $A_1 A$ that the area $A_1 A B B_1$ represents, to some heat Q_1 imparted to the copper. To take a case, let the weight of the copper be 100 pounds

and the initial temperature 500° F. absolute. To raise the temperature 50°, that is, to 550°, requires $100 \times .0951 \times 50 = 475.5$ British thermal units, or B. T. U. (The specific heat of copper is .0951.) Then choose as scales 1 inch $= 200°$, and 1 square inch $= 500$ B. T. U.; then $A_1A = \frac{500}{200} = 2.5$ inches, $B_1B = \frac{550}{200} = 2.75$ inches, and the area A_1ABB_1 must be $\frac{475.5}{500} = .951$ square inch. Assuming the side AB to be a straight line, the average height of the two ordinates A_1A and B_1B is $\frac{2.5 + 2.75}{2}$, and the distance between the ordinates to give this area is $.951 \div \frac{2.5 + 2.75}{2} = .362$ inch.

If the temperature is raised to 600°, the ordinate C_1C representing it must be $\frac{600}{200} = 3$ inches long. As the same quantity of heat, 475.5 B. T. U., is required, the area B_1BCC_1 must be equal to the area A_1ABB_1, that is, .951 square inch; and the width B_1C_1 must therefore be $.951 \div \frac{2.75 + 3}{2} = .331$ inch. Proceeding in this way, points B, C, D, E, etc. may be located successively.

It is clear that, having chosen the scales, the points B, C, D, etc. must lie in fixed definite positions.

2. When solids and liquids are heated, provided that the solids do not melt or the liquids vaporize, the curve that represents the rise in temperature as heat is added has the general form shown in Fig. 2. The heat imparted is nearly all expended in performing vibration work, and therefore the addition of heat must be accompanied by a rise in temperature.

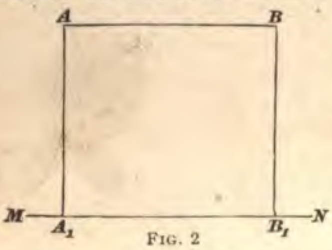


FIG. 2

In the case of gases, however, the conditions are entirely different. Because of the large amount of external work that may be done by or on the gas, the temperature may remain unchanged when heat is imparted, and it may even fall. Conversely, the temperature may rise while heat is

being abstracted from the gas, as in the case of ordinary air compression.

Suppose that the gas expands isothermally and let A_1A, Fig. 2, represent the initial temperature. As the temperature remains constant, the ordinates will be of the same length; and as it is moved to the right, the end A describes the straight line AB parallel to MN. If the gas expands adiabatically, its temperature falls; and according to the definition of an adiabatic change, no heat is conveyed to or from the gas as heat. This expansion is represented, therefore, by a contraction of the ordinate from A_1A to A_1B, Fig. 3, without motion horizontally. Since no heat is added, the area swept over must be zero.

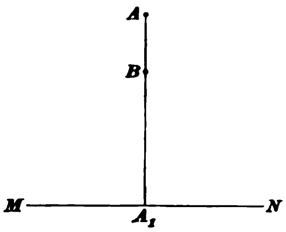


FIG. 3

If the gas in expanding follows the law pv^n = a constant, as explained in *Heat*, Part 2, and n lies between 1 and k, the temperature falls as heat is added; for if $n = 1$, the expansion is isothermal and heat is added in sufficient quantity to maintain constant temperature; and if $n = k$, the expansion is adiabatic and no heat is added. Hence, if n has a value between 1 and k, some heat will be added, but not enough to maintain a constant temperature. If the gas is compressed according to the same law, the temperature rises and heat is abstracted. The graphic representation of expansion under these conditions is shown in Fig. 4. Starting with the ordinate A_1A, the ordinate is moved to the right and at the same time shortened, so as to show the decreasing temperatures. In the final state, B_1B represents the final temperature, and the area A_1ABB_1 represents the heat imparted during the change of state. In the case of compression, suppose that B_1B represents the initial temperature. Since heat is abstracted, the ordinate is

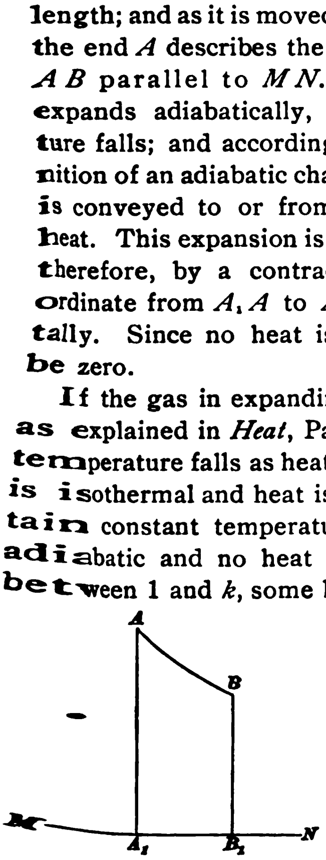


FIG. 4

moved to the left and at the same time lengthened to show the rise of temperature.

3. Analogy Between Heat Areas and Work Areas. Repeated examples of the representation of work by areas have already been considered. The work done by the expansion of the gas in each case is represented by the area lying between the expansion curve and the horizontal axis. The ordinates of points on the curves represent pressures, and the distances of the same points from the vertical axis represent the corresponding volumes.

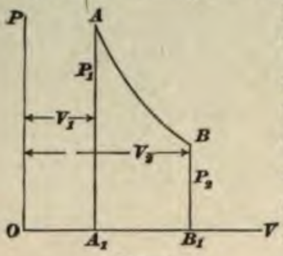


FIG. 5

In Fig. 5, OP and OV are the axes, OA_1 represents the initial volume of a gas, OB_1 the final volume, and the ordinates A_1A and B_1B represent, respectively, the initial and final pressures P_1 and P_2. In expanding from state A to state B, the gas does external work, which is represented by the area A_1ABB_1 under the curve AB. The amount of this work is W, which is equal to the average pressure multiplied by the increase of volume $(V_2 - V_1)$.

When heat is represented by an area, the ordinates represent temperatures instead of pressures. The horizontal distances of the ordinates from some assumed axis, OT, Fig. 6, represent the changing values of some quantity that has not yet been named. The heat imparted is represented by the area A_1ABB_1, Fig. 6, below AB, and the amount of heat imparted is Q = average temperature $\times$ increase of the unnamed quantity.

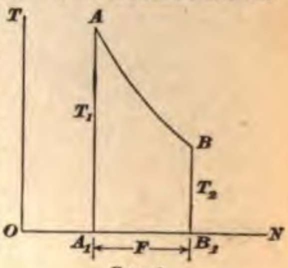


FIG. 6

Comparing Figs. 5 and 6, it appears that heat in one diagram corresponds to work in the other, that temperatures are analogous to pressures, and that the quantity or property whose initial and final values are represented by OA_1 and OB_1 in Fig. 6 corresponds to the volumes in Fig. 5.

4. Entropy.—The quantity represented by the horizontal dimensions in Figs. 1, 2, 4, and 6 is called **entropy.** The horizontal distance between the ordinates representing the initial and final temperatures, as F, Fig. 6, is the **change of entropy.** In the initial state A, Fig. 6, it is assumed that the gas has definite entropy, represented by $O A_1$; in the final state, it has a different entropy, represented by $O B_1$. The actual value of the entropy for a given state of the substance is not important; only changes of entropy enter into heat changes. Thus, in Fig. 6, the location of the point O is not important; the essential quantity is the length $A_1 B_1$, or F.

Entropy will be denoted by the letter N. The initial value of the entropy is denoted by N_1, the final value by N_2, and the difference by $N_2 - N_1$. In Fig. 6, for example, N_1 is represented by $O A_1$, N_2 by $O B_1$, and $N_2 - N_1$ by $O B_1 - O A_1 = A_1 B_1$.

5. Approximate Calculation of the Change of Entropy.—If a gas is heated isothermally, the computation of the change of entropy is a simple matter. Referring to Fig. 2, which represents the isothermal case, the heat Q is represented by the area $A_1 A B B_1$, the constant absolute temperature T by $A_1 A$, and the change in entropy $N_2 - N_1$ by $A_1 B_1$. Thus, area $A_1 A B B_1 = A_1 A \times A_1 B_1$; hence, $Q = T \times (N_2 - N_1)$; and

$$N_2 - N_1 = \frac{Q}{T}$$

that is, *the change of entropy is the quotient obtained by dividing the heat added to the gas by the absolute temperature.*

When the change of state is not isothermal, the temperature varies during the heating, and the preceding formula cannot be used; it is, however, approximately true when the change of entropy is small.

6. Exact Formula for Change of Entropy.—The following formula gives the exact change of entropy when the heat is added according to some definite law, so that the specific heat remains constant during the process. It can be derived only by the use of higher mathematics.

$dQ = G \cdot s \cdot dT$

$dQ = T \cdot dN$

$dN = \frac{dQ}{T}$

$dN = Gs \frac{dT}{T}$

$N_1 - N_2 = Gs(\log_e T_1 - \log_e T_2) = Gs \log_e \frac{T_1}{T_2}$

Let T_1 = initial absolute temperature;
T_2 = final absolute temperature;
s = specific heat;
G = weight of substance;
$N_2 - N_1$ = change in entropy.

Then, $$N_2 - N_1 = 2.3026\, G s \log \frac{T_2}{T_1}$$

For solids and liquids, the values of s, given in *Heat*, Part 1, are to be used. In the case of gases, the specific heat depends on the conditions; for heating at constant volume, it is c_v; for constant pressure, it is c_p; and for heating during an expansion following the general law $p v^n$ = a constant, it is given by the formula found in *Heat*, Part 2,

$$s = c_v \frac{n-k}{n-1}$$

EXAMPLE.—Find, by the exact formula, the change of entropy in heating 1 pound of water from an initial temperature of 500° to 600°, absolute, the specific heat being 1.

SOLUTION.— $T_1 = 500°$, $T_2 = 600°$, $G = 1$, and $s = 1$. Hence, $N_2 - N_1 = 2.3026 \times 1 \times 1 \times \log \frac{600}{500} = 2.3026 \times \log 1.2 = .1823$. Ans.

7. Construction of Temperature-Entropy Curves. The law governing the simultaneous changes of temperature and entropy is expressed by the formula in Art. **6**, and is represented graphically by a curve, as shown in Fig. 1. To obtain this curve, it is necessary to take, arbitrarily, several temperatures between the initial and final temperatures T_1 and T_2, and by the formula calculate the change of entropy for each increase or decrease of temperature. In the case shown in Fig. 1, the initial temperature is represented by $A_1 A$; some other temperature, as $B_1 B$, is then chosen, and by the formula in Art. **6** the increase of entropy $N_2 - N_1$ is calculated; then, choosing a scale of entropy, $A_1 B_1$ is laid off to represent $N_2 - N_1$ and $B_1 B$ to represent the assumed temperature; then B is a point of the desired curve. Choosing another temperature, as $C_1 C$, $A_1 C_1$ is computed and another point C on the curve obtained; and so on until the required number of points are found.

As the curvature of temperature-entropy curves is small, it is usually sufficient to obtain a few points and draw a smooth curve through them.

8. Efficiency of the Carnot Cycle.—The efficiency of the Carnot cycle is readily shown by a temperature-entropy diagram. This cycle was explained in *Heat*, Part 2. Let $ABCD$, Fig. 7, represent the heat change for such a case. The adiabatic changes are represented by the vertical lines AB and CD, and the isothermal changes by the horizontal lines DA and BC. The constant entropy during the adiabatic expansion AB will be denoted by N_2, and that during the adiabatic compression CD by N_1. The constant temperature of the expansion DA is the temperature T_1 of the source, and that of the compression BC is the temperature T_2 of the refrigerator.

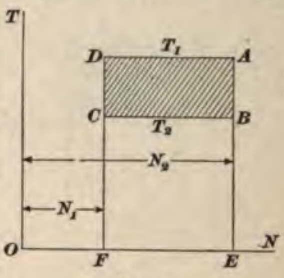


FIG. 7

The heat Q_2 rejected to the refrigerator during the compression BC is represented by the area $BCFE$; hence, Q_2 = area $BCFE = FC \times FE = T_2 \times (N_2 - N_1)$, when Q_2 is the heat given up to the refrigerator.

The heat Q_1 absorbed from the source during the expansion DA is represented by the area $DAEF$, and therefore Q_1 = area $DAEF = FD \times FE = T_1 \times (N_2 - N_1)$.

The heat transformed into work is the difference $Q_1 - Q_2 = (T_1 - T_2)\ (N_2 - N_1)$, and is represented by the shaded area $ABCD$. Then the efficiency e is the ratio of the heat transformed into work to the total heat added, or

$$e = \frac{Q_1 - Q_2}{Q_1} = \frac{(T_1 - T_2)\ (N_2 - N_1)}{T_1\ (N_2 - N_1)} = \frac{T_1 - T_2}{T_1}$$

That is, *the efficiency of the Carnot cycle is the ratio of the range of temperature $T_1 - T_2$ to the temperature T_1 of the source.*

9. Maximum Efficiency.—The efficiency of the Carnot cycle is greater than any other for a gas expanding according to the law $p v^n$ = a constant. For any expansion other

than isothermal or adiabatic, the value of n lies between 1 and k.

Assume the original state of the gas to be represented by the point A, Fig. 8. Let the gas expand isothermally from A to B. The heat added during this expansion will be represented by the area $A\,B\,G\,E$, or Q_1. If the gas expands still further, and a quantity of heat, q, is added, but not in sufficient quantity to maintain constant temperature, the expansion will be represented by some such line as $B\,C$, between $B\,I$ and $B\,G$, and the heat added, q, by the area $B\,C\,H\,G$. Such an expansion would be according to the law $p\,v^n$ = a constant, in which n lies between 1 and k.

From C, the gas is compressed isothermally to D, so that $C\,D$ shall equal $A\,B$, and the heat Q_2, equal to the area $D\,C\,H\,F$, is given up by the gas. Then the gas is compressed from D to A, according to the law of its expansion from B to C; hence, $A\,D$ will be analogous to $B\,C$, and the heat $E\,A\,D\,F$ that the gas gives up will equal the heat $B\,C\,H\,G$, or q. The shaded area $A\,B\,C\,D$ is then the heat which is transformed into useful work.

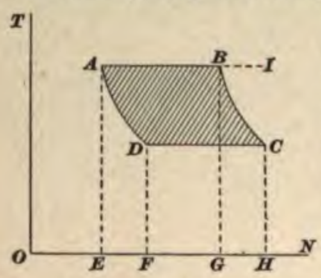


Fig. 8

This heat is equal to $(Q_1 + q) - (Q_2 + q) = Q_1 - Q_2$, and the ratio of this heat to the total heat added is the efficiency e, which is $e = \dfrac{Q_1 - Q_2}{Q_1 + q}$. But $\dfrac{Q_1 - Q_2}{Q_1 + q}$ is less than $\dfrac{Q_1 - Q_2}{Q_1}$; hence, *the Carnot cycle has a greater efficiency than any other cycle in which expansion and compression take place according to the law* $p\,v^n$ *= a constant.*

STEAM

PROPERTIES OF SATURATED STEAM

10. Behavior of Saturated Steam.—The laws governing the change of state of steam or of any vapor in contact with the liquid from which it is formed are quite different from those governing the change of state of a perfect gas. Generally speaking, the laws relating to vapors are more complicated; they are, however, of very great importance because of their application to the steam engine, refrigerating machines, etc.

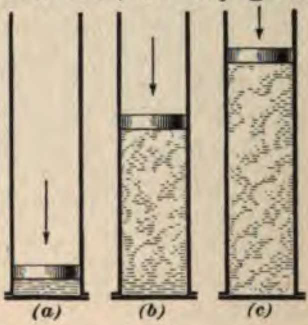


FIG. 9

To gain a clear idea of the action of steam in comparison with the action of a perfect gas, the process of generating steam from water must be understood. Suppose that a cylinder, Fig. 9 (*a*), of indefinite length is fitted with a piston, beneath which is a quantity of water. The atmospheric pressure on the piston, together with the weight of the piston, cause a definite downward pressure on the water. As the piston is free to move up and down, the pressure remains always the same. Let heat be applied to the cylinder and the contained water. The temperature of the water gradually rises, which means that the molecules of the water vibrate with gradually increasing speeds. The average length of the path of a molecule also increases, and on the whole the molecules move farther apart, as is shown by the slight increase of the volume of the water as the temperature rises. Finally, the vibrations become so rapid that the pressure on the lower side of the piston, due to the impacts of the molecules near the upper surface of the

water, slightly exceeds the downward pressure; in consequence, the piston rises a little, and thus leaves a space into which will be projected those molecules that are moving with the highest speeds. As more heat is applied, more molecules break loose and emerge from the surface, and the piston rises to make room for them. The molecules that are thus separated from the body of the water form the vapor which is commonly called **steam.**

It is found that for a given pressure, the weight per cubic foot, sometimes called the density, of the vapor has a fixed unvarying value as long as the vapor is in immediate contact with its liquid. Suppose that the piston rises, as shown in Fig. 9 (*b*). The space occupied by the vapor increases, and it might be supposed, as with a gas, that the molecules would separate farther, and that the weight per cubic foot would decrease. Such, however, is not the case. The extra space is at once filled with molecules from the liquid, and the weight per cubic foot of the steam remains the same. If heat is withdrawn and the piston descends, the weight per cubic foot is not increased. Molecules of the steam pass back into the water, or, as it is usually expressed, some of the steam condenses, so that the number of molecules per cubic inch or per cubic foot, is the same as before.

The pressure of the vapor on the under side of the piston is due to the impact of the molecules; and as the pressure remains constant and the weight per cubic foot remains constant, it follows that the same number of molecules are striking the piston and that they have the same average speed, whatever the volume of the vapor may be. This means that the temperature of the vapor, and that of the liquid also, remains constant.

11. Effect of Increased Pressure.—If the piston, Fig. 9, has a weight placed on it so as to increase the downward pressure on the liquid, it is evident that the number of impacts per unit of time must also be increased to hold the piston in equilibrium. A greater number of molecules will

strike a unit of area and the average speed will be greater than in the first case. As before, however, the number of the striking molecules and the average molecular speed are the same for all positions of the piston, that is, for all volumes.

If more weight is added, so that the pressure is still further increased, the number of molecules per unit of volume will be increased and the average speed of the molecules will be still greater.

From the preceding statements may be derived the following principles:

1. *Steam at a given pressure, if in immediate contact with water, that is, saturated steam, can have but one temperature. For every pressure, there is a corresponding temperature, and vice versa.*

2. *Under the same conditions, steam at a given pressure can have but one weight per cubic foot. For every pressure there is a corresponding definite weight per cubic foot.*

3. *The greater the pressure of the steam the higher the temperature and the greater the weight per cubic foot.*

Principles 1 and 2 are contained in the following single statement: *The temperature and weight per cubic foot of saturated steam depend only on the pressure and not at all on the volume.*

12. Saturated Steam.—Steam in immediate contact with the water from which it is being generated is called **saturated steam.** If not in contact with water, as, for example, in the cylinder of an engine, steam is still saturated when its temperature is the same as that of boiling water subjected to the same pressure. The condition of saturated steam is such that, the pressure remaining constant, any loss of heat will be followed by condensation.

As has been shown, the weight per cubic foot of steam in immediate contact with water is kept constant by the passage of molecules from the water to the vapor, or vice versa. Referring to Fig. 9 (*b*), it is seen that such an interchange of molecules is possible as long as there is any water left in the

cylinder. If, however, the water is wholly evaporated, as shown in Fig. 9 (*c*), and more heat is added and the piston is raised farther, the increase of volume must result in an increase of temperature and a decrease of weight per cubic foot, for there is no water left to supply the steam with the additional molecules necessary to keep the temperature and weight per cubic foot constant. It must not be understood, however, that saturated steam is *wet steam*, that is, steam containing particles of water, as this is not necessarily the case.

13. Superheated Steam.—Suppose, now, that heat is imparted to the water in the cylinder of Fig. 9 (*a*) until all the water is evaporated, as shown in Fig. 9 (*c*). If more heat is added, the temperature of the steam will begin to rise and the weight per cubic foot will begin to decrease, though the pressure remains constant. The steam no longer obeys the laws stated in Art. **11** but a law resembling that for a perfect gas. The more heat is added, the higher the temperature rises above the temperature of the saturated steam of the same pressure, and the more nearly the behavior of the steam resembles that of a perfect gas. Steam in this condition is said to be **superheated.**

The following distinction is usually made between saturated and superheated steam: For a given pressure, saturated steam has one temperature and one weight per cubic foot, neither of which can change so long as the steam remains in immediate contact with water. Superheated steam at the same pressure has a greater temperature and less weight per cubic foot than saturated steam, and both the temperature and weight per cubic foot may vary while the pressure remains constant if the volume increases or decreases accordingly. In other words, both the pressure and the volume of superheated steam must be constant in order to maintain a constant temperature and a constant weight per cubic foot. Hence, superheated steam may be defined as steam at a higher temperature than saturated steam and subjected to the same pressure.

14. Heat Required for Evaporation at Different Pressures.—Suppose that the piston in Fig. 9 is loaded with a weight sufficient to make the pressure on the confined water 20 pounds per square inch, absolute, and let the initial temperature of the water be 32° F., the melting point of ice. Assume the weight of the water to be 1 pound. Let heat be applied to the water. The temperature will gradually rise from 32°. The volume of the water will decrease slightly, until the temperature reaches 39.1° F., and then will increase gradually, though almost imperceptibly, until the temperature reaches nearly 228° F. At this point, the vibrations of the molecules of the water, which have become more and more rapid as the temperature has risen, have become rapid enough not only to overcome the force of cohesion, but also to raise the piston with its load. The water begins to change to the gaseous form, or, in other words, begins to become steam. The temperature now remains at about 228°, or to be exact, 227.95°, and cannot be raised above that point so long as any water remains in the bottom of the cylinder.

It is desired to know how much heat has been imparted to the water up to this state. The temperature has been raised from 32° to 227.95°. If the specific heat of water were 1 at all points of the scale, $227.95 - 32 = 195.95$ B. T. U. would be required. By experiment, however, it is found that 196.9 B. T. U. are required, the discrepancy being due to the fact that at the higher temperatures the specific heat of water is slightly greater than 1.

After heating the water to the boiling point, suppose that more heat is added until all the water is turned into steam. Experiment shows that 954.6 B. T. U. are required for this purpose. The total quantity of heat required to change the pound of water at 32° into steam at 227.95° is therefore $196.9 + 954.6 = 1{,}151.5$ B. T. U. At the end of the process, it is found that the pound of steam occupies a volume of 19.91 cubic feet.

If the piston is loaded differently, so that the pressure is changed, these quantities are all different. Thus, for

pressures of 20, 60, 100, and 200 pounds per square inch, absolute, they have the following values:

Pressure, pounds per square inch, absolute	20	60	100	200
Temperature at which boiling begins	227.95°	292.51°	327.58°	381.73°
Heat required to raise water from 32° to the boiling point, in B. T. U.	196.9	261.9	297.9	354.6
Heat required to evaporate water, in B. T. U. . . .	954.6	909.3	884.	843.8
Total heat, in B. T. U. . .	1,151,5	1,171.2	1,181.9	1,198.4
Volume of 1 pound of steam, in cubic feet	19.91	7.096	4.403	2.294

A study of these values shows, in a general way, how the quantities vary as the pressure is increased. The temperature of the steam is higher at the higher pressures. The heat required to raise the temperature of the water, of course, increases as the final temperature is increased. On the other hand, the heat required to change the water at the temperature of the boiling point into steam at the same temperature is less the higher the pressure. There is not much difference in the total heat required; but as the pressure rises the total heat increases. The most marked change is in the volume of the pound of steam. The relation between the pressure and volume approximates to Boyle's law; thus, when the pressure is double, the volume per pound is nearly one-half.

15. Table of the Properties of Saturated Steam. In problems connected with steam engineering, it is necessary to use very frequently some or all the quantities mentioned in the preceding article. For a given steam pressure, the temperature or the volume per pound, the total heat, the heat required to raise the temperature of the water, or the heat required for the evaporation, must be known.

The values of these various quantities, and of some others not yet mentioned, have been determined either by experiment or calculation for a wide range of pressures, and these values arranged in tabular form constitute the **Steam Table,**

found at the end of this Section. In the following paragraphs, each of the properties tabulated is considered, and, as far as practicable, the methods used in determining the tabular values are given.

16. Relation Between Pressure and Temperature. The temperatures of saturated steam corresponding to various pressures were determined, experimentally, by the French physicist, Regnault. The experiments, however, were made with the greatest care and accuracy and the results are generally accepted by engineers.

The curve in Fig. 10 shows, graphically, the relation between pressures and corresponding temperatures, as given by Regnault's experiments. Pressures are measured along the horizontal scale and temperatures along the vertical scale. It will be observed that for small pressures, the temperature rises rapidly as the pressure increases; but at the greater pressures, those exceeding 150 pounds per square inch, the increase of temperature is smaller for a corresponding increase of pressure. The rate of increase is shown, graphically, by the steepness of the curve; for pressures under 50 pounds per square inch, the curve is very steep, showing the rapid rate of increase of temperature; while for pressures above 150 pounds, the curve gradually becomes more nearly horizontal, showing the slow rate at which the temperature rises.

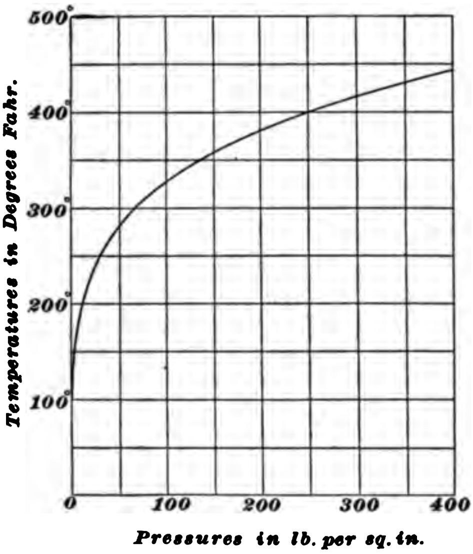


FIG. 10

The form of the curve has an important bearing on the question of steam-engine economy. It has been shown that the fraction $\frac{T_1 - T_2}{T_1}$ measures the greatest possible efficiency of an ideal engine. As T_2 has reached nearly the lowest practical limit, increase of efficiency must be effected by raising T_1, the initial temperature of the steam. Fig. 10, however, shows that with the high pressures at present used, any further increase of T_1 will be small in proportion to the increase of pressure. Hence, the economy in the use of saturated steam is limited largely by the strength of the material that forms the receptacle for the steam.

The formula given by Regnault for computing the pressures from the temperatures, or vice versa, is complicated and difficult to apply; hence, it is not given. The following formula, known as Rankine's formula, is simpler and gives results that agree quite closely with Regnault's experiments:

$$\log p = 6.1007 - \frac{2,730}{T} - \frac{393,670}{T^2}$$

in which p = absolute pressure of steam, in pounds per square inch;

T = absolute temperature, which equals $t + 460$.

EXAMPLE.—If the temperature of saturated steam is 290° F., what is the pressure?

SOLUTION.— $T = t + 460° = 290° + 460° = 750°$. $\text{Log } p = 6.1007 - \frac{2,730}{750} - \frac{393,670}{750^2} = 1.76084$. The number whose logarithm is 1.76084 = 57.66; hence, p = 57.66 lb. per sq. in. Ans.

17. Illustrations of the Use of the Steam Table. In the Steam Table, the pressures, in pounds per square inch, are given in column 1 and the corresponding temperatures are given in column 2. The temperatures are calculated from Regnault's formula. For any pressure given in the table, the corresponding temperature is found in column 2 in the same horizontal line; for a pressure lying between two pressures in the table, the corresponding temperature must be found by interpolation, as illustrated in the following examples.

EXAMPLE 1.—Find the temperature corresponding to a pressure of 147 pounds per square inch, absolute.

SOLUTION.—Referring to the Steam Table,

$$\begin{array}{lll} & \text{for } p = 150 \text{ lb.}, & t = 358.26° \\ \text{and} & \text{for } p = 145 \text{ lb.}, & t = 355.59° \\ \hline & \text{Difference, 5 lb.,} & 2.67° \end{array}$$

Difference for 1 lb. difference of pressure is $\frac{2.67°}{5} = .534°$. 147 lb. − 145 lb. = 2 lb., the given difference of pressure; and for this, the difference in temperature is $2 \times .534° = 1.068°$, or 1.07°, taking two decimal places. Hence, the increase of 2 lb. from 145 lb. to 147 lb. is accompanied by an increase in temperature of 1.07°. Therefore, adding the increase 1.07° to the temperature 355.59° corresponding to 145 lb., the temperature for 147 lb. is $355.59° + 1.07° = 356.66°$. Ans.

EXAMPLE 2.—The pressure in a steam boiler as shown by the gauge is 87 pounds per square inch. What is the temperature of the steam?

SOLUTION.—The absolute pressure is $87 + 14.7 = 101.7$ lb. per sq. in. This pressure, in the Steam Table, lies between the values 100 and 105.

$$\begin{array}{ll} \text{for } p = 105 \text{ lb.}, & t = 331.13° \\ \text{for } p = 100 \text{ lb.}, & t = 327.58° \\ \hline \text{Difference, 5 lb.,} & 3.55° \end{array}$$

For 1 lb. change of pressure, the difference in temperature is $\frac{3.55°}{5} = .71°$. From 100 lb. to 101.7 lb., the change of pressure is 1.7 lb., and the corresponding change of temperature is $.71° \times 1.7 = 1.207°$, or 1.21° as the values in the Steam Table contain but two decimal places. For 101.7 lb., therefore, the temperature is $327.58° + 1.21° = 328.79°$. Ans.

EXAMPLE 3.—What is the pressure of steam at a temperature of 285° F.?

SOLUTION.—From the Steam Table,

$$\begin{array}{ll} \text{for } t = 285.72°, & p = 54 \text{ lb.} \\ \text{for } t = 283.32°, & p = 52 \text{ lb.} \\ \hline \text{Difference, } 2.40°, & 2 \text{ lb.} \end{array}$$

From $t = 283.32°$ to $t = 285°$, the increase of temperature is 1.68°. Now, since an increase of temperature of 2.40° gives an increase of pressure of 2 lb., the increase of 1.68° must give an increase of pressure of $\frac{1.68}{2.40} \times 2 \text{ lb.} = 1.4 \text{ lb.}$ Hence, the required pressure is 52 lb. + 1.4 lb. = 53.4 lb. Ans.

EXAMPLES FOR PRACTICE

1. The temperature of steam being 224°, what is the corresponding pressure taken from the Steam Table? Ans. 18.577 lb. per sq. in.

2. Find the pressure in example 1 by using the formula in Art. **16.** Ans. 18.537 lb. per sq. in.

3. Find the pressure corresponding to a temperature of 312°: (*a*) by means of the Steam Table; (*b*) by the formula in Art. **16.**

Ans. $\begin{cases} (a) \text{ 80.23 lb. per sq. in.} \\ (b) \text{ 80.15 lb. per sq. in.} \end{cases}$

4. Find, from the Steam Table, the temperature corresponding to a gauge pressure of 210 pounds per square inch. Ans. 391.67°

---

18. The Energy Equation Applied to the Vaporization of Water.—The heat imparted to a substance is expended in doing three kinds of work: (1) vibration work; (2) disgregation work; (3) external work. (See *Heat*, Part 1.) This statement, of course, holds true for the heat imparted to 1 pound of water at 32° F. to change it to steam at the temperature of the boiling point.

Taking the case given in Art. **14,** 1 pound of water at 32° is heated to 227.95°, the boiling point under the given pressure of 20 pounds per square inch, and at that temperature it is changed into saturated steam. The heat required for the entire process is 1,151.5 B. T. U. This heat is divided among the three kinds of work mentioned.

To raise the temperature of the water from 32° to 227.95° requires 196.9 B. T. U.; hence, of the total heat this amount is spent in performing the vibration work, that is, the work of increasing the temperature. After the boiling point is reached, no more vibration work is done, because there is no further increase of temperature until the water is all changed to steam. The remaining 954.6 B. T. U. is expended in disgregation work and external work. To find how the heat is divided between these two works, it is necessary to determine one of them. The external work is easily determined; after the water is changed to steam, its volume is 19.91 cubic feet, while the original volume of the water was about .016 cubic foot. The increase in volume is therefore

19.91 cubic feet − .016 cubic foot = 19.894 cubic feet. Referring to Fig. 9, this is the volume that the piston will sweep through in passing from its initial to its final position. The expansion of the steam from the initial to the final volume has been effected against a constant pressure of 20 pounds per square inch = 20 × 144 = 2,880 pounds per square foot, and the external work done is therefore $W = P(V_2 - V_1)$ or 2,880 × 19.894 = 57,294.7 foot-pounds. The heat that must be expended in doing this work is

$$\frac{W}{J} = 57{,}294.7 \div 778 = 73.6 \text{ B. T. U.}$$

Subtracting this from 954.6 B. T. U., the heat left for the performance of disgregation work will be 954.6 − 73.6 = 881.0 B. T. U. Hence, of the 1,151.5 B. T. U. supplied for the entire process, 881 B. T. U, or 76½ per cent. of the whole, is expended in separating the molecules against their mutual attractions, that is, in changing the substance from the liquid to the gaseous state; about 17 per cent. is used for doing vibration work, that is, raising the temperature and about 6½ per cent. is used to do the external work. Hence, the energy equation may be written in the form,

$$Q = K + D + \frac{W}{J}$$

in which Q = total heat imparted, or 1,151.5 B. T. U.
K = vibration work, or increase of kinetic energy, or 196.9 B. T. U.
D = disgregation work, or increase of potential energy, or 881.0 B. T. U.
$\frac{W}{J}$ = heat equivalent of external work, or 73.6 B.T.U.

19. Definitions and Symbols.—It would be quite practicable to use the symbols Q, K, etc., and the terms vibration work or kinetic energy, disgregation work or potential energy, etc.; but in connection with steam and other vapors, special symbols and special names for the quantities that those symbols represent are generally used.

The **total heat of saturated steam**, or simply the **total heat,** for any given pressure is the number of heat units

required to raise the temperature of 1 pound of water from 32° F. to the boiling point for the given pressure and change it into steam at that pressure. The letter H is used to denote the total heat.

The **heat of the liquid** is the heat required to raise the temperature of 1 pound of water from 32° to the boiling point. It is the vibration work or increase of kinetic energy denoted by K in the general equation. The special symbol used for this quantity is the letter q. Sometimes, the expression **sensible heat** is used instead of heat of the liquid.

The **heat of vaporization**, or, simply, the **latent heat**, is the heat required to change 1 pound of water at the boiling point into steam at the same temperature; it includes both the heat expended in disgregation work and that expended in external work. The symbol r is used for this quantity.

The **internal latent heat** is that part of the latent heat that is expended in disgregation work; it is denoted by the letter i.

The **external latent heat** is that part of the latent heat that is expended in doing the external work. It is denoted by the expression $\frac{Pu}{J}$, which is derived as follows: The pressure of the steam, in pounds per square foot, is denoted by P; and the difference between the volume, in cubic feet, of the pound of steam and the volume, in cubic feet, of the pound of water from which it is formed is denoted by u. The external work, in foot-pounds, is the product of the constant pressure P and the increase of volume u, that is, $W = Pu$; hence, the heat required for the performance of this external work is $\frac{Pu}{J}$.

The **entropy of the liquid**, denoted by n, will be discussed later.

The **specific volume** of the steam, represented by V, is the volume, in cubic feet, of 1 pound of steam.

The weight, in pounds, of 1 cubic foot of steam is denoted by the symbol w; this is frequently termed the **density** of the steam.

20. Relations Between H, q, r, i, and $\frac{Pu}{J}$.—The fundamental energy equation $Q = K + D + \frac{W}{J}$ becomes, when the special symbols are introduced,

$$H = q + i + \frac{Pu}{J} \qquad (1)$$

For, by definition,

$Q = H$ = total heat imparted;
$K = q$ = vibration work, in B. T. U., or heat of the liquid;
$D = i$ = disgregation work, in B. T. U., or internal latent heat;
$\frac{W}{J} = \frac{Pu}{J}$ = heat required for external work, or external latent heat.

When the internal and external latent heats are added together, their sum is the heat required to evaporate the water after it has been raised to the boiling point; that is, it is the heat of vaporization. Then,

$$r = i + \frac{Pu}{J} \qquad (2)$$

whence and

$$\left.\begin{aligned} H &= q + r \\ r &= H - q \end{aligned}\right\} \qquad (3)$$

The quantity q is the vibration work and measures the increase of kinetic energy, and the quantity i is the disgregation work and measures the increase of potential energy. The sum $q + i$ is therefore the total increase of energy expressed in B. T. U., and $J(q + i)$ is the same quantity expressed in foot-pounds. When 1 pound of water at 32° is changed into steam of a given pressure, the pound of steam possesses $J(q + i)$ foot-pounds of energy more than the original pound of water at 32°. Denoting by E the energy, in B. T. U., of a pound of steam above water at 32°,

$$E = q + i$$

Substituting in formula **1**,

$$H = E + \frac{Pu}{J} \qquad (4)$$

That is, the total heat imparted is expended in increasing the energy by the amount E and in doing the work Pu.

21. Total Heat.—The total heat of saturated steam at different temperatures was determined experimentally by Regnault. The following formula was given by him as representing closely the results of his experiments:

$$H = 1{,}081.94 + .305\, t$$

in which t denotes the temperature of the steam, in degrees F.

Values of H corresponding to different pressures are given in column 4 of the Steam Table, in B. T. U. per pound.

Owing to the unavoidable inaccuracies in the most careful experiments, it is not be expected that the values given in the Steam Table are reliable beyond five significant figures. It will be observed that not more than five figures are given in any case, and in some of the columns only three or four figures are given. In giving results of solutions, therefore, no more figures should be retained than are used in the Steam Table.

EXAMPLE 1.—Find, from the Steam Table, the total heat of a pound of saturated steam at a pressure of 63 pounds per square inch, gauge.

SOLUTION.—The absolute pressure is $63 + 14.7 = 77.7$ pounds per square inch. From the Steam Table,

for p = 78 lb., H = 1,176.5 B. T. U.
for p = 76 lb., H = 1,176.0 B. T. U.

Difference, 2 lb., .5 B. T. U.
Difference, 1 lb., .25 B. T. U.

The difference between the given pressure and 76 lb. is $77.7 - 76 = 1.7$ lb. For a difference of 1.7 lb., the change of total heat is $1.7 \times .25 = .425$ B. T. U. Hence, for 77.7 lb., $H = 1{,}176.0 + .425 = 1{,}176.425$, say 1,176.4 B. T. U. Ans.

EXAMPLE 2.—Find by the formula the total heat of 1 pound of saturated steam at a temperature of 267° F.

SOLUTION.—Using the above formula, $H = 1{,}081.94 + .305 \times 267 = 1{,}163.375$ B. T. U., say 1,163.4 B. T. U. Ans.

22. Heat of the Liquid.—The specific heat of water varies slightly at different temperatures, but it is not necessary to take this variation into account except in cases in which extreme accuracy is desired. For ordinary calculations, sufficient accuracy is obtained by taking the specific heat of water as 1 throughout the entire range of temperature.

Hence, the heat required to raise 1 pound of water from 32° to a temperature t is

$$q = s(t-32) = 1 \times (t-32) = t-32 \qquad (1)$$

In other words, the heat of the liquid is found, approximately, by subtracting 32 from the temperature. If more accurate values of q are desired, they may be obtained from the Steam Table.

It is frequently necessary to find the heat required to raise 1 pound of water from some temperature other than 32°, as 50°, 70°, or 110°, to the boiling point and evaporate it at the boiling temperature. H B. T. U. are required if the process is started at the initial temperature 32°. If, instead, the process is started at some initial temperature t higher than 32°, the heat required in the process is evidently less than H by the quantity of heat required to raise the temperature of the pound of water from 32° to t. As has been shown, the number of B. T. U. required for this purpose is approximately $t - 32$; hence, the total heat for the process is

$$H - (t-32) = H - t + 32 \qquad (2)$$

EXAMPLE 1.—What quantity of heat is required to raise the temperature of 1 pound of water from 32° to 212°?

SOLUTION.—From formula **1**, $q = 212 - 32 = 180$ B. T. U. Ans.

EXAMPLE 2.—Feedwater enters a steam boiler at a temperature of 65° F.; the pressure, as registered by the gauge, is 80 pounds per square inch. How much heat is required to heat 1 pound of this feedwater to the boiling point and evaporate it?

SOLUTION.—For a pressure of $80 + 14.7 = 94.7$ lb. absolute, H, by interpolating from the Steam Table, is 1,180.6 B. T. U. The heat required is therefore

$$H - t + 32 = 1{,}180.6 - 65 + 32 = 1{,}147.6 \text{ B. T. U. Ans.}$$

23. Heat of Vaporization.—From formula **3** of Art. **20**, $r = H - q$. Having determined H and q from the formulas in Arts. **21** and **22**, the value of r may be found by subtraction. In the Steam Table, column 3 contains the heat of the liquid and column 5 the heat of vaporization, in B. T. U. per pound of steam, for the pressures tabulated. It will be observed that the values in column 5 are obtained by merely subtracting those in column 3 from those in column 4.

24. External and Internal Latent Heat.—The heat of vaporization r is divided into two parts: $\frac{Pu}{J}$, the heat required for external work; and i, the internal latent heat. Since, from Art. **20**, $r = i + \frac{Pu}{J}$, $i = r - \frac{Pu}{J}$.

In the Steam Table, the volume V of a pound of steam is given in column 9. By subtracting m, the volume of a pound of water, from V, the difference u is obtained. From column 1, the pressure is obtained, in pounds per square inch; hence, the external latent heat is

$$\frac{Pu}{J} = \frac{144 \times p \times (V - m)}{778}$$

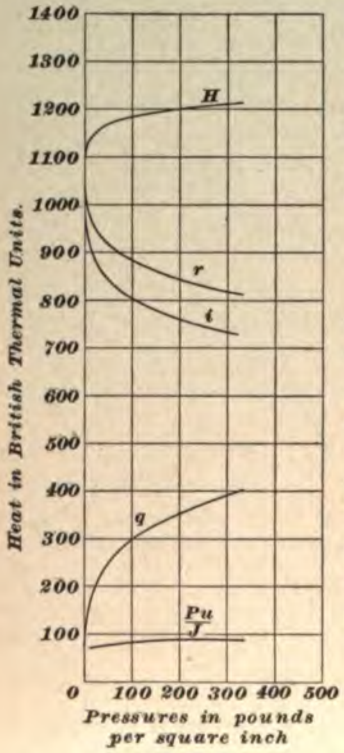


FIG. 11

For example, by taking a pressure of 40 pounds per square inch, absolute, the volume of 1 pound of steam from column 9 is 10.37 cubic feet. At ordinary temperatures, the volume m of 1 pound of water is .016 cubic foot, but at higher temperatures it is slightly increased, and may be taken as .017 cubic foot. Hence, $u = V - m = 10.37 - .017 = 10.353$ cubic feet, and the external latent heat is $\frac{144\,p\,u}{J} = \frac{144 \times 40 \times 10.353}{778} = 76.7$ B. T. U., which is the value given in the Steam Table for a pressure of 40 pounds. The heat of vaporization r is 927 B. T. U.; therefore, the internal latent heat is $i = r - \frac{Pu}{J} = 927 - 76.7 = 850.3$ B. T. U.

In the Steam Table, i and $\frac{Pu}{J}$, in B. T. U. per pound, are tabulated in columns 6 and 7, respectively.

25. Curves for H, q, r, i, **and** $\frac{Pu}{J}$.—The variation of the total heat, heat of the liquid, etc. with the pressure is shown by the curves of Fig. 11. The pressures are laid off on the horizontal base line and the different quantities of heat H, q, r, etc. are erected as ordinates.

26. Entropy of the Liquid.—The addition of the heat q to the water in raising the temperature from 32° to the boiling point is accompanied by an increase of entropy. The process of heating the water is shown graphically in Fig. 12. The ordinate FA represents, to some scale, the initial absolute temperature $32° + 460° = 492°$, and the ordinate EB represents, to the same scale, the absolute temperature of the boiling point. Starting with FA, move the ordinate to the right and at the same time lengthen it in such a way that the length of the ordinate always represents the absolute temperature of the water, while the area swept over from the beginning represents the heat that has been imparted. The end of the ordinate travels along the curve AB, and the area $ABEF$ between this curve and the axis ON represents the heat of the liquid q; that is, the heat imparted in raising the temperature from 32° to the boiling point. The horizontal distance FE through which the ordinate moves represents the increase of entropy during the heating of the water. This increase is called the entropy of the liquid.

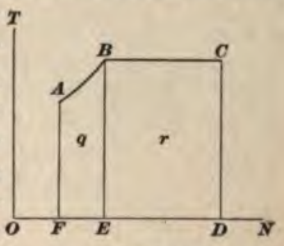


FIG. 12

Accurate computations of the entropy of the liquid are made by the method explained in Art. 6.

For water the specific heat s varies slightly at different temperatures, and for exact results the variation must be taken into account.

In the Steam Table, the values of the entropy of the liquid for the different pressures are given in column 8.

27. Entropy of the Steam.—In Fig. 12, the point B represents the state of the water at the boiling point before any of it has been changed to steam. Up to this point, the heat q has been added. The second step in the process is the vaporization of the water at the constant temperature represented by EB. During this process, the heat of vaporization r is imparted; hence, to represent this addition of heat at constant temperature, the ordinate EB is moved to the right, keeping it the same length, since the temperature does not change, until the rectangular area swept over represents r to the same scale that area $ABEF$ represents q. Suppose that DC is the final position of the ordinate; then $r = \text{area } BCDE = EB \times ED = T \times ED$, and $ED = \frac{r}{T}$. Hence, the increase of entropy during vaporization is obtained by dividing the heat of vaporization r by the absolute temperature T at which the vaporization occurs.

The total change of entropy N from the initial state A, of water at 32°, to the final state C, of steam at the temperature of the boiling point, is called the **entropy of the steam**, and is, of course, represented by FD. Now, $FD = FE + ED$; $FE = n$, and $ED = \frac{r}{T}$; hence, the entropy of the steam $N = n + \frac{r}{T}$.

The entropy of the steam is not given in the Steam Table, but as n, r, and T can be found for any pressure, the entropy can easily be calculated.

EXAMPLE.—Calculate the entropy of saturated steam for a pressure of 200 pounds per square inch, absolute.

SOLUTION.—From the Steam Table, $t = 381.73$, $r = 843.8$, and $n = .5429$. Then,

$$N = n + \frac{r}{T} = .5429 + \frac{843.8}{460 + 381.73} = 1.5453. \text{ Ans.}$$

28. Specific Volume of Steam.—The volume V of 1 pound of saturated steam at different pressures is given in

column 9 of the Steam Table. The values of V have been determined by experiment and also by calculation. The results from the different methods agree well, but those obtained by calculation are considered the most reliable.

29. Weight of Saturated Steam.—The weight of saturated steam in pounds per cubic foot for the different pressures and temperatures, is given in column 10 of the Steam Table.

The weight is the reciprocal of the specific volume. For example, 1 pound of steam at 44 pounds absolute pressure occupies a volume of 9.484 cubic feet; therefore, 1 cubic foot of steam at 44 pounds pressure must weigh $\frac{1}{9.484} = .1054$ pound. Evidently, the values in column 10 may be obtained by taking the reciprocals of those in column 9.

30. Examples in the Use of the Steam Table.—The following examples, in addition to those already given, illustrate the use of the Steam Table. These should be studied carefully, and all the Examples for Practice that follow the illustrative examples should be solved.

It must always be borne in mind, in using the Steam Table, that the pressures are reckoned from vacuum, and not from atmospheric pressure. That is, 14.7 pounds must be added to all gauge pressures to make them available for use in the Steam Table. Again, the heat of the liquid, the total heat, and the latent heat are calculated from 32° F., not from 0° F. A great deal of trouble will be saved by remembering these points.

EXAMPLE 1.—Find the heat required to change 6 pounds of water at 32° into steam at 84 pounds pressure, absolute.

SOLUTION.—The total heat of 1 lb. at 84 lb. pressure is from column 4, by interpolation, 1,178.1 B. T. U., nearly.

$$1{,}178.1 \times 6 = 7{,}068.6 \text{ B. T. U. Ans.}$$

EXAMPLE 2.—How many heat units are required to raise $8\frac{1}{2}$ pounds of water from 32° to 250° F.?

SOLUTION.—Looking in column 3, the heat of 1 lb. of water at 250.27° is seen to be 219.4. Hence, the value of q for 250° will be practically $219.4 - .27 = 219.13$ B. T. U., since for an increase of 1°

in the temperature of the water approximately 1 B. T. U. is required. Then, for $8\frac{1}{2}$ lb., the heat required is

$$219.13 \times 8\tfrac{1}{2} = 1{,}862.6 \text{ B. T. U.} \quad \text{Ans.}$$

EXAMPLE 3.—How many foot-pounds of work will be required to change 60 pounds of water at the temperature corresponding to the boiling point under a pressure of 80 pounds, absolute, into steam at the same temperature?

SOLUTION.—Looking in column 5, the latent heat of vaporization, r, is 895.6; that is, it takes 895.6 B. T. U. to change 1 lb. of water at 80 lb. into steam at the same pressure. Therefore, it takes $895.6 \times 60 = 53{,}736$ B. T. U. to perform the same operation with 60 lb. of water. The work required is therefore

$$W = Jr = 53{,}736 \times 778 = 41{,}806{,}608 \text{ ft.-lb.} \quad \text{Ans.}$$

EXAMPLE 4.—Find the volume occupied by 14 pounds of steam at 30 pounds gauge pressure.

SOLUTION.—Absolute pressure $= 30 + 14.7 = 44.7$ lb. per sq. in. From the Steam Table,

$$\begin{array}{lll} & \text{for } p = 44 \text{ lb.,} & V = 9.484 \text{ cu. ft.} \\ - & \text{for } p = 46 \text{ lb.,} & V = 9.097 \text{ cu. ft.} \\ \hline & \text{Difference, } 2 \text{ lb.,} & \quad .387 \text{ cu. ft.} \end{array}$$

The difference for 1 lb. is $\frac{.387}{2} = .1935$. $44.7 - 44 = .7$ lb. actual difference in pressure. $.1935 \times .7 = .135$ difference in volume. As the pressure increases, the volume decreases; and to obtain the volume at 44.7 lb., it is necessary to subtract the difference .135 from the volume at 44 lb.; thus, for $p = 44.7$, $V = 9.484 - .135 = 9.349$ cu. ft. The volume of 14 lb. is 14×9.349 cu. ft. $= 130.89$ cu. ft. Ans.

EXAMPLE 5.—Find the weight of 40 cubic feet of steam at a temperature of 254° F.

SOLUTION.—From column 10 of the Steam Table, the weight w of 1 cu. ft. of steam at 253.98 is .07820 lb. $254 - 253.98 = .02$. Neglecting the .02°, the weight of 40 cu. ft. is therefore $.07820 \times 40 = 3.128$ lb. Ans.

EXAMPLE 6.—How many pounds of steam at 64 pounds pressure, absolute, are required to raise the temperature of 300 pounds of water from 40° to 130° F., the water and steam being mixed together?

SOLUTION.—The number of heat units required to raise 1 lb. from 40° to 130° is $130° - 40° = 90$ B. T. U. Actually, a little more than 90 would be required, but the above is near enough for all practical purposes. Then, to raise 300 lb. from 40° to 130° requires $90 \times 300 = 27{,}000$ B. T. U. This quantity of heat must necessarily come from the steam. Now, 1 lb. of steam at 64 lb. pressure gives up, in condensing, its latent heat of vaporization, or 906.2 B. T. U.; but, in

addition to its latent heat, each pound of steam on condensing must give up an additional amount of heat in falling to 130°. Since the original temperature of the steam was 296.74° F. (see Steam Table), each pound gives up by its fall of temperature 296.74 − 130 = 166.74 B. T. U. Consequently, each pound of the steam gives up a total of 906.2 + 166.74 = 1,072.94 B. T. U., and $\frac{27,000}{1,072.94}$ = 25.16 lb. of steam will therefore be required to accomplish the desired result. Ans.

EXAMPLES FOR PRACTICE

1. Find the total heat of 25 pounds of saturated steam at a gauge pressure of 113 pounds per square inch. Ans. 29,685 B. T. U., nearly

2. Find, from the Steam Table: (*a*) the internal latent heat of 1 pound of steam at a pressure of 90 pounds gauge; (*b*) the external latent heat; (*c*) the entropy of the liquid.

$$\text{Ans.}\begin{cases}(a)\ 800\text{ B. T. U., nearly}\\(b)\ 81.4\text{ B. T. U., nearly}\\(c)\ .4777\end{cases}$$

3. What is the latent heat of vaporization corresponding to a pressure of 85 pounds, absolute? Ans. 892.5 B. T. U.

4. Find, from the Steam Table, the heat of the liquid of 10 pounds of water at a temperature of 297.6°. Ans. 2,670.9 B. T. U.

5. What is the latent heat of vaporization of the water in example 4? Ans. 9,056 B. T. U.

6. How many B. T. U. are required to convert 25 pounds of water at 32° into 109.6 cubic feet of saturated steam? Ans. 29,550 B. T. U.

7. Find the number of heat units required to change 11 pounds of water at 50° into steam at 100 pounds absolute pressure. Ans. 12,803 B. T. U., nearly

8. Find the weight of 712 cubic feet of steam at a pressure of 33 pounds, gauge. Ans. 81 lb., nearly

9. How many pounds of steam at 47.3 pounds pressure, gauge, are required to raise 120 pounds of water from 55° to 160° at atmospheric pressure? Ans. 12.07 lb.

10. Find the volume of 19 pounds of steam at a temperature of 274°. Ans. 177.27 cu. ft.

STEAM AND WATER MIXTURES

31. Total Heat of a Mixture.—Suppose that 1 pound of water is placed in a cylinder, as in Fig. 9, and that heat is applied to it until a part, but not all, of the water is changed to steam, as shown in Fig. 9 (*b*). Let the weight of the

water that is changed to steam be denoted by x; then $1-x$ will denote the weight of water that remains in the liquid state. If the total weight of the mixture is G pounds, then Gx denotes the weight of the steam and $G(1-x)$ the weight of the water. Evidently, x is a proper fraction—the fraction of the whole weight that exists as steam; thus, if one-fourth of the whole weight is evaporated, $x = \frac{1}{4}$ or .25, or 25 per cent.

To heat 1 pound of water at 32° from the initial state to the final state, in which the part x is steam and the part $(1-x)$ is water, requires how much heat? Evidently, the same heat q is required to raise the temperature of 1 pound of water from 32° to the boiling point, whether all or only a part is evaporated; hence, q B. T. U. must be imparted. To evaporate the whole of 1 pound of the water, the additional heat r B. T. U. must be supplied; hence, to evaporate the fraction x of it, only xr B. T. U. are required. Therefore, the heat required per pound is $q + xr$; and for G pounds, it is

$$Q = G(q + xr)$$

EXAMPLE 1.—How many B. T. U. are required to raise the temperature of 20 pounds of water from 32° to the boiling point and evaporate 60 per cent. of it at a pressure of 210 pounds per square inch, absolute?

SOLUTION.—From the Steam Table, for the given pressure, $q = 358.9$ B. T. U. and $r = 840.7$ B. T. U., $x = .60$, and $G = 20$; hence,

$$Q = 20 \times (358.9 + .60 \times 840.7) = 17{,}266.4 \text{ B. T. U. Ans.}$$

EXAMPLE 2.—One pound of mixture at 60 pounds, absolute, contains 3 per cent. water: (*a*) If it changes into water at 110°, how many B. T. U. will be given up? (*b*) How many pounds of this mixture would need to be run into 200 pounds of water at 56° to raise the temperature of the water to 110°?

SOLUTION.—(*a*) Substituting in the formula the values from the Steam Table, $Q = 1\,(261.9 + .97 \times 909.3) = 1{,}143.9$ B. T. U. In changing to water at 32°, the pound of mixture gives up 1,143.9 B. T. U.; but in changing to water at 110°, it gives up $110 - 32 = 78$ B. T. U. less, or $1{,}143.9 - 78 = 1{,}065.9$ B. T. U. Ans.

(*b*) To raise 200 lb. of water from 56° to 110° requires $200 \times (110 - 56) = 10{,}800$ B. T. U. Since each pound of the mixture

gives up 1,065.9 B. T. U., 10,800 ÷ 1,065.9 = 10.13 lb. of steam would be required. Ans.

32. Volume of a Mixture.—Since 1 pound of saturated steam has a volume denoted by V, the weight $G x$ of steam has a volume of $G x V$. The volume of $G(1-x)$ pounds of water has the volume $G(1-x)m$, where m, as usual, denotes the volume, in cubic feet, of 1 pound of water. According to Art. **24**, in steam problems, the value of m is generally taken as .017. The total volume of the mixture is, therefore,

$$v = G x V + G(1-x)m = G x V - G x m + G m$$
$$= G x (V - m) + G m$$

But, $V - m = u$. Hence,

$$v = G x u + G m = G(x u + m)$$

The volume of 1 pound of the mixture is evidently $x u + m$ or $x(V-m) + m = x V + m(1-x)$.

EXAMPLE.—What is the volume of the 10.13 pounds of the mixture in example 2 of Art. **31**?

SOLUTION.— $G = 10.13$, $x = .97$, $u = V - m = 7.096 - .017 = 7.079$, and $m = .017$; hence, using the above formula,

$$v = 10.13(.97 \times 7.079 + .017) = 69.73 \text{ cu. ft. Ans.}$$

33. Energy of a Mixture.—Of the total heat supplied to 1 pound of mixture, part is expended in increasing the energy and part in doing external work. The volume of 1 pound of the mixture is $x u + m$, and that of the water in the initial state is m; therefore, the increase in volume is $x u + m - m = x u$, and the external work done is $P \times x u = P x u$. The heat required for this work is $\frac{P x u}{J}$.

Since the total heat supplied per pound is $q + x r$, the difference, $q + x r - \frac{P x u}{J} = q + x\left(r - \frac{P u}{J}\right)$, is the heat expended in the increase of energy, expressed in B. T. U. Now, $r - \frac{P u}{J} = i$, from formula **2** of Art. **20**, and the increase of energy is therefore $(q + x i)$ B. T. U. For G pounds of mixture the increase of energy is given by the formula,

$$E = G(q + x i)$$

EXAMPLE 1.—Calculate the energy above that of water at 32° contained in 5 pounds of a mixture of steam and water at 160 pounds pressure, absolute; the mixture is 70 per cent. steam. Give results in B. T. U. and also in foot-pounds.

SOLUTION.—Referring to the Steam Table, at the given pressure $q = 335.4$, and $i = 774$; hence, from the formula,

$$E = 5 \times (335.4 + .70 \times 774) = 4{,}386 \text{ B. T. U. Ans.}$$

Also, $$E = 4{,}386 \times 778 = 3{,}412{,}308 \text{ ft.-lb. Ans.}$$

EXAMPLE 2.—A volume of 5 cubic feet of a mixture that is 80 per cent. steam and the pressure of which is 60 pounds per square inch, absolute, expands until its pressure reduces to 40 pounds, absolute; in the final state, the mixture contains 75 per cent. steam. Calculate: (*a*) the weight of the mixture; (*b*) the final volume; (*c*) the energy above 32° in the first state; (*d*) the energy above 32° in the second state; (*e*) the change of energy in foot-pounds.

SOLUTION.—(*a*) The volume of 1 lb. of the mixture under the given conditions is, by the formula in Art. **32**,

$$x V + m (1 - x) = .80 \times 7.096 + .017 \times (1 - .80) = 5.68 \text{ cu. ft.}$$

hence, the weight of 5 cu. ft. is $5 \div 5.68 = .8803$ lb. Ans.

(*b*) In the second state, the volume, from the formula in Art. **32**, is

$$v = .8803 \times [.75 (10.37 - .017) + .017] = 6.85 \text{ cu. ft. Ans.}$$

(*c*) In the first state, $q = 261.9$ and $i = 830.7$; hence, from the above formula,

$$E_1 = .8803 \times (261.9 + .80 \times 830.7) = 815.56 \text{ B. T. U. Ans.}$$

(*d*) In the final state, $q = 236.4$ and $i = 850.3$; hence,

$$E_2 = .8803 \times (236.4 + .75 \times 850.3) = 769.49 \text{ B. T. U. Ans.}$$

(*e*) The decrease of energy is

$$815.56 - 769.49 = 46.07 \text{ B. T. U., or } 46.07 \times 778 = 35{,}842 \text{ ft.-lb.}$$

Ans.

34. Entropy of a Mixture.—According to Art. **27**, the entropy of the steam is given by the expression $n + \frac{r}{T}$, where n is the entropy of the liquid. The way in which the entropy of the steam varies as the temperature rises may now be examined, and for ease of calculation temperatures corresponding to the pressures given in the Steam Table, and differing by about 100°, may be used. The data and results are given in Table I.

Assume axes $O\,T$ and $O\,N$, Fig. 13, and lay off on $O\,T$ the lengths OA_1, OA_2, etc., to represent the absolute temperatures in column T, Table I. From these points, lay off horizontally

A_1B_1, A_2B_2, etc. to represent the entropies of the liquid given in column n, and A_1C_1, A_2C_2, etc. to represent the entropies of the steam, as given in column $n + \frac{r}{T}$. Thus, $OA_2 = 702.21$, $A_2B_2 = .3570$, and $A_2C_2 = 1.7022$. The

TABLE I

Pressure	t	T	r	n	$\frac{r}{T}$	$n+\frac{r}{T}$
3	141.62	601.62	1,015.3	.2013	1.6876	1.8889
26	242.21	702.21	944.6	.3570	1.3452	1.7022
120	341.05	801.05	874.	.4911	1.0911	1.5822
220	389.84	849.84	837.8	.5529	.9858	1.5387

points B_1, B_2, etc. lie on a curve EF, and the points C_1, C_2, etc. lie on a second curve RS. The first curve passes through a point E on OT such that $OE = 460° + 32° = 492°$. The curve EF is called the **liquid curve**, because the points on it represent the substance in a liquid state at different pressures; the curve RS is called the **saturation curve**, because points on it represent the substance at different pressures in the state of a saturated vapor. It may be noted that the curve RS approaches OT at the higher pressures, for the reason that the entropy of the steam decreases as the pressure increases. Having these curves, it is easy to determine, graphically, the entropy

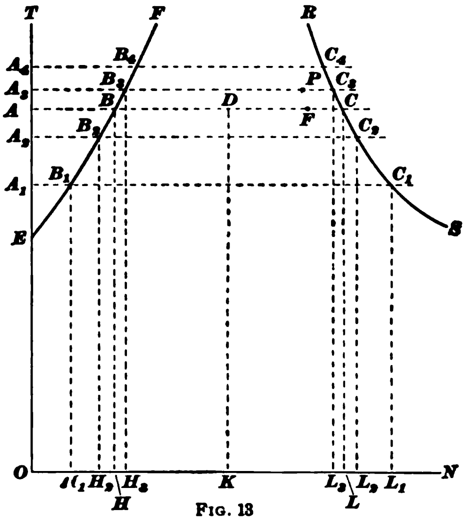


FIG. 13

of the liquid or the entropy of the steam for any given temperature. Lay off OA to represent the temperature to the proper scale, and draw a horizontal line through A cutting the curves in B and C, respectively; then $AB = n$, the entropy of the liquid, $BC = \frac{r}{T}$, and

$$N = n + \frac{r}{T} \qquad \textbf{(1)}$$

As previously shown, area $OEBH = q$, and area $HBCL = r$. (See Arts. **26** and **27**.)

Next consider the case of 1 pound of mixture of steam and water, of which the part x is steam and the part $1 - x$ is water. When the whole pound is vaporized, the heat r is imparted during the vaporization, and the change of entropy is $\frac{\text{heat imparted}}{\text{absolute temperature}} = \frac{r}{T}$; if, now, only the fraction x is vaporized, only the heat xr is imparted after the boiling point is reached, and the change of entropy is $\frac{xr}{T}$. The change of entropy during the heating of the water from 32° to the boiling point is n, and the entropy of the mixture is therefore

$$N = n + \frac{xr}{T} \qquad (2)$$

In Fig. 13, $AB = n$ and $BC = \frac{r}{T}$; hence, if the point D is located on BC so that $BD = x \times BC$, then AD will represent the entropy N of the mixture; for $AB = n$, and $BD = x \times BC = x\frac{r}{T}$. If $x = \frac{1}{4}$, D will lie one-fourth of the distance from B to C; if $x = \frac{3}{4}$, D will lie nearer C, so that $BD = \frac{3}{4}\,BC$ and $DC = \frac{1}{4}\,BC$; and so on.

Conversely, if a point representing the state of the mixture is given, the value of x can be found. Thus, suppose that the state is represented by the point F, Fig. 13; the height of F above ON gives the temperature, and the ratio $BF : BC$ gives the value of x.

EXAMPLE.—(*a*) Calculate the entropy of a mixture of which 85 per cent. is steam at a pressure of 120 pounds, absolute. (*b*) Locate, on Fig. 13, the point representing this state.

SOLUTION.—(*a*) From Table I, $n = .4911$ and $\frac{r}{T} = 1.0911$.

$$N = n + \frac{xr}{T} = .4911 + .85 \times 1.0911 = 1.4185. \quad \text{Ans.}$$

(*b*) Since OA_3 represents the temperature corresponding to the given pressure, the point P, representing the state, must lie on A_3C_3, and in such a position that $\frac{B_3P}{B_3C_3} = .85$. Ans.

35. Wet Steam and Quality of Steam.—Steam from a boiler usually contains a small percentage of water in the form of mist or fine water particles held in suspension; sometimes the moisture present is considerable. Steam in this state is called **wet steam.** Evidently wet steam is merely a mixture of steam and water, and in calculations relating to wet steam the formulas in Arts. **31** to **33** must be used.

The **quality of the steam** is the ratio of the weight of the steam to the weight of the mixture. The quality is numerically equal to x, as defined in Art. **31.** Thus, if in 1 pound of wet steam .85 is steam and 15 is water, the quality is .85, or 85 per cent.

EXAMPLES FOR PRACTICE

1. Compute the heat required to raise 1 pound of water from a temperature of 52° F., and evaporate 83 per cent. of it at a pressure of 62 pounds per square inch, gauge. Ans. 1,003.5 B. T. U.

2. Calculate the energy of the mixture in the final state, example 1, above that of water at 32°. Give results in B. T. U. and in foot-pounds. Ans. $\begin{cases} 957.3 \text{ B. T. U.} \\ 744{,}779.4 \text{ ft.-lb.} \end{cases}$

3. Calculate the volume of 8 pounds of wet steam, quality 96 per cent., at a pressure of 200 pounds per square inch, absolute. Ans. 17.624 cu. ft.

SUPERHEATED STEAM

36. Preliminary Statement.—Superheated steam has recently received much more attention than formerly, largely because of the increasing importance of the steam turbine as a heat motor. It has been found that for the best results, the steam turbine must be supplied with superheated steam.

It has been found also that the use of steam moderately superheated in ordinary reciprocating engines gives a marked increase in economy, and the practice of superheating is becoming quite common, especially in Europe, where even locomotives are occasionally equipped with superheating devices.

Unfortunately, the properties of superheated steam have not yet been carefully determined, and much doubt exists as to some of the important quantities, as specific heat, total heat, etc. The formulas usually employed are based on experiments performed a number of years ago, and future experiments may show them to be unreliable. They are, however, the only ones at command, and they must be used until more accurate knowledge of the action of superheated steam is obtained.

37. Relation Between Pressure, Volume, and Temperature.—The formula connecting the pressure, volume, and temperature of superheated steam is as follows:

Let p = absolute pressure, in pounds per square inch;
V = volume of 1 pound, in cubic feet;
v = volume of G pounds, in cubic feet:
T = absolute temperature, in degrees Fahrenheit.

Then for 1 pound

$$V = .591\frac{T}{p} - .135 \qquad (1)$$

and for G pounds

$$v = .591\frac{G\,T}{p} - .135\,G \qquad (2)$$

EXAMPLE.—(*a*) What is the volume of 1 pound of superheated steam at 540° F. and having a pressure of 80 pounds per square inch, absolute? (*b*) What is the weight of 1 cubic foot under the same conditions?

SOLUTION.—(*a*) $T = 540 + 460 = 1{,}000$ and $p = 80$. Substituting in formula **1**,

$$V = .591 \times \tfrac{1000}{80} - .135 = 7.25 \text{ cu. ft. Ans.}$$

(*b*) $$w = \frac{1}{V} = \frac{1}{7.25} = .138 \text{ lb. Ans.}$$

38. Specific Heats.—Regnault's experiments on the specific heat of superheated steam show that at constant

pressure the specific heat is constant and has the value .48. The specific heat at constant volume, as calculated from Doctor Fenner's equations, is variable, being .351 at a pressure of 5 pounds and .341 at 200 pounds.

39. Total Heat.—The **total heat** of superheated steam is the heat required to change 1 pound of water at 32° into superheated steam of a given pressure and temperature. Assuming that the heat is added at constant pressure, the process may be divided into three stages: (1) The liquid is heated from 32° to the boiling temperature, during which operation the heat of the liquid q is imparted. (2) The water is vaporized, and in vaporizing absorbs the heat r. (3) The steam is heated from its saturated state to the final temperature; this operation requires the heat .48 × (range of temperature). If t_s denotes the final temperature and t the temperature of the boiling point for the given pressure, then the total heat imparted is

$$Q = q + r + .48(t_s - t) = H + .48(t_s - t) \qquad (1)$$

Another formula, in which values from the Steam Table are not required, is the following:

$$Q = .48(T - 10.27\sqrt[4]{P}) + 857.2 \qquad (2)$$

in which P = pressure in pounds per square foot.

Example.—Calculate the total heat in the example of Art. **37.**

Solution.—At a pressure of 80 lb., the Steam Table gives t = 311.8 and H = 1,177. From formula **1**,

$Q = 1{,}177 + .48 \times (540 - 311.8) = 1{,}286.5$ B. T. U. Ans.

Or, using formula **2**,

$Q = .48(1{,}000 - 10.27\sqrt[4]{80 \times 144}) + 857.2 = 1{,}286.2$ B. T. U. Ans.

40. Entropy of Superheated Steam.—Having considered the specific heat and total heat of superheated steam, its entropy may now be considered. For the first two stages, the graphic representation of the heat change is the same as that for saturated steam, as shown in Fig. 12. The heat q imparted to the liquid is represented by the area $OABL$, Fig. 14, and the change of entropy is $OL = n$. In the second stage, the water is vaporized, the heat r = the area $LBCM$ is imparted, and the increase of

entropy is $LM = \frac{r}{T}$, where T is the absolute temperature at which vaporization takes place. During the third stage, that is, while the steam is being superheated, the temperature rises from T, represented by MC, to the final temperature of the superheat T_s, represented by ND. The heat imparted is $.48\ (T_s - T) = .48\ (t_s - t)$ B. T. U., and is represented by the area $MCDN$. The increase of entropy is represented by MN.

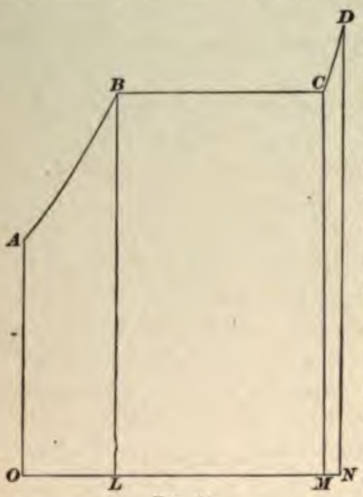


FIG. 14

The three areas represent, respectively, the three parts of the total heat in formula **1** of Art. **39**, and the total area represents the total heat. Thus,

$$\text{Area } OABCDN = OABL + LBCM + MCDN$$

$$\text{or} \qquad Q \quad = \quad q \quad + \quad r \quad + .48(t_s - t)$$

The increase of entropy denoted by MN can be calculated from the general formula for change of entropy in Art. **6**. Using .48 for the specific heat s, then, for 1 pound,

$$MN = 2.3026 \times .48 \log \frac{T_s}{T} = 1.105 \log \frac{T_s}{T} \qquad \textbf{(1)}$$

The total entropy N_s of superheated steam in the state D, Fig. 14, is represented by ON. Hence,

$$ON = OL + LM + MN$$

$$\text{or,} \qquad N_s = n + \frac{r}{T} + 1.105 \log \frac{T_s}{T} \qquad \textbf{(2)}$$

in which n = entropy of liquid;
r = heat of vaporization

as given in the Steam Table.

But the expression $n + \frac{r}{T}$ gives the entropy of saturated steam. Hence, the entropy of superheated steam is equal

to the entropy of saturated steam plus the entropy of the superheat, 1.105 log $\frac{T_s}{T}$.

EXAMPLE.—Find the entropy of 1 pound of superheated steam at an absolute pressure of 110 pounds per square inch and superheated to 480° F.

SOLUTION.—For 110 lb. pressure, from the Steam Table, t = 334.56, r = 878.8, and n = .4826. Hence, substituting in formula **2**,

$$N_s = .4826 + \frac{878.8}{334.56 + 460} + 1.105 \log \frac{480 + 460}{334.56 + 460} = 1.669. \quad \text{Ans.}$$

41. Adiabatic Expansion of Superheated Steam. Let p_1, v_1 denote the initial pressure and volume, and p_2, v_2 the final pressure and volume. Then, during an adiabatic expansion of superheated steam, the following equation gives the relation between them:

$$p_1 v_1^{\frac{4}{3}} = p_2 v_2^{\frac{4}{3}}$$

It will be observed that this formula is the same as that derived for the adiabatic expansion of a perfect gas, except that the exponent 1.405 is replaced by $\frac{4}{3}$.

During the adiabatic expansion, the superheated steam approaches more nearly the saturated state, and if the expansion is carried far enough, the steam becomes saturated and then wet.

EXPANSION OF STEAM

42. Consider a closed cylinder with a piston, as shown in Fig. 15. The left end of the cylinder is open to the entrance of steam from the boiler and the right end is open to the atmosphere, so that the steam on the right side of the piston may freely escape. The pressure of the steam from the boiler will drive the piston to the right end of the cylinder, pushing the steam on the right of the piston into the atmosphere.

FIG. 15

When the piston arrives at the right of the cylinder, the right end is put in communication with the boiler and the left end with the atmosphere; the piston then moves back to the left end. The operation is thus repeated indefinitely.

43. Suppose that, during the entire movement of the piston from left to right, the left end of the cylinder is in communication with the boiler, and, further, that the steam on the right of the piston escapes freely into the atmosphere without requiring any work on the part of the piston to push it out. It is necessary to know the relation between the pressure and the volume at all points of the stroke, and the

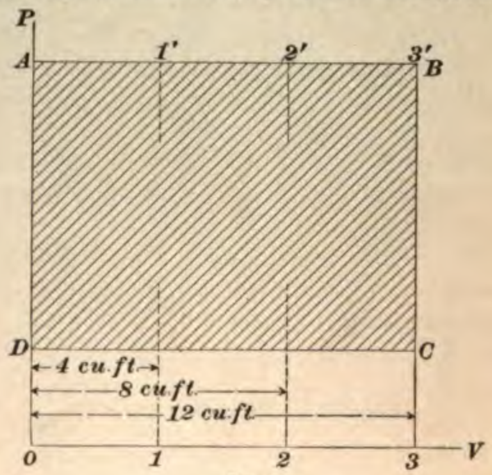


FIG. 16

work done by the steam. The relation between pressures and volumes may be shown graphically as follows: Draw two lines OV and OP, at right angles to each other, as shown in Fig. 16. Then, the volume of the steam for any position of the piston may be represented by some distance on the line OV, and likewise the pressure of the steam at any position of the piston may be represented by the length of a vertical line parallel to OP. Suppose that the area of the piston in Fig. 15 is 2 square feet, and that the distance moved through by the piston is 6 feet. The length $O3$ of the diagram then represents a piston travel of 6 feet, or a

steam volume of $6 \times 2 = 12$ cubic feet. Likewise, $O\,1$ represents a piston travel of 2 feet and a steam volume of 4 cubic feet, and $O\,2$ a piston travel of 4 feet, and a steam volume of 8 cubic feet.

Now, through *1*, *2*, and *3* draw lines parallel to $O\,P$, and on these lines lay off the length $1\text{–}1'$, $2\text{–}2'$, $3\text{–}3'$, representing, respectively, the pressures of the steam at the volumes $O\,1$, $O\,2$, and $O\,3$. Connect the points $1'$, $2'$, and $3'$ by a line, and the diagram gives a complete graphic representation of the relation between the pressures and volumes.

44. Since the left end of the cylinder is in communication with the boiler during the whole motion of the piston from left to right, and since the temperature and pressure of the steam in the boiler is constant, it follows that the temperature and pressure of the steam in the cylinder will be the same for all positions of the piston. Suppose that the pressure of the steam, when the piston is just starting from the left end of the cylinder, to be 60 pounds per square inch, absolute. Let 1 inch on the diagram represent a pressure of 30 pounds. Then the length $O\,A$, which represents the steam pressure in question, must have a length of $\frac{60}{30} = 2$ inches; and since the pressure is constant for all positions of the piston, the lengths $1\text{–}1'$, $2\text{–}2'$, and $3\text{–}3'$, etc., must each be equal to 2 inches. In other words, the line of pressures $A\,B$ must be parallel to the line $O\,V$. It was observed that the temperature of the steam is also constant throughout the stroke of the piston. Hence, the above line $A\,B$ is the isothermal (constant temperature line) of saturated steam; therefore, it follows that the isothermal of saturated steam is a straight line parallel to $O\,V$, the axis of volumes.

45. The pressure $O\,A$ on the left of the piston is taken above absolute zero. There is also a pressure on the right of the piston of 14.7 pounds per square inch, since the right end of the cylinder is open to the atmosphere. Let this be represented by the height $O\,D = \frac{14.7}{30} = .49$ inch. Since this atmospheric pressure is constant throughout the stroke

of the piston, it may be represented by the straight line DC parallel to OV. The net pressure on the piston is then represented by $OA - OD = DA = 60 - 14.7 = 45.3$ pounds per square inch. The work done by the steam during one stroke of the piston may now be easily found.

There is a constant net pressure throughout the stroke of 45.3 pounds per square inch $= 45.3 \times 144 = 6{,}523.2$ pounds per square foot. The total pressure on the piston is, therefore, $6{,}523.2 \times 2 = 13{,}046.4$ pounds. Work is defined as pressure, force, or resistance multiplied by the distance through which it acts. Consequently, the work spent in moving the piston from one end of the cylinder to the other, a distance of 6 feet, is $13{,}046.4 \times 6 = 78{,}278.4$ foot-pounds.

46. The work may be obtained in a still more convenient way. As shown above, the pressure on the piston per square foot is 6,523.2 pounds. The volume of the cylinder is area $\times$ length $= 2$ square feet $\times$ 6 feet $= 12$ cubic feet. Now, multiplying the pressure, in pounds per square foot, by the volume of the cylinder, in cubic feet, the result is $6{,}523.2 \times 12 = 78{,}278.4$ foot-pounds, as before.

Let P = pressure on piston, in pounds per square foot;
p = pressure on piston, in pounds per square inch;
V = volume, in cubic feet, swept through by piston;
W = work, in foot-pounds.

Then, $$W = PV = 144\,pV$$

Referring to Fig. 16, it will be remembered that the line DA represents the pressure p, and the line DC represents the volume V. The product $DA \times DC$ = area of the rectangle $ABCD$. But $DA \times DC = pV$. Therefore, the area of the rectangle is proportional to the work of the steam.

$DA = \dfrac{45.3}{30} = 1.51$ inches. $DC = 2$ inches. Then, the area $ABCD = 1.51 \times 2 = 3.02$ square inches. But 1 inch in height equals 30 pounds per square inch, and 1 inch in length is $\frac{12}{2} = 6$ cubic feet. Hence, $pV = 3.02 \times 30 \times 6 = 543.6$ foot-pounds, and $W = 144\,pV = 543.6 \times 144 = 78{,}278$ foot-pounds.

47. In the same way, it can be shown that, no matter what the form of the area $ABCD$ may be, it will represent the work done by the steam, provided that the upper line AB represents the relation between the pressures and volumes of the steam on one side of the piston, and the lower line CD represents the relation between the pressures and volumes on the other side. For the area is equal to the length of the diagram multiplied by the mean ordinate, and the mean ordinate simply represents the average net pressure on one side of the piston.

Hence, in general, to find the work of the steam from the diagram, multiply the area, in square inches, by the vertical scale of pressures, by the horizontal scale of volumes, and by 144. The result is the work, in foot-pounds.

EXAMPLE.—The area of a diagram of the character shown in Fig. 16 is 7.34 square inches; the vertical scale of pressures is 36 pounds to the inch, and the horizontal scale of volumes is $2\frac{1}{2}$ cubic feet to the inch. What is the work of the steam per stroke of piston?

SOLUTION.—Work = $7.34 \times 36 \times 2\frac{1}{2} \times 144 = 95{,}126.4$ ft.-lb. Ans.

48. Suppose now that the left end of the cylinder is shut

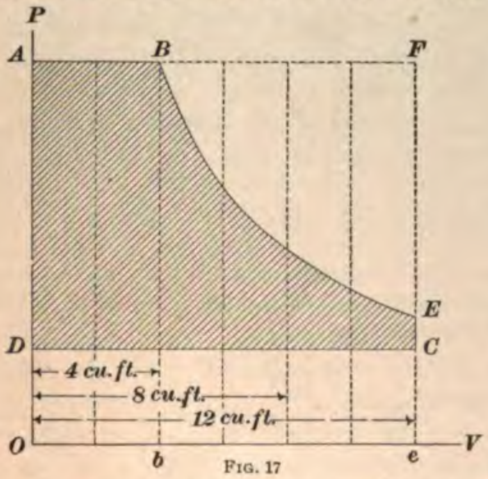


FIG. 17

off from the boiler just as the piston has completed one-third of the stroke—that is, when it is just 2 feet from the left end of the cylinder. The pressure on the left of the piston, as

in Fig. 16, will be constant so long as the cylinder is in communication with the boiler—that is, during the first 2 feet of the stroke. Therefore, the pressure line for the first one-third of the diagram will be a straight line AB parallel to OV, as shown in Fig. 17, and, as before, $\frac{60}{30} = 2$ inches above it. At this point, the steam supply is shut off, and the 4 cubic feet of steam in the cylinder must expand as the piston moves along, until it fills the entire volume of the cylinder. Its pressure, then, must fall somewhat after the manner of a perfect gas. This fall in pressure is graphically represented by the curved line BE.

Assume that the steam expands according to the law pv = a constant, which is the assumption commonly made in steam-engine calculations. The work then may be calculated as follows: The height of Fig. 17 is 2 inches; the length AB is $\frac{2}{3}$ inch; therefore, the area $ABbO = 2 \times \frac{2}{3} = 1.3333$ square inches.

49. The area $BEeb$ may be found by the formula

$$A = 2.3026 \times p_1 V_1 \log \frac{V_2}{V_1} \qquad (1)$$

in which A = area of diagram, in square inches;

p_1 = initial pressure (as OA), in inches;

V_1 = volume at cut-off, in inches;

V_2 = volume at end of stroke, in inches.

Substituting the values of p_1, V_1, and V_2 in the above formula,

$$A = 2.3026 \times 2 \times \frac{2}{3} \times \log \frac{2}{\frac{2}{3}} = 2.3026 \times 2 \times \frac{2}{3} \times \log \frac{6}{2}$$

$$= 1.4648 \text{ square inches}$$

Then, area $ABEeO$ is $1.3333 + 1.4648 = 2.7981$ square inches; $OD = .49$ inch; area $CDOe = .49 \times 2 = .98$ square inch; area $ABECD = ABEeO - CDOe = 2.7981 - .98 = 1.8181$ square inches.

Then, $$W = 144\, a h h_1 \qquad (2)$$

in which W = work, in foot-pounds;

a = net area of diagram (as $ABECD$);

h = scale used to lay off pressures;

h_1 = scale used to lay off volumes.

The volumes are always expressed in cubic feet and the pressures in pounds per square inch. In the diagram, $W = 1.8181 \times 30 \times 6 \times 144 = 47{,}125$ foot-pounds. The work done in the first case was 78,278 foot-pounds; so by shutting off the steam at one-third of the stroke, the work of the steam has been decreased by $78{,}278 - 47{,}125 = 31{,}153$ foot-pounds. But, in the first case, 12 cubic feet of steam at boiler pressure was used, while, in the last case, only 4 cubic feet was used; hence, the work per cubic foot of steam was, in the first case, $\frac{78{,}278}{12} = 6{,}523.2$ foot-pounds, and, in the second case, $\frac{47{,}125}{4} = 11{,}781$ foot-pounds, or nearly twice as much. This shows the economy of cutting off the steam early in the stroke and allowing it to expand.

EXAMPLES FOR PRACTICE

1. The mean ordinate of a diagram, similar to that shown in Fig. 16, is 1.2 inches long. The vertical scale of pressures is 1 inch = 40 pounds per square inch, and the horizontal scale of distances is 1 inch = 10 inches. The length of the diagram is 3 inches, and 1 foot of actual length of the vessel that contains the steam represents a volume of 452 cubic inches. What is the work done in one stroke of the piston? Ans. 4,520 ft.-lb.

2. The mean pressure in the cylinder of a steam engine is 38.7 pounds per square inch; the diameter of the cylinder is 26 inches, the stroke 32 inches, and the number of strokes per minute 120. Find the horsepower by means of the formula in Art. **45**. Ans. 199.24 H. P.

3. The mean ordinate of a diagram is .89 inch; the length, 3.2 inches; the vertical scale of pressure, 1 inch = 50 pounds per square inch; the horizontal scale of volumes, 1 inch = .56 cubic foot. Find the work done in twelve strokes. Ans. 137,798 ft.-lb.

---

50. The Expansion Line.—The character of the expansion curve BE, Fig. 17, depends on a variety of circumstances. Suppose, first, that the steam expands without any condensation. The equation representing the relation between the pressures and volumes will then be

$$p v^{\frac{17}{16}} = \text{some constant}$$

This is known as Rankine's formula for constant steam weight. The curve for this expansion may be plotted by laying off the volumes, taken from column 9 of the Steam Table, along the line OV, and the corresponding pressures from column 1 along the line OP. It will now be necessary to consider the conditions under which the steam will follow the above expansion line.

51. Suppose that 1 pound of steam at 60 pounds pressure, absolute, is confined in a non-conducting cylinder; that is, a cylinder that can neither absorb, transmit, nor give up heat. Its volume, according to the Steam Table, is 7.096 cubic feet. Let the pound of steam expand until the pressure reaches 54 pounds, absolute; the volume, then, is 7.829 cubic feet. It may be assumed with little error that the portion ab, Fig. 18, of the expansion line, which indicates the drop of pressure from 60 to 54 pounds, is straight. Then, the average pressure is $\frac{60+54}{2} = 57$ pounds. The volume dc is $7.829 - 7.096 = .733$ cubic foot.

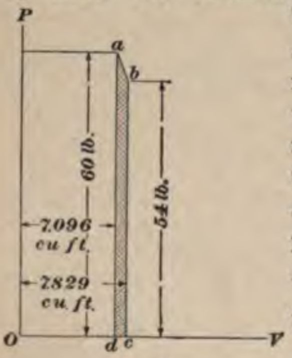


FIG. 18

The work is $144\,p\,V = 144 \times 57 \times .733 = 6{,}016.5$ foot-pounds. The total heat of the steam at 60 pounds pressure is 1,171.2 B. T. U. per pound, and at 54 pounds pressure, 1,169.1 B. T. U. per pound. Hence, the pound of steam during its expansion gives up $1{,}171.2 - 1{,}169.1 = 2.1$ B. T. U. $= 1{,}633.8$ foot-pounds.

52. The work done by the expanding steam is 6,016.5 foot-pounds, while the work given up by the steam during its expansion is only 1,633.8 foot-pounds, or a little over one-fourth as much. It is evident that the work could not have been done by the expansion of the steam alone. There remains $6{,}016.5 - 1{,}633.8 = 4{,}382.7$ foot-pounds of work that must have been obtained in some other manner. The cylinder is non-conducting, so no heat could have been

obtained from without. The only way in which this extra amount of work can be accounted for is to suppose that a small amount of steam is condensed while expanding, and thus gave up its latent heat of vaporization. 4,382.7 foot-pounds = 5.633 B. T. U. The average pressure was 57 pounds, and the latent heat of steam at that pressure is 911.55 B. T. U. Hence, $\frac{5.633}{911.55} = .0062$, nearly, of the pound of steam must have been condensed to furnish the extra 4,382.7 foot-pounds of work. Thus, the conclusion is reached that saturated steam expanding in a non-conducting cylinder must of necessity condense somewhat.

In other words, the condensation of steam, when expanding in a non-conducting cylinder, is not due in any way to the cylinder itself, but is caused by the loss of that portion of the heat that is converted into work.

In the same manner, it can be shown that, if saturated steam is compressed in a non-conducting cylinder, it must become superheated.

53. It is thus apparent that steam in a non-conducting cylinder cannot expand according to Rankine's formula $p v^{\frac{17}{16}}$ = a constant. The curve is really the adiabatic for steam, and is usually represented by the formula,

$$p v^{\frac{10}{9}} = \text{a constant}$$

The cylinder of the actual engine is never perfectly non-conducting. In fact, it is made of cast iron, which is a very good conductor of heat. This fact leads to great complications in the study of the action of the steam in the cylinder.

54. Suppose, first, that the cylinder is simply exposed to the atmosphere—that is, it is not covered with any non-conducting material. The steam, entering one end of the cylinder with a pressure of say 80 pounds absolute, has a temperature of about 312°. As it meets the cold cylinder walls, it will raise them to 312°, nearly; but, at the same time, the steam, meeting a surface cooler than itself, must partially condense. At some point near the middle of the stroke, the communication with the boiler will be cut off, and the steam

will expand, the pressure lowering to say 20 pounds, absolute, and the temperature, consequently, lowering to about 228°. Therefore, at the end of the stroke, the cylinder walls will be at a temperature of about 228°. The fresh steam now enters the other end of the cylinder, with a pressure of 80 pounds and a temperature of 312°, comes in contact with the cylinder walls, and partially condenses, this action being repeated at every stroke. Of course, this condensation is almost a total loss, since the water can exert no pressure on the piston. In addition to the condensation due to the alternate heating and cooling of the cylinder walls, there is the condensation that must always accompany expansion, even in a non-conducting cylinder.

Some of the condensed steam, however, may be, and generally will be, evaporated again toward the end of the stroke. Just before the steam supply is cut off, the temperature of the cylinder walls is 312°; as the steam expands, it partially condenses and its temperature falls. The temperature of the cylinder walls also falls, and in doing so, heat is given up, part of which is radiated and lost, while part reevaporates some of the condensed steam. As a consequence of this action, the real expansion line falls below the theoretical line ($p\,v^{\frac{17}{16}}$ = a constant) just after cut-off, but rises above it toward the end of the stroke.

55. A very common way of avoiding the loss due to the condensation of steam in the cylinder is to surround the latter with a **steam jacket;** that is, the cylinder is kept continually immersed in steam at boiler pressure. Heat is then transferred from the steam outside the cylinder to the expanding steam inside, and condensation is thus transferred; that is, it takes place in the jacket instead of in the cylinder. The expansion curve in this case approaches more nearly the theoretical curve, but usually rises a little above it toward the end of the stroke, due to the reevaporation of moisture, near the end of the stroke, by the heat from the jacket.

56. Fig. 19 shows the three curves above considered. The upper one is the **equilateral hyperbola,** $p\,v$ = a constant;

the second is the **curve of constant steam weight,** $p v^{\frac{17}{16}}$ = a constant, and the third is the **adiabatic** for steam $p v^{\frac{10}{9}}$ = a constant. The exponent $\frac{10}{9}$, for adiabatic expansion of steam, is generally used for ordinary calculations. When extreme accuracy is desired, the exponent $\frac{10}{9}$ is frequently displaced by the value of k as found by the formula $k = 1.035 + \frac{1}{10} x$, when x is the quality of the steam.

It will be seen how marked may be the difference between the expansion curves under varying circumstances. It is usual, in practice, to assume that the expansion curve is the

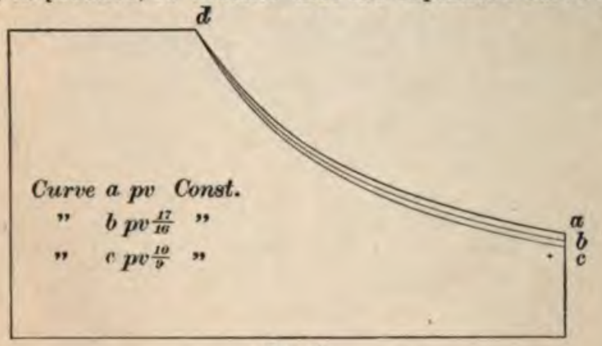


FIG. 19

equilateral hyperbola; and, though it is in no sense the theoretical curve, it is generally nearer than the others, and rather easier to deal with. Hence, in the study of the steam engine, it will be assumed that expanding steam follows the law $p v$ = a constant.

FLOW OF STEAM

57. For rough computations, the following equations derived from the experiments of R. D. Napier may be used in the case of steam flowing through an orifice into the atmosphere:

Let p_1 = absolute pressure, in pounds per square inch, in the reservoir;

p_a = atmospheric pressure, in pounds per square inch;

a = area of orifice, in square inches;

G = flow, in pounds per second.

Then if p_1 is equal to or exceeds $\frac{5}{3}\,p_a$,

$$G = \frac{p_1 a}{70} \qquad \textbf{(1)}$$

but if p_1 is less than $\frac{5}{3}\,p_a$,

$$G = \frac{p_1 a}{42}\sqrt{\frac{3(p_1 - p_a)}{2p_a}} \qquad \textbf{(2)}$$

Example.—What weight of steam will flow per hour from a reservoir, in which the gauge pressure is 70.3 pounds per square inch, into the atmosphere, the area of the orifice being .06 square inch?

Solution.—Using formula **1**, since p_1 is greater than $\frac{5}{3}\,p_a$, and substituting,

$$G = \frac{(70.3 + 14.7) \times .06}{70} = .0729 \text{ lb. per sec.}$$

Flow per hour is $.0729 \times 60 \times 60 = 262.44$. lb. Ans.

TABLE II

TABLE OF THE PROPERTIES OF SATURATED STEAM

Compiled from Prof. C. H. Peabody's Steam Tables, published by John Wiley & Sons, with permission of the author and publisher

Pressure, Absolute Pounds per Square Inch p	Fahrenheit Temperature t	Heat of the Liquid q	Total Heat H	Heat of Vaporization r	Internal Latent Heat i	External Latent Heat $\frac{Pu}{J}$	Entropy of the Liquid n	Volume in Cubic Feet of 1 Pound V	Weight in Pounds of 1 Cubic Foot w
1	2	3	4	5	6	7	8	9	10
1	101.99	70.0	1,113.1	1,043.0	981.1	61.9	.1329	334.6	.00299
2	126.27	94.4	1,120.5	1,026.1	961.9	64.2	.1754	173.6	.00576
3	141.62	109.8	1,125.1	1,015.3	949.5	65.8	.2013	118.4	.00844
4	153.09	121.4	1,128.6	1,007.2	940.4	66.8	.2203	90.31	.01107
5	162.34	130.7	1,131.5	1,000.8	933.1	67.7	.2353	73.22	.01366
6	170.14	138.6	1,133.8	995.2	926.7	68.5	.2480	61.67	.01622
7	176.90	145.4	1,135.9	990.5	921.4	69.1	.2587	53.37	.01874
8	182.92	151.5	1,137.7	986.2	916.5	69.7	.2682	47.07	.02125
9	188.33	156.9	1,139.4	982.5	912.4	70.1	.2766	42.13	.02374
10	193.25	161.9	1,140.9	979.0	908.4	70.6	.2842	38.16	.02621
11	197.78	166.5	1,142.3	975.8	904.8	71.0	.2912	34.88	.02866
12	201.98	170.7	1,143.6	972.9	901.5	71.4	.2976	32.14	.03111
13	205.89	174.6	1,144.7	970.1	898.4	71.7	.3035	29.82	.03355
14	209.57	178.3	1,145.8	967.5	895.5	72.0	.3091	27.79	.03600
14.7	212.0	180.8	1,146.6	965.8	893.5	72.3	.3127	26.60	.03760
16	216.32	185.1	1,147.9	962.8	890.0	72.8	.3192	24.59	.04067
18	222.40	191.3	1,149.8	958.5	885.3	73.2	.3282	22.00	.04547
20	227.95	196.9	1,151.5	954.6	881.0	73.6	.3363	19.91	.05023
22	233.06	202.0	1,153.0	951.0	877.0	74.0	.3438	18.20	.05495
24	237.79	206.8	1,154.4	947.6	873.2	74.4	.3506	16.76	.05966
26	242.21	211.2	1,155.8	944.6	869.9	74.7	.3570	15.55	.06432
28	246.36	215.4	1,157.1	941.7	866.7	75.0	.3629	14.49	.06899
30	250.27	219.4	1,158.3	938.9	863.6	75.3	.3685	13.59	.07360
32	253.98	223.1	1,159.4	936.3	860.7	75.6	.3737	12.78	.07820
34	257.50	226.7	1,160.4	933.7	857.8	75.9	.3787	12.07	.08280
36	260.85	230.0	1,161.5	931.5	855.3	76.2	.3834	11.45	.08736
38	264.06	233.3	1,162.5	929.2	852.8	76.4	.3878	10.88	.09191
40	267.13	236.4	1,163.4	927.0	850.3	76.7	.3921	10.37	.09644
42	270.08	239.3	1,164.3	925.0	848.1	76.9	.3962	9.906	.1009
44	272.91	242.2	1,165.2	923.0	845.9	77.1	.4001	9.484	.1054

TABLE II—*Continued*

Pressure, Absolute Pounds per Square Inch p	Fahrenheit Temperature t	Heat of the Liquid q	Total Heat H	Heat of Vaporization r	Internal Latent Heat i	External Latent Heat $\frac{Pu}{J}$	Entropy of the Liquid n	Volume in Cubic Feet of 1 Pound V	Weight in Pounds 1 Cubic Foot w
1	2	3	4	5	6	7	8	9	10
46	275.65	245.0	1,166.0	921.0	843.7	77.3	.4038	9.097	.1099
48	278.30	247.6	1,166.8	919.2	841.7	77.5	.4074	8.740	.1144
50	280.85	250.2	1,167.6	917.4	839.7	77.7	.4109	8.414	.1188
52	283.32	252.7	1,168.4	915.7	837.8	77.9	.4143	8.110	.1233
54	285.72	255.1	1,169.1	914.0	835.9	78.1	.4175	7.829	.1277
56	288.05	257.5	1,169.8	912.3	834.0	78.3	.4207	7.568	.1321
58	290.31	259.7	1,170.5	910.8	832.4	78.4	.4237	7.323	.1366
60	292.51	261.9	1,171.2	909.3	830.7	78.6	.4267	7.096	.1409
62	294.65	264.1	1,171.8	907.7	828.9	78.8	.4295	6.882	.1453
64	296.74	266.2	1,172.4	906.2	827.3	78.9	.4323	6.680	.1497
66	298.78	268.3	1,173.0	904.7	825.6	79.1	.4350	6.490	.1541
68	300.76	270.3	1,173.6	903.3	824.1	79.2	.4376	6.314	.1584
70	302.71	272.2	1,174.3	902.1	822.7	79.4	.4402	6.144	.1628
72	304.61	274.1	1,174.9	900.8	821.3	79.5	.4428	5.984	.1671
74	306.46	276.0	1,175.4	899.4	819.7	79.7	.4452	5.834	.1714
76	308.28	277.8	1,176.0	898.2	818.4	79.8	.4476	5.691	.1757
78	310.06	279.6	1,176.5	896.9	817.0	79.9	.4499	5.554	.1801
80	311.80	281.4	1,177.0	895.6	815.5	80.1	.4522	5.425	.1843
82	313.51	283.2	1,177.6	894.4	814.2	80.2	.4545	5.301	.1886
85	316.02	285.8	1,178.3	892.5	812.1	80.4	.4579	5.125	.1951
90	320.04	290.0	1,179.6	889.6	808.9	80.7	.4633	4.858	.2058
95	323.89	294.0	1,180.7	886.7	805.8	80.9	.4683	4.619	.2165
100	327.58	297.9	1,181.9	884.0	802.8	81.2	.4733	4.403	.2271
105	331.13	301.6	1,182.9	881.3	799.9	81.4	.4780	4.206	.2378
110	334.56	305.2	1,184.0	878.8	797.1	81.7	.4826	4.026	.2484
115	337.86	308.7	1,185.0	876.3	794.4	81.9	.4869	3.862	.2589
120	341.05	312.0	1,186.0	874.0	791.9	82.1	.4911	3.711	.2695
125	344.13	315.2	1,186.9	871.7	789.4	82.3	.4951	3.572	.2800
130	347.12	318.4	1,187.8	869.4	786.9	82.5	.4990	3.444	.2904
135	350.03	321.4	1,188.7	867.3	784.7	82.6	.5027	3.323	.3009
140	352.85	324.4	1,189.5	865.1	782.3	82.8	.5064	3.212	.3113
145	355.59	327.2	1,190.4	863.2	780.2	83.0	.5099	3.107	.3218
150	358.26	330.0	1,191.2	861.2	778.1	83.1	.5133	3.011	.3321
155	360.86	332.7	1,192.0	859.3	776.0	83.3	.5166	2.919	.3426
160	363.40	335.4	1,192.8	857.4	774.0	83.4	.5198	2.833	.3530

TABLE II—*Continued*

Fahrenheit Temperature t	Heat of the Liquid q	Total Heat H	Heat of Vaporization r	Internal Latent Heat i	External Latent Heat $\frac{Pu}{J}$	Entropy of the Liquid n	Volume in Cubic Feet of 1 Pound V	Weight in Pounds of 1 Cubic Foot w
2	3	4	5	6	7	8	9	10
365.88	338.0	1,193.6	855.6	772.0	83.6	.5230	2.751	.3635
368.29	340.5	1,194.3	853.8	770.1	83.7	.5260	2.676	.3737
370.65	343.0	1,195.0	852.0	768.2	83.8	.5290	2.603	.3841
372.97	345.4	1,195.7	850.3	766.4	83.9	.5319	2.535	.3945
375.23	347.8	1,196.4	848.6	764.6	84.0	.5347	2.470	.4049
377.44	350.1	1,197.1	847.0	762.9	84.1	.5375	2.408	.4153
379.61	352.4	1,197.7	845.3	761.1	84.2	.5402	2.349	.4257
381.73	354.6	1,198.4	843.8	759.5	84.3	.5429	2.294	.4359
383.82	356.8	1,199.0	842.2	757.8	84.4	.5454	2.241	.4461
385.87	358.9	1,199.6	840.7	756.2	84.5	.5480	2.190	.4565
387.88	361.0	1,200.2	839.2	754.6	84.6	.5504	2.142	.4669
389.84	363.0	1,200.8	837.8	753.1	84.7	.5529	2.096	.4772
391.79	365.1	1,201.4	836.3	751.6	84.7	.5553	2.051	.4876
393.69	367.1	1,202.0	834.9	750.1	84.8	.5576	2.009	.4979
395.56	369.0	1,202.6	833.6	748.7	84.9	.5599	1.968	.5082
397.41	371.0	1,203.2	832.2	747.3	84.9	.5621	1.928	.5186
399.21	372.8	1,203.7	830.9	745.9	85.0	.5643	1.891	.5289
400.99	374.7	1,204.2	829.5	744.5	85.0	.5665	1.854	.5393
402:74	376.5	1,204.8	828.3	743.2	85.1	.5686	1.819	.5496
404.47	378.4	1,205.3	826.9	741.7	85.2	.5707	1.785	.5601
406.17	380.2	1,205.8	825.6	740.4	85.2	.5728	1.753	.5705
407.85	381.9	1,206.3	824.4	739.2	85.2	.5748	1.722	.5809
409.50	383.6	1,206.8	823.2	737.9	85.3	.5768	1.691	.5913
411.12	385.3	1,207.3	822.0	736.7	85.3	.5787	1.662	.602
412.72	387.0	1,207.8	820.8	735.5	85.3	.5806	1.634	.612
414.32	388.6	1,208.3	819.7	734.3	85.4	.5826	1.607	.622
415.87	390.3	1,208.8	818.5	733.1	85.4	.5844	1.580	.633
417.42	391.9	1,209.3	817.4	732.0	85.4	.5863	1.554	.644
418.92	393.5	1,209.7	816.2	730.8	85.4	.5880	1.529	.654
420.42	395.0	1,210.2	815.2	729.8	85.4	.5898	1.505	.664
421.92	396.6	1,210.6	814.0	728.5	85.5	.5916	1.481	.675
423.37	398.1	1,211.1	813.0	727.5	85.5	.5933	1.459	.685
424.82	399.6	1,211.5	811.9	726.4	85.5	.5950	1.437	.696
426.24	401.1	1,211.9	810.8	725.3	85.5	.5967	1.415	.707
427.64	402.6	1,212.4	809.8	724.3	85.5	.5984	1.395	.717

STEAM-ENGINE MECHANISM

ELEMENTS OF THE STEAM ENGINE

FUNDAMENTAL PARTS

1. The Four-Link Slider Crank.—The steam engine is a mechanism for transforming the energy of steam into work. In *Entropy and Steam*, it was shown that steam may do work by lifting weights against the pressure of the atmosphere. As it is not generally desired to do work in this manner, it is essential that some method of changing the to-and-fro motion of the piston into a continuous motion in one direction should be devised. The form of mechanism used for this purpose in most types of engines is shown in Fig. 1.

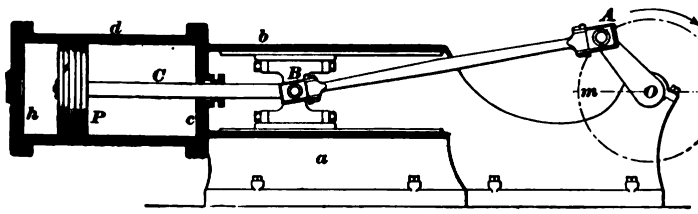


Fig. 1

It is composed of four parts or **links**: the link OA, called the **crank**; BA, the **connecting-rod**; CB, the **piston rod**; and the stationary link a, called the **frame**. The part shown at b is called the **guide**, and that at d, the **cylinder**. The cylinder, guide, and frame are rigidly connected to one another, and form the fourth or fixed link. These four links form what is known as the **four-link slider crank.**

2. The piston P moves first to one end and then to the other end of the cylinder d. The steam from the boiler enters one end—say, in this case, the end h—of the cylinder, and pushes the piston to the other end. By means of another mechanism called the **valve**, the steam is now admitted to the end c of the cylinder, while the end h is at the same time allowed to communicate with the atmosphere or with a condenser. The steam in h escapes, while that in c pushes the piston back again to its original position, whence the same operation is repeated.

Attached to the piston, and forming a part of it, is the piston rod CB; to the end of the piston rod is fastened, by a joint, one end of the connecting-rod BA. The other end of BA is joined to the crank OA; and the other end of OA terminates in a shaft O, which rotates in stationary bearings. It is evident that the end of BA, which is attached to CB, can move only in a straight line; and since the shaft O can rotate only in its bearings, the end of OA, which is attached to BA, can move only in a circle.

When the piston P is at one extreme end of the cylinder, say at h, the joint A is at the point m, and all three links, OA, BA, and CB, lie in a straight line. As the piston moves to the right, the link CB also moves to the right, while the joint A is constrained to move in the upper semicircle mn; when P reaches the other end of the cylinder, the joint A is at n, and again OA, BA, and CB are in a straight line. The piston now moves back to the end h of the cylinder, the joint A moving in the lower semicircle from n to m.

3. Those parts of the four-link slider crank that have a to-and-fro, or reciprocating, motion are called the **reciprocating parts**. They are the piston, piston rod, crosshead, and connecting-rod.

The end h of the cylinder is called the **head end**, and c the **crank end**. That is, the end of the cylinder farther from the crank is the head end, and the one nearer the crank is the crank end.

The distance passed through by the piston in moving from one end of the cylinder to the other while the crank is making half a revolution is called the **stroke**; the stroke is evidently equal to the diameter $m\,n$ of the circle described by the end A of the crank.

The engine may run in the direction shown by the arrows in the figure, or it may run in the reverse direction. In the former case, it is said to **run over**; and in the latter case, to **run under**. In other words, an engine runs over when the crank passes through the upper half of its circle as the piston moves from the head end to the crank end of the cylinder; and the engine runs under when the crank passes through the lower half of its circle as the piston moves from the head end of the cylinder to the crank end.

The stroke from the head end to the crank end of the cylinder—that is, from left to right in the figure—is called the **forward stroke**; the one from crank end to head end, the **return stroke**.

The foregoing mechanism gives a continuous motion in one direction. A pulley is keyed to the shaft O, and the power is transmitted by belting to shafting, or directly to the machinery to be run.

THE PLAIN SLIDE-VALVE ENGINE

4. Types of Engines.—There are many types of steam engines, most of which are covered by the following classification:

1. According to the kind of service, as	Stationary Locomotive Marine
2. According to number and arrangement of cylinders, as	Simple Compound Triple expansion Quadruple expansion Duplex
3. According to the type of valve used, as	Plain slide valve Automatic cut-off Corliss

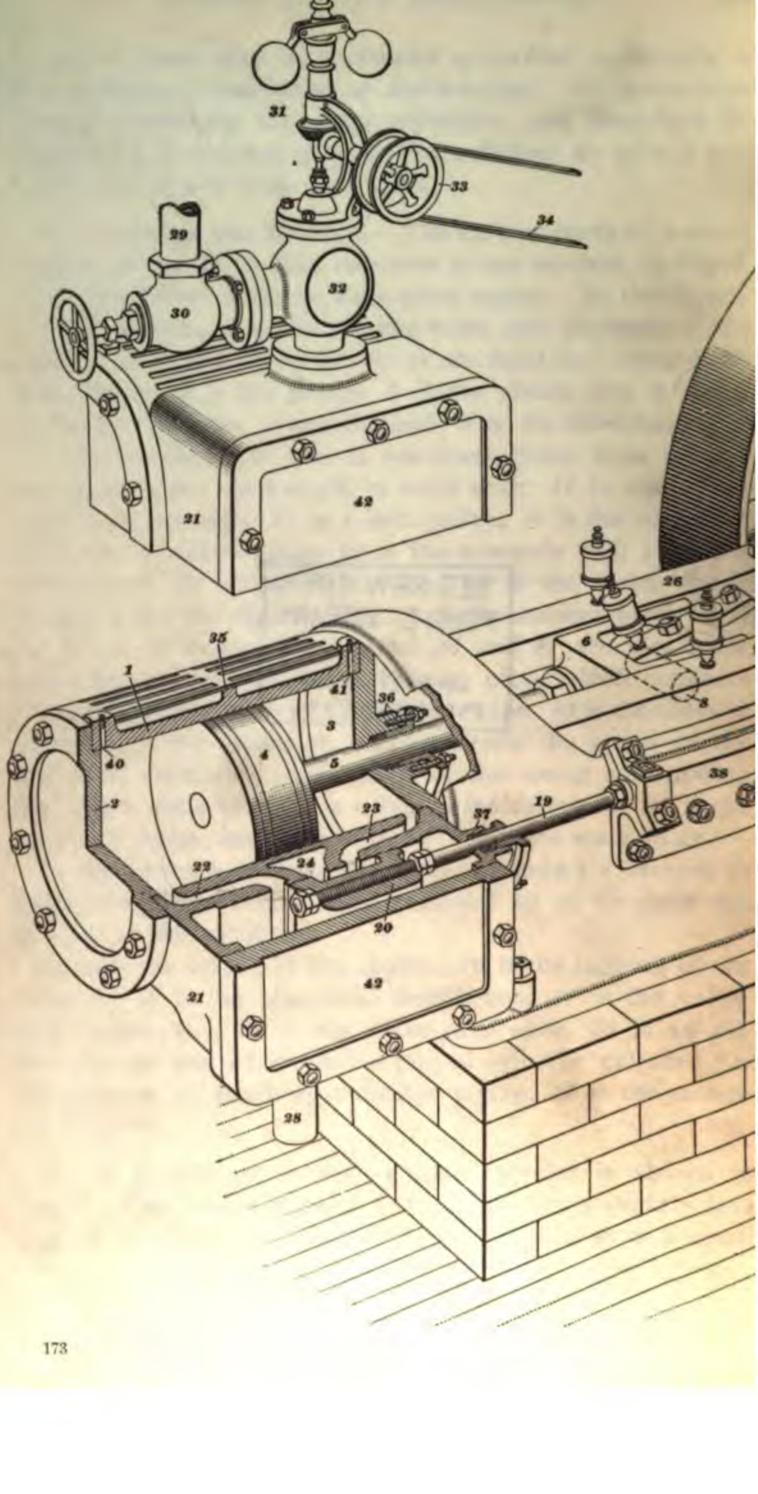
Any of these may be *horizontal* or *vertical*, *condensing* or *non-condensing*, *single-acting* or *double-acting*. All these types involve essentially the same principles, and therefore the description of a single type will be sufficient to give a general knowledge of these principles.

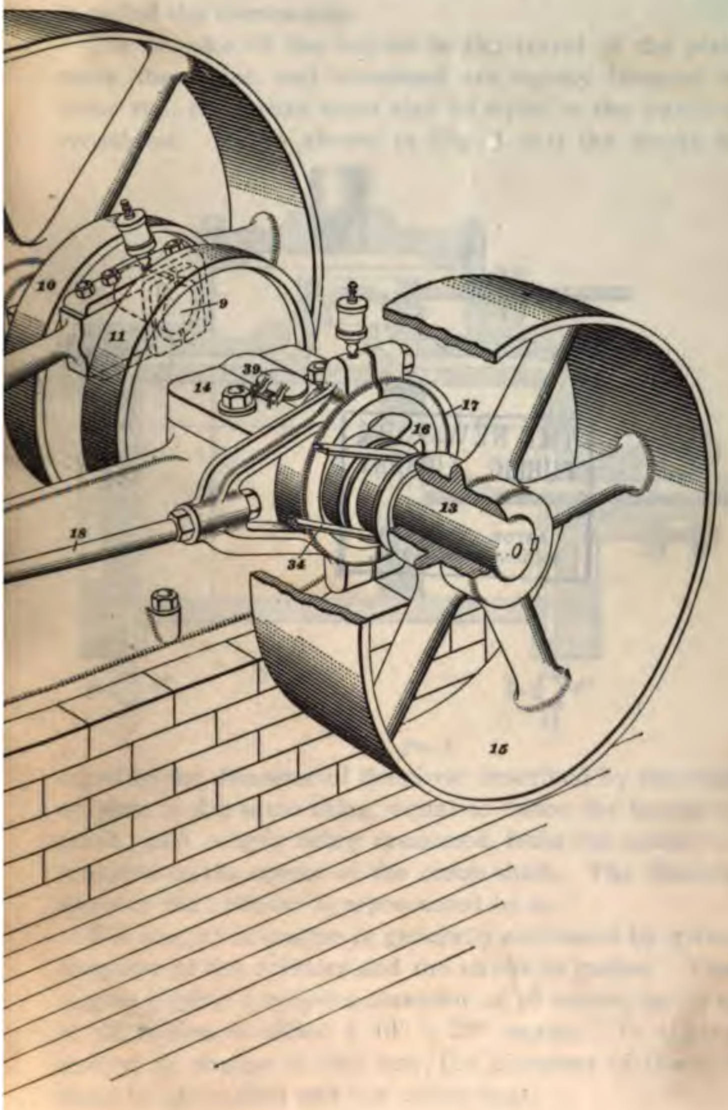
5. Parts of the Engine.—The various parts of a steam engine are shown, in their relations to one another, in Fig. 2, which represents a simple slide-valve engine. In this figure, *1* is the cylinder; *2* is the cylinder head, and the ends of the cylinder at *2* and *3* are known as the head and crank ends, respectively; *4* is the piston; *5* is the piston rod; *6* is the crosshead; *7* is the connecting-rod; *8* is the crosshead pin; *9* is the crankpin; *10* and *11* are crank-disks; *12* is the fly-wheel; *13* is the crank-shaft, or main shaft; *14* is one of the main-shaft bearings; *15* is a belt pulley; *16* is the eccentric; *17* is the eccentric strap; *18* is the eccentric rod; *19* is the valve stem; *20* is the slide valve; *21* is the steam chest; *22* and *23* are the steam ports; *24* is the exhaust port; *25* is the frame or bed of the engine; *26* and *27* are the upper crosshead guides; *28* is the exhaust pipe, which connects with the exhaust port; *29* is the steam pipe; *30* is the throttle valve; *31* is the governor, which controls the speed of the engine by regulating the pressure of the steam admitted to the steam chest through a valve in the casing *32*; *33* is the governor pulley, driven by a belt *34* from the main shaft.

In the upper portion of the figure is shown a section of the cylinder and steam chest removed so as to show the interior of the cylinder.

Among the details of the engine, *35* is the lagging of the cylinder; *36* is the piston-rod stuffingbox; *37* is the valve-stem stuffingbox; *38* is the valve-stem slide; *39* is an oil-hole cap; *40* and *41* are holes tapped into the cylinder for the purpose of attaching indicator piping; *42* is the steam-chest cover.

6. A section of a steam-engine cylinder is shown in Fig. 3. The working length l of the cylinder is slightly less than the distance between the cylinder heads, since a small


31
33
34
29
32
30
21
42
26
6
8
35
1
41
36
3
40
4
5
38
2
23
19
37
22
24
20
42
21
28


10
9
11
39
14
17
16
13
18
34
15

space must be left between the head and the piston when the latter is at the end of its stroke. The volume of this space, together with the volume of the steam port that leads to it, is called the **clearance.**

The **stroke** of the engine is the travel of the piston *p*; since the piston and crosshead are rigidly fastened to the same rod, the stroke must also be equal to the travel of the crosshead. It was shown in Fig. 1 that the stroke is also

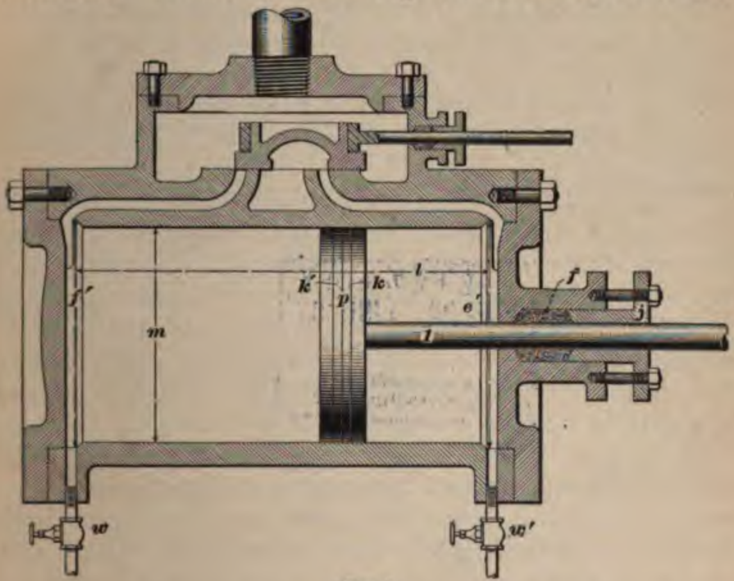


FIG. 3

equal to the diameter of the circle described by the crankpin, or what is the same thing, equal to twice the length of the crank, this length being measured from the center of the crankpin to the center of the crank-shaft. The diameter or **bore** of the cylinder is represented by *m*.

The size of an engine is generally expressed by giving the diameter of the cylinder and the stroke in inches. Thus, an engine having a cylinder diameter of 16 inches, and a stroke of 22 inches, is called a 16″ × 22″ engine. In stating the size of an engine in this way, the diameter of the cylinder must be given first and the stroke next.

At the ends e' and f', the cylinder is **counterbored**; that is, for a short distance the bore is greater than m. The piston projects partly into this counterbore at the end of each stroke. Were it not for the counterbore, the piston would not wear the cylinder walls their entire length, and shoulders would be formed at each end of the cylinder. When the wear of the joints in the connecting-rod is taken up, the length of the connecting-rod is changed, and the piston is moved slightly from its original position. In this case, a shoulder would cause an undesirable pounding of the piston.

Drain cocks w, w' are fitted in each end of the cylinder, through which any condensed steam may be discharged.

7. The piston fits loosely in the cylinder and has split rings k, k' inserted, which springs out so as to press against the wall of the cylinder and prevent leakage of steam between the wall of the cylinder and the piston. Pistons are usually supplied with a follower plate, which is bolted to the head end of the piston in order to hold the split rings k, k' in place. The piston rod is a round bar rigidly connected to both the piston p and the crosshead; t is a stuffingbox in which packing is placed, and is fitted with a gland j, which, when bolted down, compresses the packing around the piston rod and makes a steam-tight joint. This packing is often made in the form of split rings, which are so placed that the split of the first ring is covered by the solid part of the next ring. When repacking, care should be taken not to cause unnecessary friction by too much pressure from the gland. The crosshead, shown at *6*, Fig. 2, is given an easy sliding fit between the guide bars *26*, *27*, which are in line with the path of the piston rod, and, with the crosshead, relieve the piston rod of all bending strains.

The connecting-rod *7* forms the connecting link between the crosshead and the crank-disks. The joint between the crosshead *6* and the connecting-rod *7* is made by the crosshead pin *8*; and the joint between the connecting-rod and the crank-disks is made by the crankpin *9*. Connecting-rods are

usually made from four to six times the length of the crank, or from *4 to 6 cranks* in length, as it is called.

8. The eccentric, which imparts motion to the slide valve, is shown in Fig. 4. It consists of a circular disk of

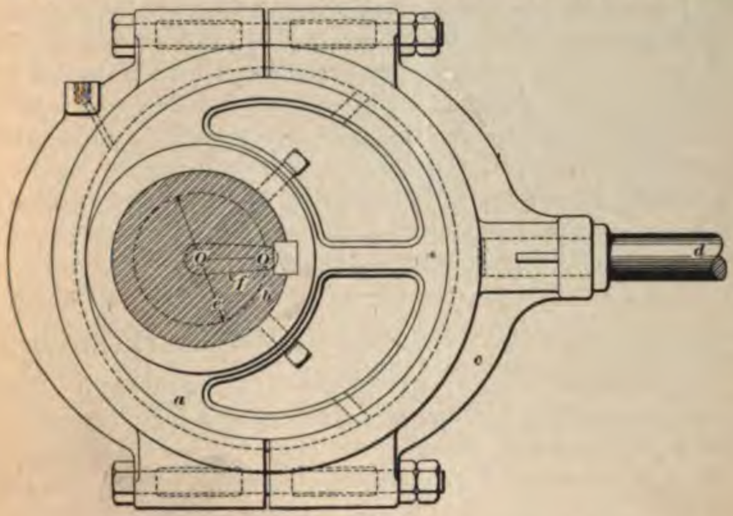


FIG. 4

iron *a*, which is keyed or fastened by setscrews to the shaft and revolves with it. The center *O* of the eccentric is at some distance from the center *Q* of the shaft, so that, as the shaft turns, the center *O* of the eccentric describes the circle *b* whose diameter is *e*. The **eccentric strap** *c*, which surrounds the eccentric, is fastened to the eccentric rod *d*. Hence, for each half revolution of the shaft, the eccentric rod is moved horizontally a distance equal to the diameter *e*. This distance is called the **throw** of the eccentric. The distance *O Q* from the center of the eccentric to the center of the shaft is called the radius of the eccentric, or the **eccentricity.** Evidently the throw is twice the eccentricity. Some engineers consider the throw as being equal to the radius *O Q*, but throughout the succeeding pages the definition as given above will be observed.

The eccentric is equivalent to a crank whose length is equal to the eccentricity. Thus, if the end of the eccentric-rod *d* were attached at *O* to the crank *f*, shown by dotted lines, the motion imparted by the crank would be precisely like that given by the eccentric.

In practice, the diameter of the shaft generally exceeds the diameter *e* of the circle described by the eccentric. In plain slide-valve engines, the eccentric is usually keyed to the shaft after being properly adjusted. The connection between the eccentric rod and the valve stem is accomplished in a variety of ways. In Fig. 2 a slide *38* is used to support the joint between the eccentric rod *18* and the valve stem *19*. The latter must be supported in some such manner to prevent it from binding in its stuffingbox.

THE D SLIDE-VALVE AND STEAM DISTRIBUTION

9. The Valve and Valve Seat.—Of the different kinds of valves used to distribute the steam in the engine cylinder, the D slide valve is the most common. A section of such a valve is shown in Fig. 5; *p* and *p* are the **steam ports,**

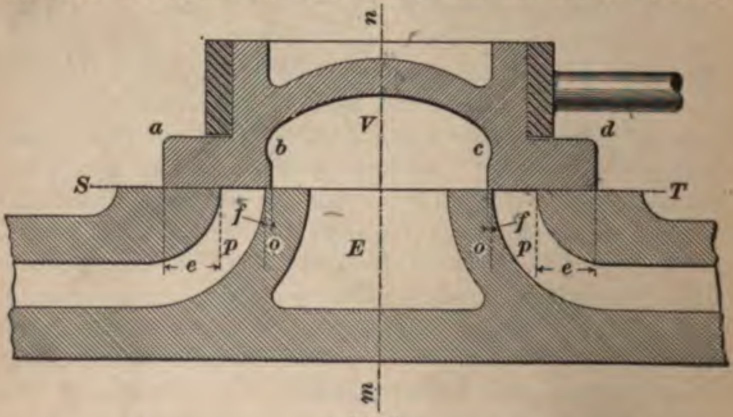


FIG. 5

o and *o* the **bridges,** *E* the **exhaust port,** and *S T* the **valve seat.** The valve flanges *a b* and *c d* extend beyond the edges of the steam ports in both directions. The distance

by which the edge of the flange extends beyond the outer edge of the steam port, when the valve is in its central position, is called the **outside lap.** In the figure, *e*, *e* represent the outside laps for the two ends of the cylinder. The distance by which the edge of the flange extends beyond the inner edge of the steam port, when the valve is in its central position, is called the **inside lap** and it is represented in the figure by *f*, *f* for the two ends of the cylinder.

The valve is here shown in mid-position; that is, the center line *n* of the valve coincides with the center line *m* of the exhaust port. As the motion of the valve is caused by the eccentric, the valve is in mid-position when the radius *Q O* of the eccentric, Fig. 4, is in a vertical position. When *Q O* lies horizontally on the right side of *Q*, the valve is in its position nearest the head end of the steam chest, and when *Q O* lies horizontally on the left side of *Q*, the valve is at the end of its stroke in the crank end of the steam chest. A valve is said to have **lead** when the steam port is opened slightly before the piston reaches the end of its stroke.

10. Relative Position of Valve and Piston.—In order to show how the steam is distributed in the cylinder by means of the valve and eccentric, a series of skeleton diagrams have been drawn showing the relative positions of the valve and piston for different points of a double stroke. Fig. 6 shows five diagrams representing a **D** slide valve without lap or lead. *O a* represents the crank; *O b* the eccentric; *a c* the connecting-rod; and *b d* the eccentric rod. It should be remarked that the sizes of some of the parts have been greatly exaggerated, particularly the radius of the eccentric circle and the amount of clearance. Diagram *A* represents the piston as just on the point of beginning the forward stroke. The valve is moving in the direction of the arrow and the outer edge is just about to admit steam to the left-hand port. As will be seen, the valve is in its central position and consequently the line joining the center of the shaft and the center of the eccentric, which represents

the *eccentric radius* is vertical. All the parts are moving in the direction of the arrows.

Diagram B shows the position of the parts when the crank

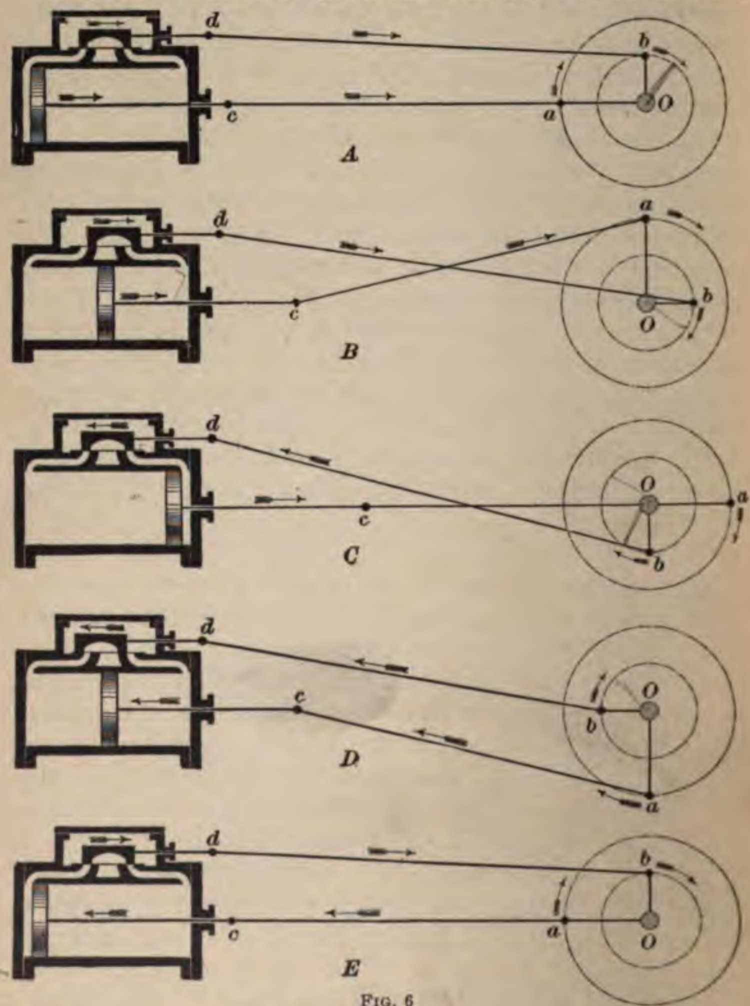


FIG. 6

has moved through 90° from its position in A. The piston is at the middle of its stroke, or very nearly there. It would be exactly at the middle of its stroke but for the fact that the

connecting-rod makes an angle with the horizontal. The angularity of the connecting-rod will be treated of later; for the present it will be assumed that it has no effect on the position of the piston. The valve has reached the extreme limit of its travel to the right, and the eccentric radius Ob is horizontal. The left steam port is fully opened for the live steam, and the right steam port is fully opened for exhaust.

Another crank-movement of 90° places the different parts as shown in diagram C. The piston has reached the end of its forward stroke; the valve, which is in its central position, moving toward the left, has just closed the left steam port and right exhaust port, and is just about to open the right port for the admission of live steam, and the left port for the release of exhaust steam. The piston has now traveled one full stroke.

Diagram D shows the piston in its central position on the return stroke. The crank is in the position Oa; the eccentric is horizontal, as represented by Ob, and the valve is at the farthest point of its travel to the left, the right port being fully open for live steam and the left port fully open for exhaust.

In diagram E, the piston has reached the extreme point of the return stroke, the piston-rod, connecting-rod, and crank being all in one straight line; this also occurs in diagrams A (which is the same as E) and C. The valve has been moving to the right, and is now in its central position, just on the point of admitting steam to the left port.

11. These diagrams show conclusively that, with no lap or lead, the steam is admitted to the cylinder for the full stroke of the engine, consequently there can be no cut-off, and, therefore, no expansion of steam.

The following conclusion is now evident: *With an ordinary* **D** *slide valve, operated by one eccentric, there can be no cut-off until the end of the stroke, and, therefore, no expansion of steam, unless the valve has outside lap.*

12. Distribution of Steam.—The effect of lap on the movement of the valve relatively to the piston, and also on

the movement of the eccentric and crank, is shown in Figs. 7 to 14. In these figures, the valve has both outside and inside lap, but no lead. These diagrams have been distorted, as was done in Fig. 6, in order that the eccentric radius might be long enough to show the relative positions of the moving parts. In Figs. 7 to 14, the eccentric radius is three times as long as it should be for the amount of valve movement shown by the figure. The diameter of the crank-circle is also a little greater than the stroke of the piston for the same reason. In order to show the distribution of steam by the valve, a diagram has been drawn above and below each cylinder, those above being marked *M*, and those below, *N*. These diagrams are supposed to be drawn in the following manner: Imagine it to be possible to connect two small pipes to the piston, one on each side. Suppose that each pipe has a steam-tight piston working in it, the lower sides of the pistons being subjected to the steam pressure in the cylinder, and the upper sides to the atmospheric pressure. Suppose, further, that there is a coiled spring on top of the piston; that a piston rod passes through the center of the spring; and that a pencil is attached to the end of the piston rod. If a pressure of 10 pounds is required to compress the spring 1 inch, it is evident that for every 10 pounds pressure in the cylinder the pencil will move upwards 1 inch, and that if it touched a sheet of paper it would mark a line on that paper. It will now be presumed that an arrangement like that just described is attached to the steam-engine piston, and that the pencil touches a sheet of paper, which is held stationary. Then, when the steam piston moves ahead, the pencils will make straight lines at heights corresponding to the steam pressure on the under sides of the little pistons, except when the pressure of the steam in the cylinder varies, in which case the pencil will move up or down, according as the pressure increases or diminishes.

Having made these suppositions clear, let *Q X*, Figs. 7 to 14, represent the line that the pencil would trace if there were a perfect vacuum in the cylinder; that is, *Q X* is the line of no pressure; also, let *A B* represent the line that the

pencil would trace if the pressure in the cylinder were just equal to that of the atmosphere, and *Q Y* were the line of no volume. Then the point *Q* represents no volume and no pressure. Finally, let *1 D* represent the volume of the clearance.

13. In Fig. 7, the piston is represented as just beginning

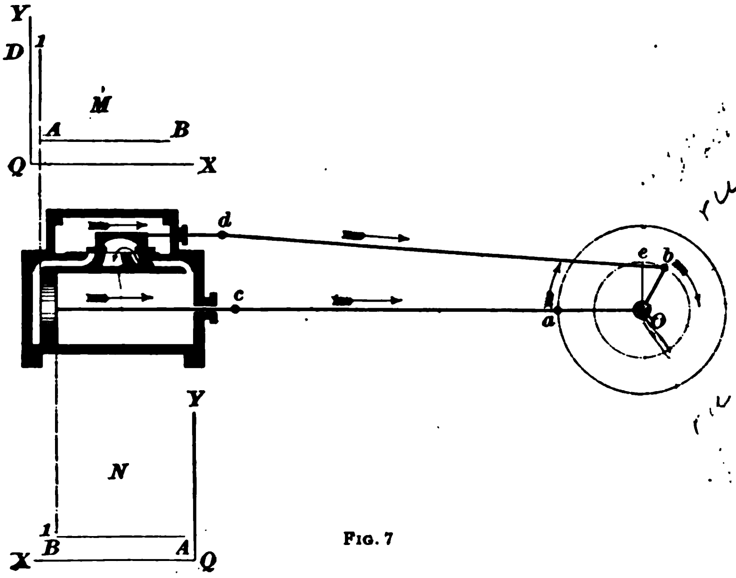


FIG. 7

the forward stroke, and the valve as just commencing to open the left steam port, both moving in the same direction, as shown by the arrows. If the valve had no outside lap, the position of the eccentric center would be at *e*; but on account of the lap, the valve has moved ahead of its central position in order to bring its edge to the edge of the port. To accomplish this, the eccentric center has been moved from *e* to *b*, *O b* being the position of the eccentric radius. The angle *b O e* that the eccentric radius makes with the position it would be in if there were no lap or lead, is called the **angle of advance.** In other words, the angle of advance is the angle between the position of the eccentric radius when the valve is in mid-position, and its position when the

piston is at the end of the stroke. It is equal to the angle due to the lap plus the angle due to the lead; it is frequently called **angular advance.**

Assume that the piston and valve have moved a very small distance, just sufficient to admit steam to fill the clearance space on the left of the piston, so that the steam acts on the piston at full boiler pressure. If the length of the line *A 1* represents the boiler pressure (gauge), the pencil that registers the pressure on the left side of the piston will be at *1*.

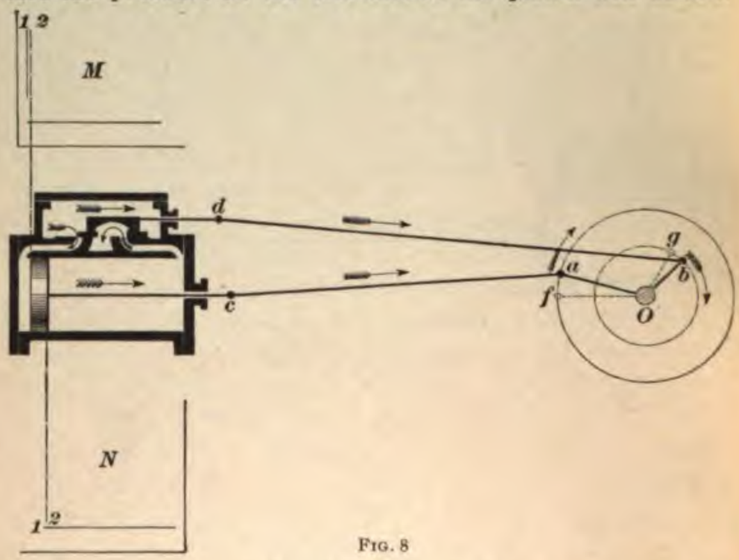


FIG. 8

The steam on the right side of the piston is exhausting into the atmosphere through the exhaust port, as shown by the arrow. As the size of the exhaust port is limited by practical considerations, the exhaust is not perfectly free, and there is consequently a pressure on the exhaust side of the piston; this is termed **back pressure.** Therefore, in the diagram *N*, let *1* be the position of the second pencil; then *1 B* is the back pressure.

14. Fig. 8 shows the position of the piston and valve when the exhaust is fully open. The crank has moved from the position *O f*, shown by the dotted line, to *O a*, and the

eccentric center from *g* to *b*. Steam is entering the head end of the cylinder and exhausting at the crank end. The pencils have moved from *1* to *2* on both diagrams *M* and *N*.

15. In Fig. 9, the piston has advanced far enough to enable the valve to reach the end of its stroke and open the port to its full width. The crank and eccentric have moved to the positions *O a* and *O b*, the dotted lines showing their

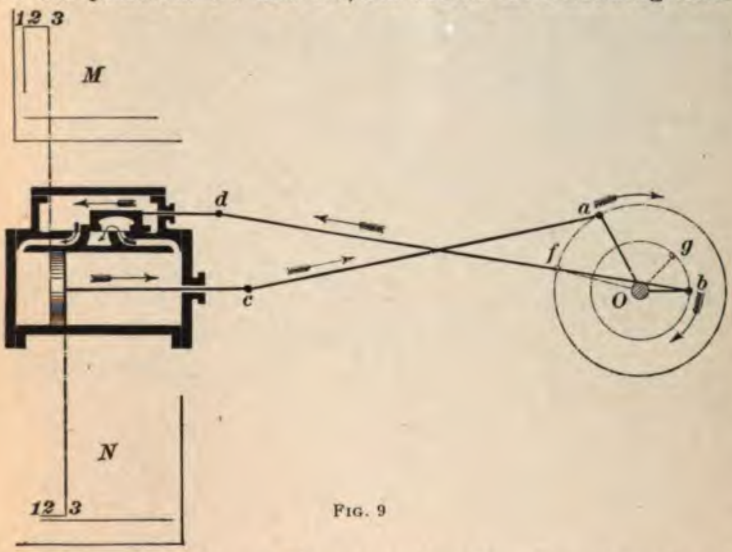


FIG. 9

last, Fig. 8, position. The eccentric radius is horizontal and any further movement of the crank will cause the eccentric to travel in the lower half of its circle and make the valve move back. In diagrams *M* and *N*, the pencil has traced the lines *2–3*.

16. Fig. 10 shows the piston still further advanced on its stroke, and the valve as having its inside edge in line with the outside edge of the right steam port. The left end of the valve has partially closed the steam port. The amount of advancement of the crank and eccentric from their last positions is shown by their distances from the dotted lines. The pencils have traced the lines *3–4* on the diagrams *M* and *N* during this movement of the piston from the last position.

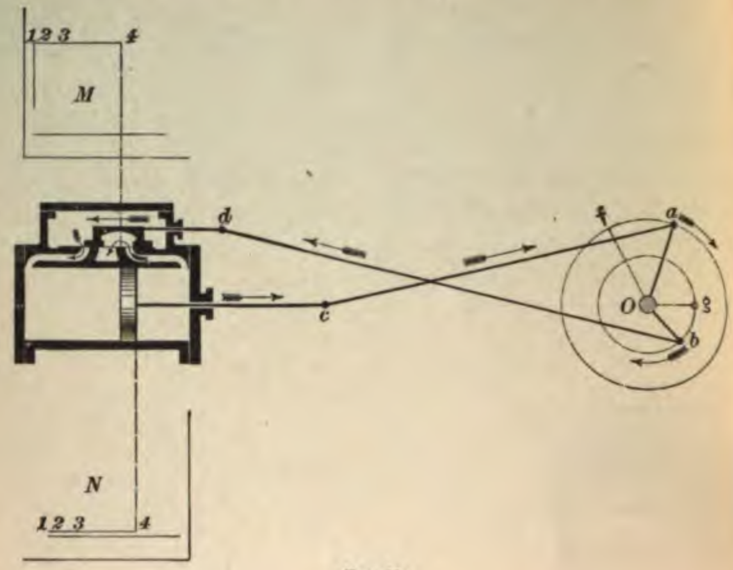


FIG. 10

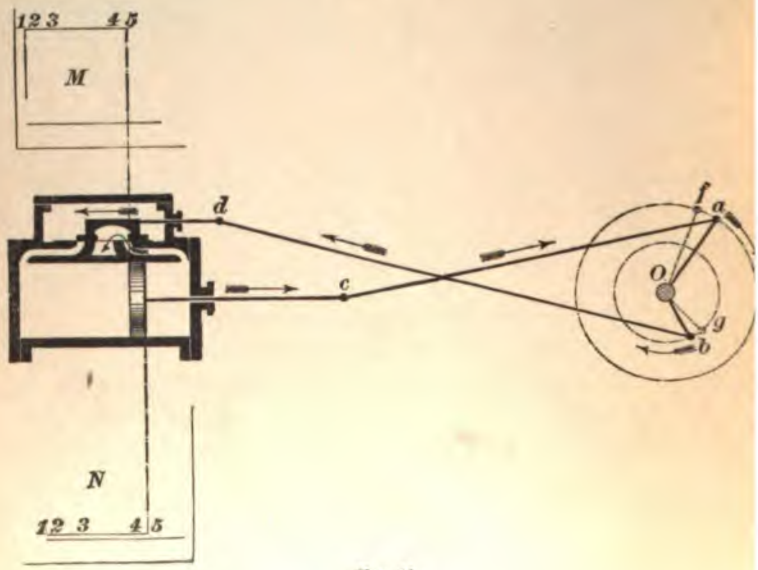


FIG. 11

Fig. 11 marks one of the most important points of the stroke. Here the valve has closed the steam port, that is, cut off the steam, and from here to the end of the stroke the steam in the cylinder expands. The exhaust is now partially closed. The crank and eccentric have moved through the angles indicated. During this movement the pencils have traced the lines *4–5*.

17. Fig. 12 shows another very important valve position.

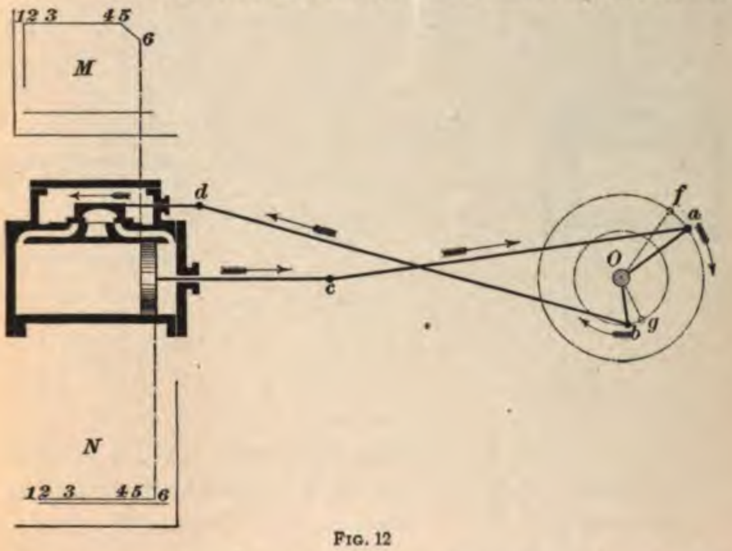


FIG. 12

Here the inside edge of the valve closes the exhaust, and from now on to the end of the stroke the steam in front of the piston is compressed. In the diagrams *M* and *N*, the lines *5–6* are traced by the pencils. The line *5–6* on the diagram *M* is an expansion line, the pressure falling as the piston moves ahead.

18. In Fig. 13, the piston has advanced far enough to cause the left inside edge of the valve to be in line with the inside edge of the left port. The slightest movement of the valve to the left will open the left port to exhaust. Expansion really ends here, although, on account of the limitation

in the size of the ports, there will still be a slight further expansion owing to the inability of the steam to escape instantly. During this last movement of the piston, the pencils trace the lines 6–7 on the diagrams *M* and *N*. On the diagram *M*, the line 6–7 is a continuation of the expansion

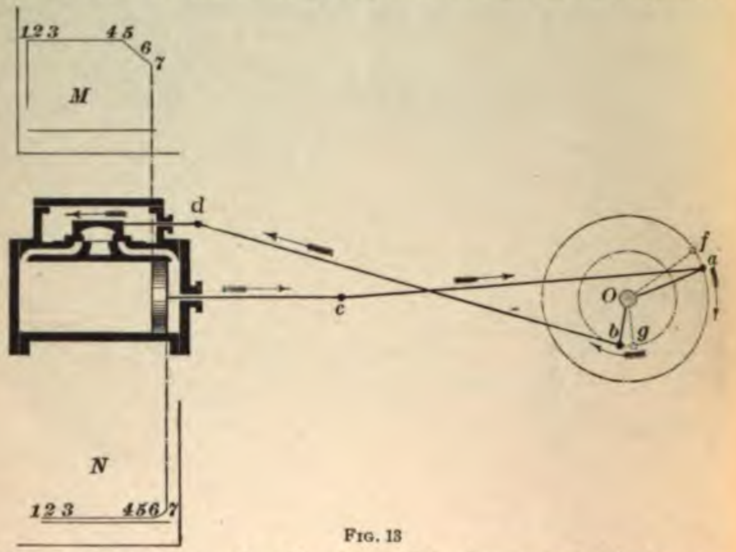


FIG. 13

line 5–6; while on the diagram *N* it shows part of the compression line, the pressure rapidly increasing as the piston nears the end of the stroke.

19. In Fig. 14, the piston has reached the end of its forward stroke and is about to begin the return stroke. The right outside edge of the valve is in line with the outside edge of the right port. The steam is exhausting from the head end of the cylinder, as shown by the arrows. The crank and eccentric are both diametrically opposite their positions in Fig. 7. In the diagrams *M* and *N*, the pencils have traced the lines 7–8. *M* shows that the pressure has fallen very rapidly from 7 to 8; while in *N*, it has risen from 7 to 8. The very slightest movement of the piston to the left will admit steam to the crank end of the cylinder and cause the pencil to rise to the point 1′.

20. During the return stroke, the actions of the steam just described will be repeated, the pencils tracing the dotted lines on the diagrams *M* and *N*, Fig. 14, the exhaust going through the left port and the steam through the right port. As the process is so nearly like the preceding, the diagrams have not been drawn, but the valve should be followed

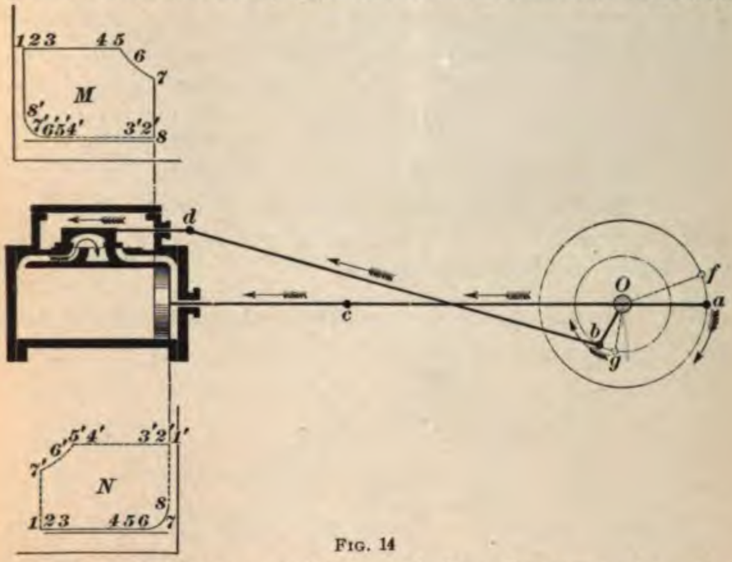


Fig. 14

through the different positions and the effects noted on the diagrams; to assist in this, the corresponding points have been numbered as in the foregoing figures.

21. Effects of Lap.—The study of Figs. 7 to 14 should show the effects caused by varying the lap. Thus, in Fig. 11, it is evident that if the outside lap were less, the valve would not close the left port when its center was in the position shown; consequently, the piston would have to move farther ahead before the valve could move back far enough to close the port. This, of course, would make the cut-off take place later in the stroke and shorten the expansion. It is likewise evident that if the valve had more lap, this extra lap would extend beyond the port when the center of the valve was in the position shown. Therefore, the valve would cut off

earlier in the stroke and the expansion would be lengthened. Hence, *increasing the outside lap means an earlier cut-off and an increased expansion, while decreasing the outside lap means a later cut-off and a diminished expansion.*

Considering the inside lap, it is evident, from Fig. 12, that if the inside lap had been less, the exhaust would not have closed so soon, and consequently the compression would have begun later; had the inside lap been greater, the compression would have begun earlier. Fig. 13 shows that with a diminished inside lap, the opening of the exhaust, usually termed **release**, would take place earlier; while with an increased inside lap, the release would have taken place later in the stroke. Hence, *increasing the inside lap increases compression and delays release, while diminishing the inside lap decreases compression and hastens release.*

22. Lead.—A valve is said to have **lead** when it com-

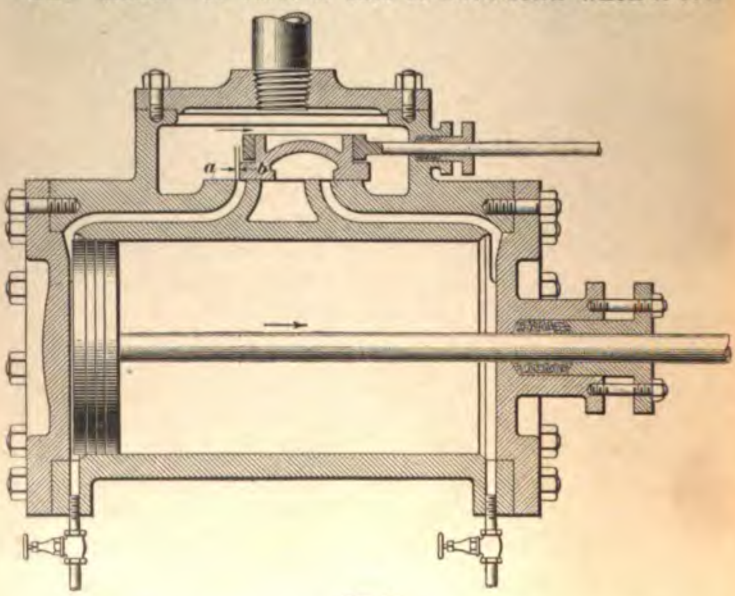


FIG. 15

mences to open the steam port just before the piston reaches the end of the stroke. The lead is the distance between the

edge of the valve and the edge of the port from which the valve is traveling when the piston is at the end of its stroke. In Fig. 15, the distance *a b* is the lead. Lead is given to a valve in order to have the clearance space filled with steam at boiler pressure when the piston begins its stroke. The effect of lead on the angular advance of the eccentric is evidently the same as an increase of lap; that is, it increases the angular advance. Its effect on the distribution of steam will be discussed further on.

23. Positions of Eccentric for Opposite Directions of Rotation.—In the preceding discussion of steam distribution, it has been assumed that the engine runs over. When the engine runs under, the steam distribution and the piston and valve movements will be precisely the same as before, but the position of the center of the eccentric relative to the crankpin will be changed. To determine the position of the eccentric in this case, draw the horizontal diameter *a e* of the crank-circle, as shown in Fig. 16. *O b'*

FIG. 16

represents the position of the eccentric radius when the piston is just beginning the forward stroke and the engine runs over. Draw *b' b* perpendicular to *a e* and through the point *b'*; it intersects the valve circle in *b*, and *b* is the position of the center of the eccentric when the engine runs under and is about to begin the forward stroke. It is easy to see that this is so, for the valve and piston must both have a forward movement at this point, whether the engine runs over or under. If the eccentric radius were placed so as to occupy the position *O f*, the forward movement of the piston and downward movement of the crank would cause the valve to move to the left, closing instead of opening the port. It

cannot be in the position Og, for that would throw the valve too far back. Ob is the only position in which the eccentric radius can be placed to give the valve the same movement when the engine runs under that it would have if placed in the position Ob' and the engine ran over. In both cases, the valve has the same forward movement, while the center of the eccentric is passing from b or b' to the horizontal position Oe.

24. Rocker-Arms.—It frequently happens that the eccentric cannot be so located on the shaft, that the eccentric rod and valve stem shall be in the same straight line. It can never be done when the valve is on top of the cylinder without inclining the valve seat, now very seldom done; and, with the valve on the side of the cylinder, other considerations, such as the location of the flywheel, may interfere. In such cases as this, a lever, or **rocker-arm**, may be employed.

There are two kinds of rocker-arms—direct and reversing. A *direct rocker-arm* is one in which the points of attachment of the valve stem and eccentric rod lie on the same side of the fulcrum of the rocker-arm, in consequence of which the direction of motion of the valve is always the same as that of the eccentric. A *reversing rocker-arm* is one having its fulcrum between the points of attachment of the valve stem and eccentric rod. With a rocker-arm of this class, the eccentric and the valve always move in opposite directions.

There are four conditions that may arise when using a rocker-arm: (1) The travel of the valve, the throw of the eccentric, and the direction of motion of the valve and eccentric may be the same as before. (2) The direction of motion of the valve and eccentric may be the same as before, but the travel of the valve may be greater than the throw of the eccentric. (3) The travel of the valve and the throw of the eccentric may be the same as before, but the eccentric may move in the opposite direction. (4) The travel of the valve may be greater than the throw of the eccentric, and the direction of motion may be opposite that in (1) and (2).

25. Sometimes the valve travel is such that if the eccentric were made to have the same throw it would be inconveniently large. In such a case the valve and its seat may be raised, the valve stem connecting to the rocker-arm at a higher point, as illustrated in Fig. 17. The direction of

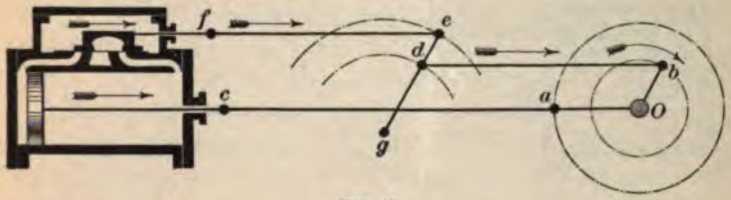


FIG. 17

motion of valve and eccentric is the same as in Figs. 7 to 14, but the throw of the eccentric is less than the travel of the valve by the ratio $gd : ge$; that is, if the valve travel is 4 inches, $gd = 12$ inches and $ge = 15$ inches, the throw of the eccentric will be $4'' \times \frac{12}{15} = 3.2$ inches. If the engine

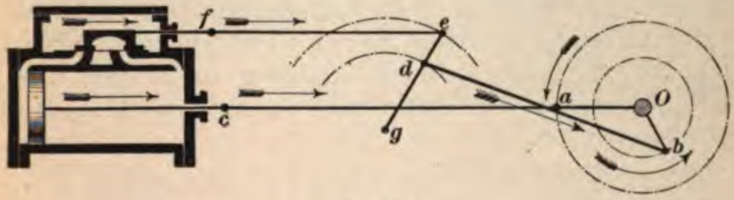


FIG. 18

runs under, the position of the center of the eccentric will be as shown in Fig. 18, and may be found by the same method as that given for finding it in the case shown in Fig. 16.

26. In the cases just described, the direction of motion of the valve and of the eccentric has remained the same as if there had been no rocker-arm, and both points of connection d and e, of the valve stem and eccentric rod, to the rocker-arm are on the same side of the pivot g. Suppose that the valve had been placed on the top of the cylinder, and it had been found more convenient to place the pivot of the rocker between the connections of the rocker-arm to the valve stem and eccentric rod, as shown in Fig. 19; then,

when d moves to the right along the dotted arc whose center is at g, e moves to the left. Consequently, if the eccentric center were in the position $O\,b'$, and the engine were running in the direction of the arrow, the valve would move backwards instead of ahead. To overcome this difficulty, the

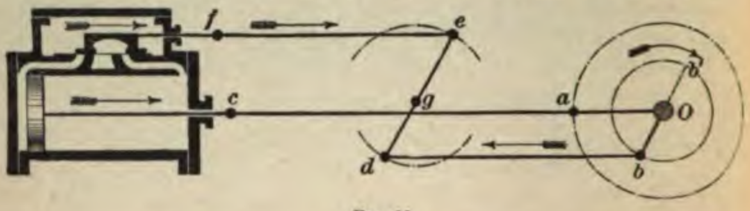


FIG. 19

eccentric is shifted around the shaft 180° to the position $O\,b$; then a movement of b in the direction of the arrow will throw d to the left and e to the right.

If $g\,d$ and $g\,e$ are equal, the valve travel and the throw of the eccentric will be equal, fulfilling condition 3. If $g\,d$ is less than $g\,e$, the throw of the eccentric will be less than the valve travel by the ratio $g\,d : g\,e$. For example, suppose that $g\,e = 20$ inches and $g\,d = 15$ inches and the valve travel is 5 inches; then the throw of the eccentric will be $5'' \times \frac{15}{20} = 3\frac{3}{4}$ inches.

27. Fig. 20 shows the position of the eccentric center when the engine runs under, and the rocker-arm is of the same design as the one shown in Fig. 19. If there were no

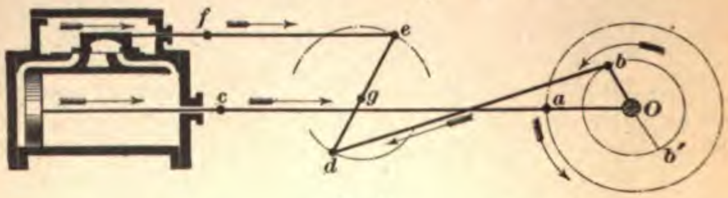


FIG. 20

rocker-arm, the eccentric center would be at b', as explained in Fig. 16, but, since the rocker-arm changes the direction of motion in this case, the eccentric is turned around 180°, to a point diametrically opposite.

The following rule may be applied to any engines whose valves cut off by their outside edges, as has been done in all the previous cases:

Rule.—*Place the crank in the position Oa, and the eccentric in the position Ob, as shown in Fig. 7, eOb being the angle of advance. If the engine runs over and the rocker-arm does not reverse the direction of motion of the eccentric, the eccentric is now correctly set. If the engine runs under, the eccentric should be placed in the position shown in Fig. 16, according to the rule given in connection with that figure. If the engine has a rocker-arm whose pivot lies between the point of connection with the valve stem and eccentric rod, and the engine runs over, place the eccentric center diametrically opposite the position shown in Fig. 7. If the engine runs under, and the pivot of the rocker-arm lies between the two points of connection, place the eccentric center diametrically opposite the position shown in Fig. 16.*

The following conveniently summarizes the instructions contained in the previous rule:

Direction of Running	Kind of Rocker-Arm	Angle Between Crank and Eccentric	Position of Eccentric Relative to Crank
Over	Direct	90° + angular advance	Ahead of crank
Over	Reversing	90° − angular advance	Behind crank
Under	Direct	90° + angular advance	Ahead of crank
Under	Reversing	90° − angular advance	Behind crank

28. Dead Centers.—When the piston has reached the end of either stroke, the piston rod, connecting-rod, and crank are all in one straight line, and the entire steam pressure on the piston is transmitted directly to the shaft and bearings, none of it being used to turn the crank. When the crank occupies this position, it is said to be on its **dead center.** This position is shown in Fig. 16. It is evident that there is no turning force on the crank due to the steam pressure when the reciprocating parts are in the position shown. There are two dead-center positions Oa and Oe,

diametrically opposite each other, corresponding to the two extreme positions of the piston. When the crank occupies the position Oa, it is said to be on its **interior** dead center; and when it occupies the position Oe, it is on its **exterior** dead center. That is, when the crank is in line with the piston rod and connecting-rod, and lies on the side of the shaft toward the cylinder, it is said to be on its interior dead center; when on the opposite side of the shaft, it is said to be on its exterior dead center.

29. Clearance.—The term *clearance* is used in two senses in connection with the steam engine. It may be the distance between the piston and the cylinder head when the piston is at the end of its stroke, or it may represent the volume between the piston and the valve when the engine is on dead center. To avoid confusion, the former is called **piston clearance**, and the latter is termed simply **clearance.** Piston clearance is always a measurement, expressed in parts of an inch. Clearance, however, is a volume. Hereafter, then, clearance will be used to represent the volume of the clearance space. Wherever piston clearance is meant, it will be so stated.

When the crank is on a dead center and the piston at the end of its stroke, there is always a space between the piston and the cylinder head. The volume of this space plus the volume of the one steam port leading into it is called the clearance. Thus, in Fig. 15, the piston is at the end of its return stroke, and the clearance is the volume of the space between the piston and the left cylinder head, plus the volume of the left steam port. In other words, the clearance may be defined as the volume of steam between the valve and the piston, when the latter is at the end of its stroke. The clearance of an engine may be found by putting the engine on a dead center and pouring in water until the space between the piston and the cylinder head, and the steam port leading into it, is filled. The volume of the water poured in is the clearance.

The clearance may be expressed in cubic feet or cubic inches, but it is more convenient to express it as a percentage

of the volume swept through by the piston. For example, suppose that the clearance volume of a 12″ × 18″ engine is found to be 128 cubic inches. The volume swept through by the piston per stroke is $12^2 \times .7854 \times 18 = 2{,}035.8$ cubic inches. Then, the clearance is $\frac{128}{2{,}035.8} = .063 = 6.3$ per cent. The clearance may be as low as $\frac{1}{2}$ per cent. in Corliss engines, and as high as 14 per cent. in high-speed engines.

30. Theoretically, there should be no clearance, since the steam that fills the clearance space does no work except during expansion; it is exhausted from the cylinder during the return stroke, and represents so much dead loss. This is remedied, to some extent, by compression. If the compression were carried up to the boiler pressure, there would be very little, if any, loss, since it would then fill the entire clearance space at boiler pressure, and the amount of fresh steam needed would be the volume displaced by the piston up to the point of cut-off, the same as if there were no clearance. In practice, however, the compression is only made sufficiently great to cushion the reciprocating parts and bring them to rest quietly.

It is not practicable to build an engine without any clearance, owing to the formation of water in the cylinder due to the condensation of steam, particularly when starting the engine. As water is practically incompressible, some part of the engine would be broken when the piston reached the end of its stroke, if there were no clearance space for the water to collect in; usually, the cylinder heads would be knocked off. Automatic cut-off high-speed engines of the best design, with shaft governors, usually compress to about half the boiler pressure, and have a clearance of from 7 per cent. to 14 per cent.

Corliss engines require but very little compression, owing to their low rotative speeds; they also have very little clearance, since the ports are short and direct.

31. Real and Apparent Cut-Off and Ratio of Expansion.—The **apparent cut-off** is the ratio between

the portion of the stroke completed by the piston at the point of cut-off, and the total length of the stroke. For example, if the length of stroke is 48 inches, and the steam is shut off from the cylinder just as the piston has completed 15 inches of the stroke, the apparent cut-off is $\frac{15}{48} = \frac{5}{16}$.

The **real cut-off** is the ratio between the volume of steam in the cylinder at the point of cut-off and the volume at the end of the stroke, both volumes including the clearance of the end of the cylinder in question. If the volume of steam in the cylinder, including the clearance, at the point of cut-off is 4 cubic feet, and the volume, including the clearance, at the end of the stroke is 6 cubic feet, the real cut-off is $\frac{4}{6} = \frac{2}{3}$.

The **ratio of expansion**, also called the **real number of expansions**, is the ratio between the volume of steam, including the steam in the clearance space, at the end of the stroke, and the volume, including the clearance, at the point of cut-off. It is the reciprocal of the real cut-off. For example, if the volume at the end of the stroke is 8 cubic feet, and at the cut-off is 5 cubic feet, the ratio of expansion is $\frac{8}{5} = 1.6$; in other words, the steam would be said to have one and six-tenths expansions. The corresponding real cut-off would be $\frac{5}{8}$.

Let e = real number of expansions;
i = clearance, expressed as a per cent. of the stroke;
k = real cut-off;
k_1 = apparent cut-off;
r = apparent number of expansions $= \frac{1}{k_1}$.

Then,
$$e = \frac{1}{k} \text{ and } k = \frac{1}{e} \qquad (1)$$

$$k = \frac{k_1 + i}{1 + i} \qquad (2)$$

EXAMPLE.—The length of stroke is 36 inches; the steam is cut off when the piston has completed 16 inches of the stroke; the clearance is 4 per cent. Find the apparent cut-off, the real cut-off, and the real number of expansions.

SOLUTION.—Apparent cut-off $= \frac{16}{36} = \frac{4}{9} = .444$. Ans.

Real cut-off $= k = \frac{k_1 + i}{1 + i} = \frac{.444 + .04}{1 + .04} = \frac{.484}{1.04} = .465$. Ans.

Real number of expansions $= e = \frac{1}{k} = \frac{1}{.465} = 2.15$. Ans.

EXAMPLES FOR PRACTICE

1. Length of stroke, 18 inches; apparent cut-off, .4; clearance, 7.5 per cent. Find: (*a*) real cut-off, (*b*) real number of expansions.

Ans. $\begin{cases} (a)\ .442 \\ (b)\ 2.262 \end{cases}$

2. Length of stroke, 66 inches; clearance, 4 per cent.; steam cuts off at $14\frac{1}{2}$ inches. Find: (*a*) real and (*b*) apparent cut-off in per cent. of stroke; (*c*) real and (*d*) apparent number of expansions.

Ans. $\begin{cases} (a)\ 24.97 \text{ per cent.} \\ (b)\ 21.97 \text{ per cent.} \\ (c)\ 4, \text{ nearly} \\ (d)\ 4.552, \text{ nearly} \end{cases}$

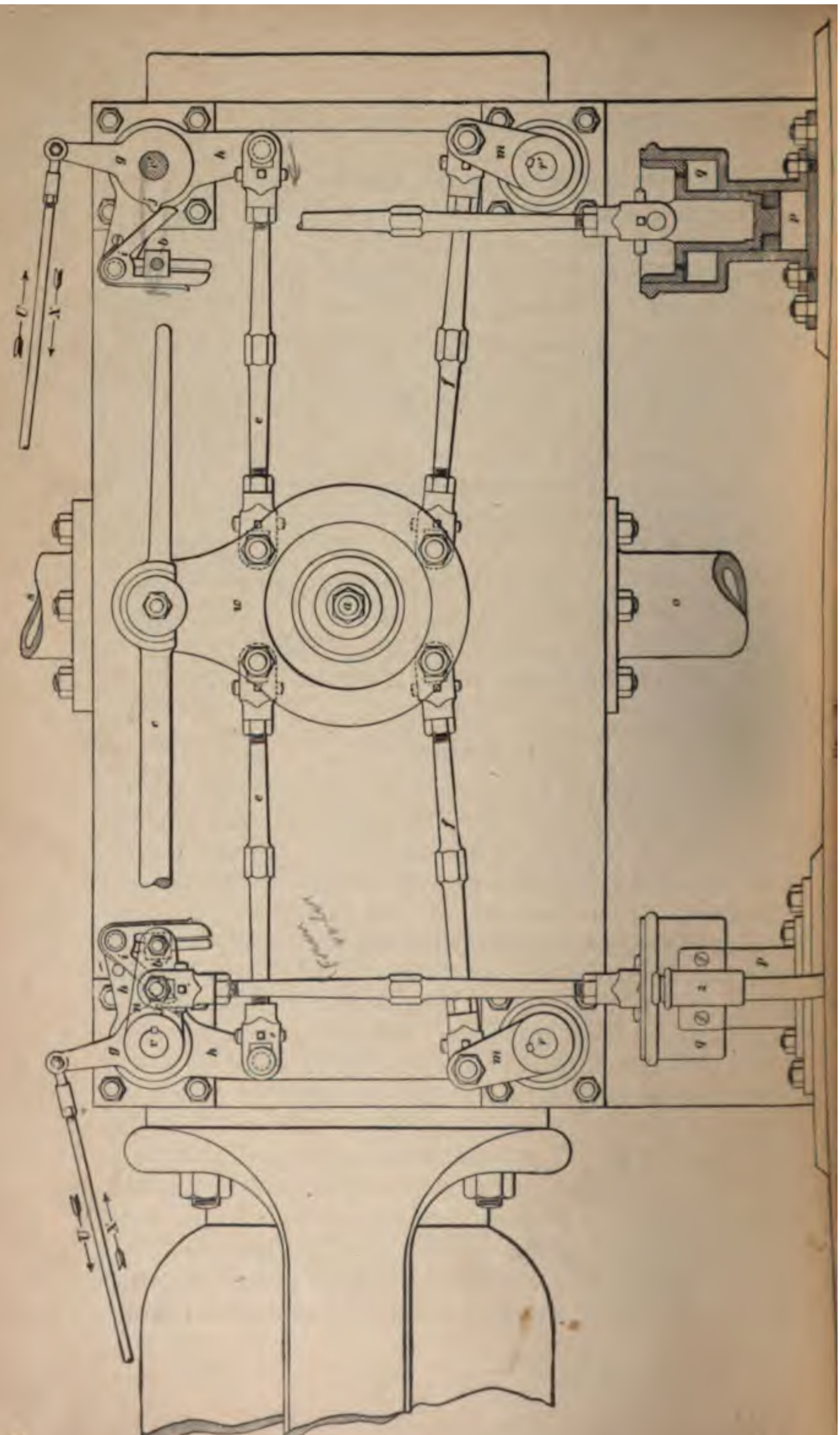
next

CORLISS VALVE GEAR

DESCRIPTION

32. The **Corliss valve gear,** which is used in a large number of engines, differs from the plain slide valve in many particulars. In Fig. 21 is shown a side elevation of this valve gear, and in Fig. 22 a section through the cylinder and and valves. It has four separate and distinct valves. Two of these v, v', Fig. 22, connect directly with the steam chest d and steam pipe s, and are called *steam valves;* they are rigidly connected with the cranks n, Fig. 21, the right-hand crank being removed in order to show more clearly the disengaging hook i. The other two valves r, r', Fig. 22, connect directly with the exhaust chest l and the exhaust pipe o, and are called *exhaust valves;* they are rigidly connected with the cranks m, m, Fig. 21. All the valves are cylindrical in form, and extend across the cylinder above and below, respectively. The wristplate w is made to rock on a stud a, by the hook rod c, connecting it with an eccentric on the crank-shaft.

Two motion rods e, e connect the wristplate w with the bell-cranks h, h of the steam valves, and two motion rods f, f



connect the wristplate with the cranks *m*, *m* of the exhaust valves. The motion rods can be lengthened or shortened as the case may require, and the action of any one valve regulated independently of the other three. As the wristplate *w* rocks backwards and forwards, the exhaust valves *r* and *r'*, which are rigidly connected with their cranks *m*, *m*, rock with it. The bell-cranks *h*, *h*, which are provided with the disengaging hooks *i*, *i*, are also given this rocking motion, and by hooking on to the blocks *b*, *b*, which are rigidly connected to the cranks *n*, open the steam valves *v*, *v'*.

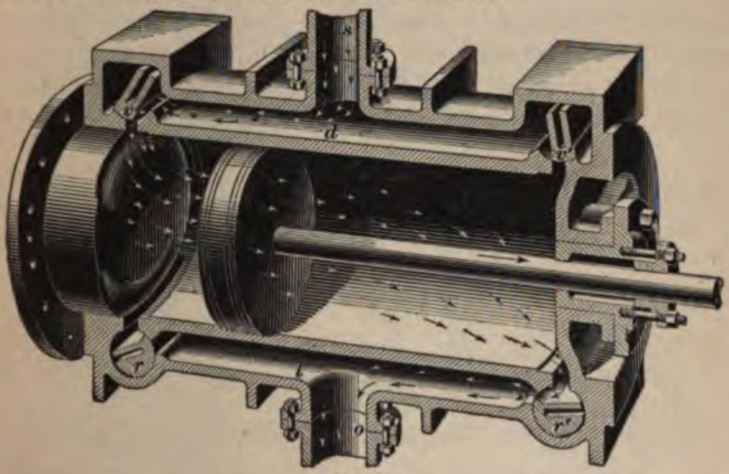


FIG. 22

The projections *j*, *j* on the two trip collars *g*, *g* unhook the disengaging hooks *i*, *i*, after they have rotated the valves *v*, *v'* through a certain angle, and the cranks *n*, *n* are pulled back to their first positions by the vacuum dashpots *p*, *p*, against the resistance of which the valve cranks *n* were raised. The governor changes the point of cut-off by moving the reach rods *U*, *X*, which are connected to the trip collars, thus enabling the projections *j*, *j* to be moved into various positions, causing the hooks *i*, *i* to disengage at any desired points. The movements of the valves open and close the steam and exhaust ports of the cylinder at the proper intervals. The pins of the motion rods are so located on the

wristplate that the steam valves v, v' have their quickest movement while the exhaust valves r, r' have their slowest movement, and the exhaust valves have their quickest movement while the steam valves have their slowest movement. As a consequence of this arrangement, the steam and exhaust valves have entirely independent movements and the inlet ports may be suddenly opened full width by the quick movement of the steam valves, while the exhaust valves are practically motionless. The advantage of this valve gear is that it permits an earlier cut-off, a greater range of cut-off, a more perfect steam distribution, and a smaller clearance space than is attained with a plain slide valve.

Engines fitted with the Corliss valve gear do not usually run at much more than 100 revolutions per minute.

RELATIVE MOTIONS OF PISTON, CRANK, AND VALVES

33. Fig. 23 shows the piston nearing the end of its

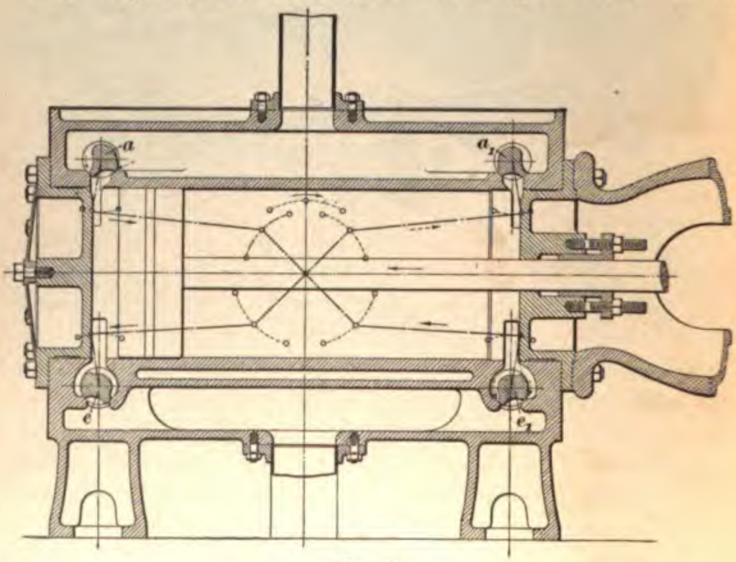


FIG. 23

return stroke and all the valves closed. The wristplate is in its middle position; hence, the exhaust valves are in

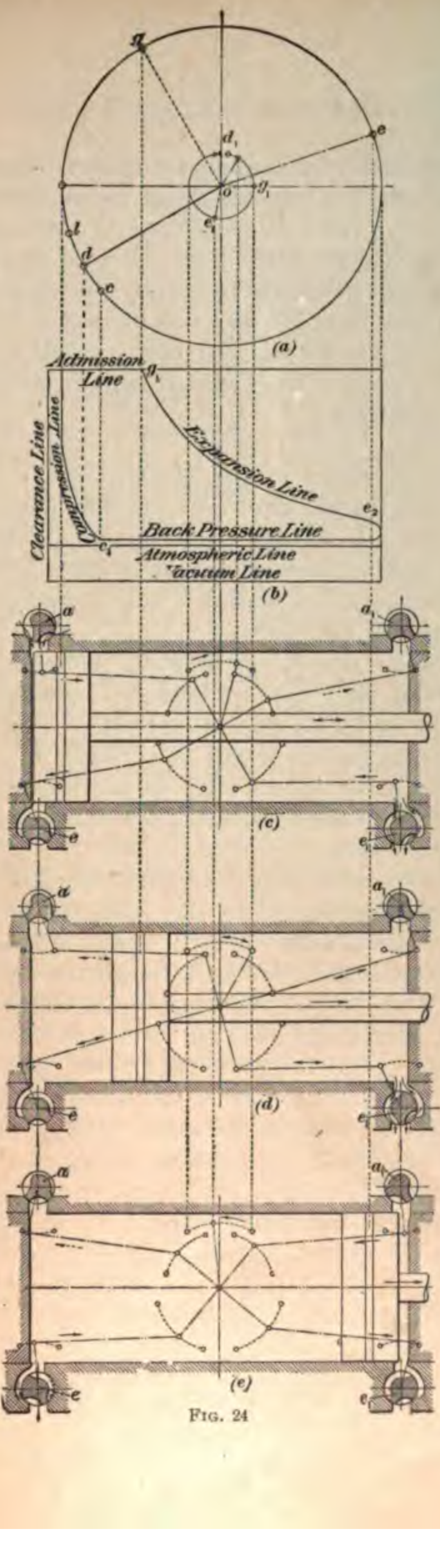

g
d₁
e
o
g₁
e₁
l
d
c
(a)
Admission
Line
g₁
Clearance Line
Compression Line
Expansion Line
e₂
Back Pressure Line
c₁
Atmospheric Line
Vacuum Line
(b)
a
a₁
(c)
e
e₁
a
a₁
(d)
e
e₁
a
a₁
(e)
e
e₁

FIG. 24

precisely the same position, relative to their respective ports as would also be the two admission valves were it not that the one a_1 is in a released condition. Observing the arrows on the various motion rods, it will be seen that the exhaust valve e_1 will soon be opened to liberate the steam that is still exerting a pressure in the direction of motion of the piston, while the other valve e has just been closed; the admission valve a will also soon be opened to admit steam against the motion of the piston.

34. The diagrams in Fig. 24 (a), (b), (c), (d), and (e) show the most important simultaneous positions of the piston and the four valves, together with a skeleton outline of the principal members of the mechanism in their various corresponding positions, during a little more than a complete forward stroke of the engine.

In all the diagrams, the directions of motion of the piston, wristplate, and valve rods are indicated by arrows, a double-headed arrow being shown when a member is in the position in which its direction of motion is being reversed, and a dotted arrow being shown when an admission-valve-motion rod is moving without affecting its valve, the connecting parts being released.

In Fig. 24 (a), the respective positions of the crank and eccentric that correspond to the positions in Fig. 23 are d and d_1, and the position c of the crank indicates when the closing of the exhaust valve e actually took place and the resulting compression commenced, as indicated at c_1 in diagram (b), while the position l indicates where the steam valve a will open—in other words, the lead position of the crank. Diagram (c) shows the piston at the end of its return stroke, or, what is the same thing, at the beginning of its forward stroke. By this time, both the exhaust valve e_1 and the admission valve a have been opened considerably, without, however, reaching the limits of their opening positions, while the exhaust valve e has nearly reached the limit of its closing position, and this because the four points of attachment of the valve rods to the wristplate are, as will be

seen, so located that the angular closing movements of the valves are very small compared with their angular opening movements.

35. In diagram (*d*), the wristplate is shown in the position in which its motion is just being reversed by the eccentric, the valve rods, in consequence, being also in the positions in which their motions are reversed, as indicated by the double-headed arrows. According to what has already been learned, this is the limiting position at which the admission valve *a* is released, which for that reason is supposed to have just occurred, as indicated by its closed position. As indicated at g_1, in diagram (*b*), this is then the moment at which in this particular case the expansion of the steam commences. The exhaust valve *e* has at the same time reached the limit of its closing position, while e_1 has reached the limit of its opening position. In this position of its motion rod, the admission valve a_1 is picked up, as indicated by the double arrow. In diagram (*a*), *g* and g_1 are, respectively, the positions of the crank and the eccentric that correspond to the valve positions of diagram (*d*).

36. In diagram (*e*), the piston is represented in the position near the end of its forward stroke, at which the exhaust valve *e* just begins to open, e_1 having been closed some time previously to produce compression, and the admission valve a_1 nearing its opening point.

Point e_2, in diagram (*b*), shows the release of the expanded steam due to the opening of *e*, and points *e* and e_1, in diagram (*a*), represent, respectively, the positions of crank and eccentric corresponding to diagram (*e*).

STEAM-ENGINE INDICATORS AND DIAGRAMS

THE INDICATOR

INDICATORS AND REDUCING MOTIONS

1. Indicators.—The arrangement described in *Steam-Engine Mechanism* for recording the steam pressure at all points of the stroke of the piston would be impossible to put into actual operation. Again, the diagram traced by the pencil would be altogether too large to be handled conveniently. The purpose in view, however, is accomplished by the use of an instrument called the **indicator.** This instrument measures the pressure in the cylinder at all points of the stroke and records it on a paper or a card. The principal reason for obtaining a diagram of this kind is that it affords a ready means of computing the mean pressure of the steam on the piston during one stroke. If the mean pressure on both sides of the piston, the length of the stroke, and the number of strokes per minute are known, the horsepower of the engine can be easily found.

2. Fig. 1 shows the general appearance of an indicator. The instrument consists essentially of a cylinder *a* containing a piston and helical spring for measuring the steam pressure, the lever *b* for transmitting the motion of the piston to the pencil point *c*, and the drum *d* that carries the paper on which

this motion is recorded. The card *e* is held close to the drum by the clips shown at *f*, *f*, so that the pencil can easily trace the outline of the diagram.

In Fig. 2 is shown a section of the steam cylinder *a* of the indicator and a partial section of the drum *d*. The piston, shown at *g*, must work in the cylinder as nearly frictionless

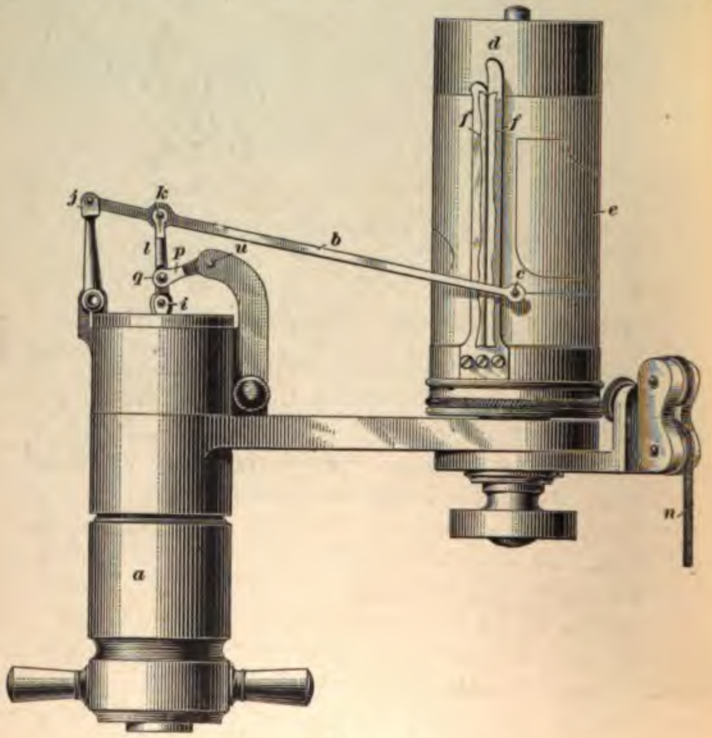


Fig. 1

as possible, the spring *h* being the only resistance to the upward motion of the piston. This spring is calibrated; that is, tested so as to determine the pressures required to move the pencil to various heights against the resistance of the spring. Hence, it is possible to find the pressure in the cylinder by the position of the pencil point. By turning a cock in the small pipe connecting the indicator with the

engine cylinder, steam may be admitted to, or shut off from, the cylinder of the indicator at pleasure. When steam is admitted through the channel *s*, its pressure causes the piston *g* to rise. The helical spring *h* is compressed, and resists the upward movement of the piston. The height to which the piston rises should then be in exact proportion

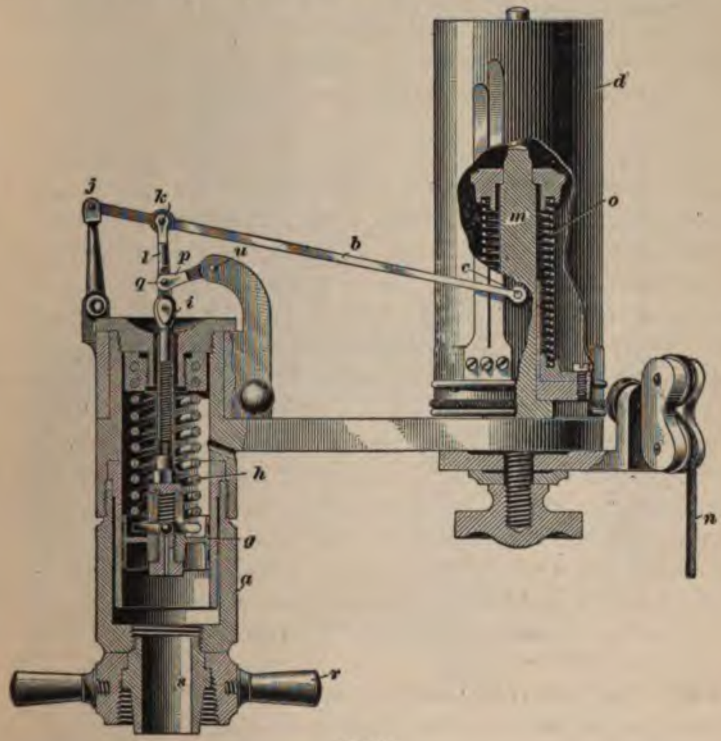


FIG. 2

to the pressure of the steam, and as the steam pressure rises and falls the piston must rise and fall accordingly.

To register this pressure, a pencil might simply be attached to the end of the piston rod, the point of the pencil being made to press against a piece of paper. It is desirable, however, to restrict the maximum travel of the piston to about $\frac{1}{2}$ inch, while the height of the card may advantageously

be 2 inches. To give a long range to the pencil while keeping the travel of the piston short, the pencil is attached at *c* to the long end of the lever *b*. The fulcrum of the lever is at *j*, and the piston rod is connected to it at *k* through the link *l*. The pencil motion is thus $\frac{jc}{jk}$ times the piston travel; for most indicators this ratio $\frac{jc}{jk}$ is either 4, 5, or 6. The point *c* is made to move in a vertical straight line by the arrangement of the links and joints *i*, *j*, *k*, *l*, *p*, *q*, and *u*.

3. The height to which the piston will rise under a given steam pressure depends on the stiffness of the spring. Indicators are usually furnished with a number of springs of varying degrees of stiffness, which are distinguished by the numbers 20, 30, 40, etc. These numbers indicate the pressure, in pounds per square inch, required to raise the pencil 1 inch. Thus, if a 40 spring is used, a pressure of 40 pounds per square inch raises the pencil 1 inch, and therefore the vertical scale of the diagram is 40 pounds per inch. That is, the vertical distance, in inches, of any point on the diagram from the atmospheric line, multiplied by 40, gives the gauge pressure per square inch at that point. The scale of the spring chosen should not be less than half the boiler pressure, since it is not desirable to have the indicator card more than 2 inches in height. For example, a 40 spring would be chosen for a steam pressure of 75 pounds per square inch.

4. The indicator, however, must not only register pressures, but it must register them in relation to the position of the piston. This is accomplished by means of the cylindrical drum shown at *d*, Fig. 2. This drum can be revolved on its axis *m* by pulling the cord *n* that is coiled around it. When the pull is released, the spring *o* turns the drum back to its original position. If the cord *n* is attached to some part of the engine that has a motion proportional to the motion of the piston, the motion of the drum also will be proportional to the motion of the piston.

5. Another indicator, differing slightly in construction from the one already described, is shown in Fig. 3 (*a*). The spring shown at *h* is a single spring, while the one shown in Fig. 2 is a double spring. The mechanism for transmitting the motion of the piston to the pencil also differs in form, but gives the pencil an almost exactly straight line motion. The spring *o*, Fig. 3 (*a*), for returning the drum is flat like a watch spring, instead of being a helical spring, as shown

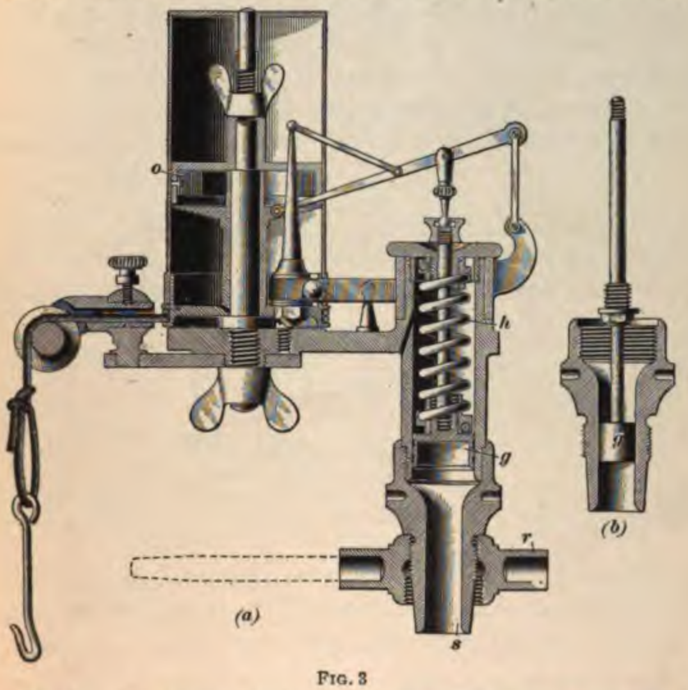


Fig. 3

in Fig. 2. The same principles, however, are involved in the operation of both indicators, and they are consequently lettered alike in both figures. For indicating a gas engine, a smaller piston than that used for steam engines must be employed. Such a piston is shown at *g*, Fig. 3 (*b*). It takes the place of the piston *g* in Fig. 3 (*a*), and works in the lower cylinder. The area of this piston is one-half that of the larger piston. Recently, a number of indicators with

springs outside the indicator cylinders have been placed on the market. In such cases, the spring is less affected by the heat than are those shown in Fig. 2 and Fig. 3 (*a*).

To attach the indicator to the engine, a hole is drilled in the clearance space of the cylinder and tapped for a ½-inch nipple, which should be as short as possible. The nipple has an elbow, into which is screwed a cock. The indicator may then be attached directly to the cock by the nut *r*, Fig. 2 and Fig. 3 (*a*), the conical projection *s* of the indicator wedging tightly into the cock to prevent the leakage of steam. It is preferable to have an indicator at each end of the cylinder, but if that is not convenient, one indicator may be connected with both ends of the cylinder by means of a three-way cock,

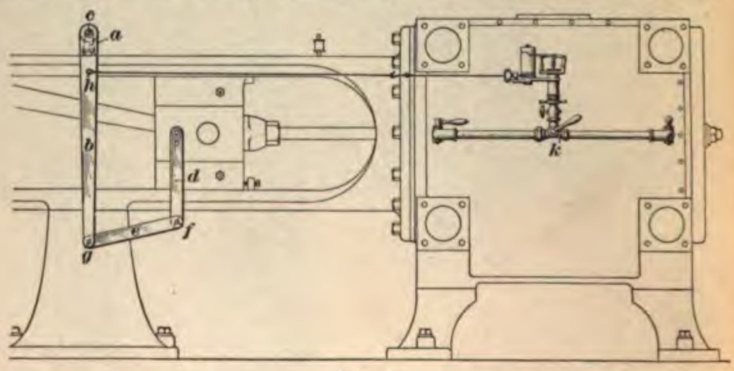


FIG. 4

as shown at *k*, Fig. 4. Before attaching the indicator, it is advisable to open the cock slightly, to blow out any dirt or rust that may have accumulated in the pipe.

6. Reducing Motions.—The motion of the drum cord is usually obtained from the crosshead. Since the stroke of the engine is nearly always greater than the circumference of the drum, the cord cannot be attached directly to the cross-head, and an arrangement called a **reducing motion** is used. A pendulum reducing motion is shown in Fig. 4. The upright *a* is fastened to the engine frame, and the lever *b* is pivoted at *c* to the upright. Another upright *d* is fastened to the crosshead or to the piston rod near the crosshead

and the link e is connected at f to the piece d and at g to the lever b. The cord, which should be parallel to the axis of the cylinder, is attached to the point h on the lever b, which point must be on the straight line connecting c and g.

When the piston is in its central position, the link b should be vertical and the link e should be so placed that it will swing equally above and below the horizontal, so that point h will move as nearly in a straight line as possible. Then, letting

L = length of stroke of piston;
l = length of indicator diagram;

it follows that

$$L : l = cg : ch, \text{ or } \frac{l}{L} = \frac{ch}{cg} \qquad (1)$$

In Fig. 5 is shown another form of reducing motion, known as the **reducing wheel**, attached to the engine, with

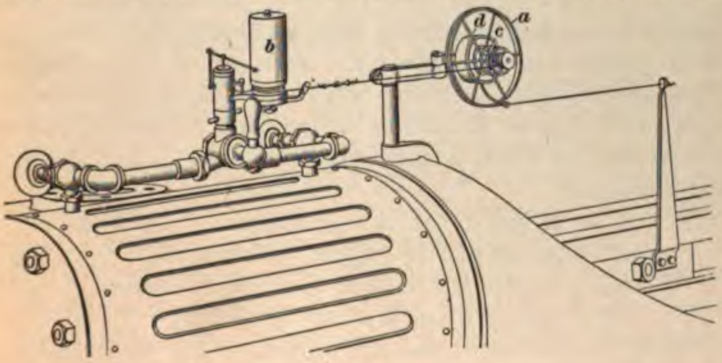


FIG. 5

the indicator in readiness for use. A rigid upright is firmly fastened to the crosshead, and to this upright is tied a cord, the other end of which is wound on the wheel a. As the crosshead moves back and forth, the cord rotates the wheel a. Evidently, the linear movement of a point on the rim of this wheel in any period is the same as that of the crosshead in that period. Fixed to the wheel a and turning with it on the same shaft is a smaller wheel c, on which is wound the

cord leading to the indicator *b*. Hence, as the wheel *a* turns, the drum of the indicator is given a rotary motion that is proportional to the motion of the wheel *c* and hence proportional to the crosshead movement also. But since the wheel *c* is so much smaller than wheel *a*, the movement of a point on the drum surface is much less than the movement of the crosshead. On the forward stroke, the wheel *a* is rotated against the resistance of a spring at *d*; but on the return stroke, this spring rotates the wheel in the opposite direction. Both wheels *a* and *c* are made as light as possible, in order that their inertia may not affect the accuracy of the reduction. The cord leading from the wheel *a* to the upright on the crosshead must be parallel to the axis of the cylinder, but the cord from the wheel *c* to the indicator may incline upwards or downwards.

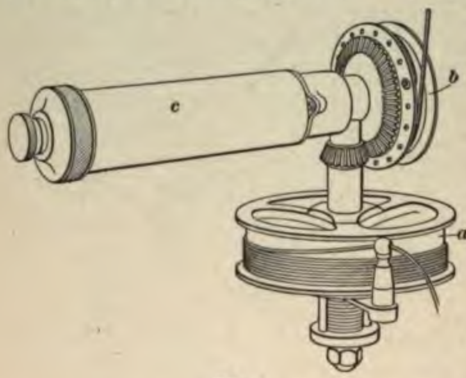


FIG. 6

Let L = length of stroke, in inches;
R = radius of large wheel, in inches;
l = length of indicator diagram, in inches;
r = radius of small wheel, in inches.

Then the length of the diagram is to the length of the stroke as the radius of the small wheel is to that of the large wheel; that is,

$$l = L\frac{r}{R} \qquad (2)$$

Reducing wheels, employing gears, are often made of aluminum for the sake of lightness. Such a wheel is shown in Fig. 6. It really consists of two wheels; on the larger one, shown at *a*, is wound the string that is attached to the arm on the crosshead, and from the smaller one *b* runs the

cord to the indicator. A spring in the horizontal case *c* takes up the slack in the string. Frequently, the reducing wheel is attached directly to the body of the indicator, thus avoiding the necessity of fastening it to the engine frame, as in Fig. 5.

The cord leading from a reducing motion to the indicator drum should be in two pieces with a hook on one of the free ends, preferably the end next the indicator, and a loop in the end fastened to the reducing motion, as shown in Figs. 4 and 5. This makes it possible to disconnect the indicator from the reducing motion when desired, and lessens the wear on the instrument. The length of the string should be carefully adjusted so as to give the drum the correct amount of motion—if the string is too short, it will be broken; and if too long, there will be lost motion and the card will not represent the true length of the engine stroke. It may also result in damage to the indicator.

INDICATOR DIAGRAMS

7. The instrument being properly attached, as just explained, a blank card is slipped over the drum so as to fit smoothly, as in Fig. 1. The hook on the indicator cord is then engaged with the loop on the cord from the reducing motion, and the drum is allowed to rotate back and forth several times, to see that it works properly, and that the cord is adjusted correctly. The cock is then opened and the indicator is allowed to work freely while the engine makes several revolutions. This warms up the parts to the working temperature. The pencil is then pressed lightly against the card during a single revolution. Next, the cock is closed and the pencil is again pressed against the card, recording the atmospheric line. Finally, the cord is unhooked, and the card is removed from the drum.

If but one indicator and a three-way cock is used, as shown in Fig. 4, the cock is opened to admit steam from one end of the cylinder, and the diagram from that end is taken; then the cock is turned to admit steam from the other

end, and another diagram is taken; finally, the steam is shut off entirely, and the atmospheric line is drawn.

8. Figs. 7 and 8 are diagrams taken from the head and

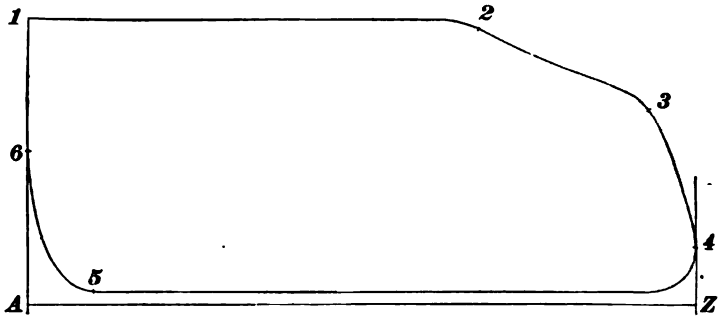


FIG. 7

crank ends of the cylinder, respectively. The different phases during the stroke are very clearly shown.

Thus: *1* is the beginning of the stroke;
2 is the point of cut-off;
3 is the point of release;
4 is the end of the stroke;
5 is the point of compression;
6 is the point of admission.

The lines included between any two of these points have received special names, which are as follows:

6–1 is the admission line;
1–2 is the steam line;
2–3 is the expansion curve;
3–4–5 is the period of release;
4–5 is the back-pressure line;
5–6 is the compression curve;
A Z is the atmospheric line.

9. If but one indicator is used, the two diagrams may be taken on the same card as shown in Fig 9. With the diagrams placed one over the other, as shown, it is very easy to tell exactly what is taking place in the cylinder at any point of the stroke. On the forward stroke, the pencil of the indicator describes the line *ABCD* of the head-end

diagram, if the cock is opened to the head end; or it describes the line KLM, if the cock is open to the crank end. Like-

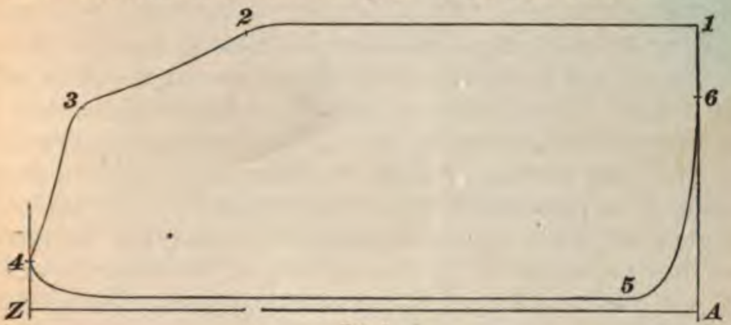


FIG. 8

wise, the lines $GHJK$ and DEF are described during the return stroke.

Suppose that the piston is at a position corresponding to r on the forward stroke; the pressure (absolute) urging the piston forwards is rS, while the pressure resisting is rt; hence the net pressure on the piston is St. Suppose, now,

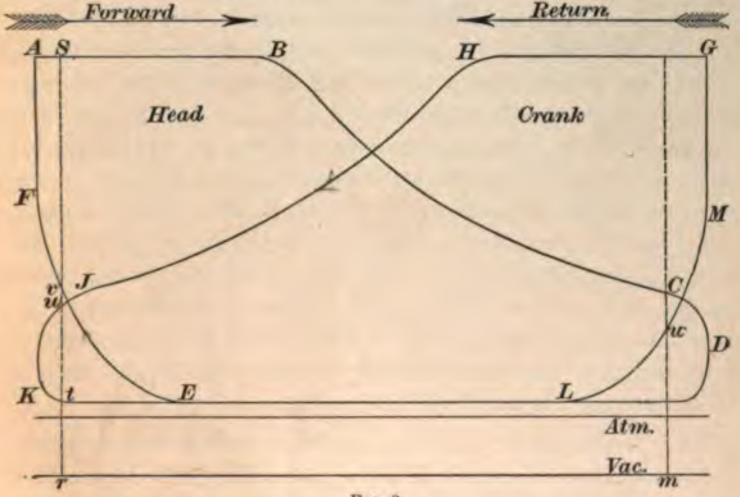


FIG. 9

that the piston is at r on the return stroke; the pressure at the right urging the piston on is ru, while the pressure on

the left is $r v$. The net pressure, therefore, is $u v$ and is negative; or, in other words, the resistance is greater than the effort.

A double diagram of this character tells at a glance what is taking place at either end of the cylinder at any point of the stroke. Thus, when the piston is on the forward stroke, in the position corresponding to m, the steam in the head end is at the point of release, as shown at C. Draw a line through m perpendicular to the vacuum line. C lies on $A B C$, and since $K L M$ is described during the forward stroke, as is $A B C$, the intersection of the line through C with the line $K L M$ is the point corresponding to C. Since w is on the compression line, compression is taking place in the crank end when release occurs in the head end.

HORSEPOWER CALCULATIONS

10. Mean Effective Pressure.—In order to find from the diagram the horsepower exerted by the engine, it is necessary to find the *mean effective pressure.*

The **mean effective pressure,** usually written M. E. P., is defined as the average pressure urging the piston forwards during its entire stroke in one direction, less the average pressure that resists its progress.

The M. E. P. may be found in two ways: (1) The area of the diagram, in square inches, may be found by an instrument called the **planimeter;** the M. E. P. is then found by dividing the area of the diagram, in square inches, by the length of the diagram, in inches, and multiplying the quotient by the scale of the spring. (2) Where a planimeter is not available, the mean ordinate can be found by measurement. This mean ordinate, multiplied by the scale of the spring, will then give the M. E. P.

11. The Amsler polar planimeter, which is one of the most common in use, is illustrated in Fig. 10. It consists of two bars a, b with a hinged joint c and roller d. At the end of the bar b is a weighted point e, which is pressed into the paper just enough to fix it in one position; the bar b then

moves about the point *e* when the planimeter is in use. The point *f* on the arm *a* is the tracing point, which is moved over the outline of the diagram. The roller *d* has on one edge a flange, which should roll on a smooth surface, and behind the flange are graduations, giving readings in square inches and tenths of a square inch. By means of a vernier *g*, the graduations on the roller may be read to hundredths of a square inch. There are a number of types of planimeters in use, differing in construction but operating in the same manner. The mode of reading may differ considerably, but complete instructions are invariably furnished with each instrument.

The planimeter should be used on a smooth level surface; a drawing board covered with a heavy, well-sized paper, or

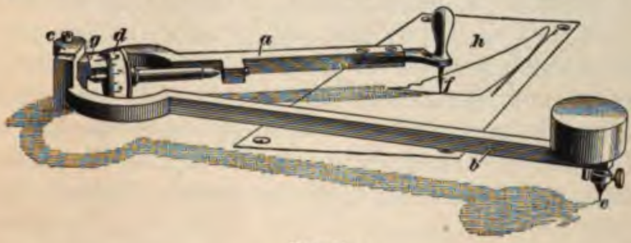


FIG. 10

with bristol board, answers very well. The indicator card *h* is fastened to the board, and the planimeter is set in about the position shown in Fig. 10. The starting point should be marked with the tracing point *f*, and the recording roller adjusted to zero. The outline of the diagram is then carefully traced with the point *f*, being sure to stop exactly on the starting point. The reading taken will be the area of the diagram, in square inches. The M. E. P. is then found by dividing this area by the length of the diagram on a line parallel with the atmospheric or vacuum line, and multiplying by the scale of the spring.

EXAMPLE.—The area of the diagram is 4.2 square inches, the length is 3.5 inches, and a 40 spring is used; find the M. E. P.

SOLUTION.— $\frac{4.2}{3.5} \times 40 = 48$ lb. per sq. in. M. E. P. Ans.

12. The area is read from the recording wheel and vernier as follows: The circumference of the wheel is divided into ten equal spaces by long lines that are consecutively numbered from 0 to 9. Each of these spaces represents an area of 1 square inch and is subdivided into ten equal spaces, each of which represents an area of .1 square inch. Starting with the zero line of the wheel opposite the zero line of the vernier, and moving the tracing point once around the diagram, the zero of the vernier will be opposite some point on the wheel; if it happens to be directly opposite one of the division lines on the wheel, that line gives the exact area in tenths of a square inch. The zero of the vernier, however, will probably be between two of the division lines on the wheel, in which case write down the inches and tenths that are to the left of the vernier zero, and from the vernier find the nearest hundredth of a square inch as follows: Find the line of the vernier that is exactly opposite one of the lines on the wheel. The number of *spaces on the vernier* between the vernier zero and this line is the number of hundredths of a square inch to be added to the inches and tenths read from the wheel. An example is presented in Fig. 11, where the 0 of the vernier lies between the lines on the wheel representing 4.7 and 4.8 square inches, respectively, showing that the area

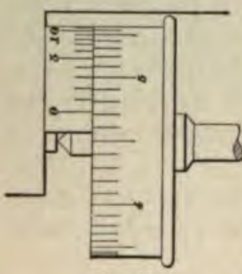


FIG. 11

FIG. 12

something more than 4.7 square inches. Looking along the vernier, it is seen that there are three spaces between the vernier zero and the line of the vernier that coincides with one of the lines on the wheel; this shows that .03 square inch is to be added to the 4.7 square inches read from the wheel, making the area 4.73 square inches, to the nearest hundredth of a square inch.

13. While the Amsler form of planimeter is very convenient, a much simpler and less expensive instrument, called the **hatchet planimeter,** shown in Fig. 12, may be used for measuring the areas of indicator diagrams. This simple instrument, if accurately made and used with proper care, will give very satisfactory results. It is made of $\frac{1}{4}$-inch steel rod bent at both ends, as shown. The end *a* is sharpened for a tracing point, and the other, *b*, is flattened like a hatchet. The distance between the tracing point and the point at which the curved hatchet end *b* touches the paper should be at least twice the length of the indicator diagram; 10 inches is a desirable length for ordinary use.

The method of using the hatchet planimeter is shown in Fig. 13. The indicator card *a* is fastened to a drawing board over a piece of smooth heavy paper *b* or bristol board of sufficient size to furnish the surface for the records made by the hatchet. The center of gravity *c* of the diagram must be located. This may be done approximately by inspection, or it may be found accurately enough by cutting out the diagram and balancing it on the point of a pin. Draw a line *A B* through the center of gravity parallel to the atmospheric line *f*, extending on the bristol board beyond the card *a*. With *c* as a center and the length of the planimeter as a radius, describe an arc *d* on the paper *b*. Then place the planimeter approximately at right angles to the atmospheric line *f*, with the tracing point at *c*, make the mark *1* on the arc *d* with the hatchet end, and proceed with the tracing point from *c* to *g* and thence over the outline of the diagram, moving clockwise and back to *c*. The hatchet will stop at some point *2* on the arc *d*. Next revolve the

card 180° about the point *c*, as shown by the dotted diagram, until the horizontal line *A B* coincides with the extensions *A′ B′* on the paper *b*.

With the hatchet at *2* move the tracing point from *c* to *j* and around the diagram in a counterclockwise direction, returning to *c*. The hatchet will stop at some point *3* near *1*. Locate the

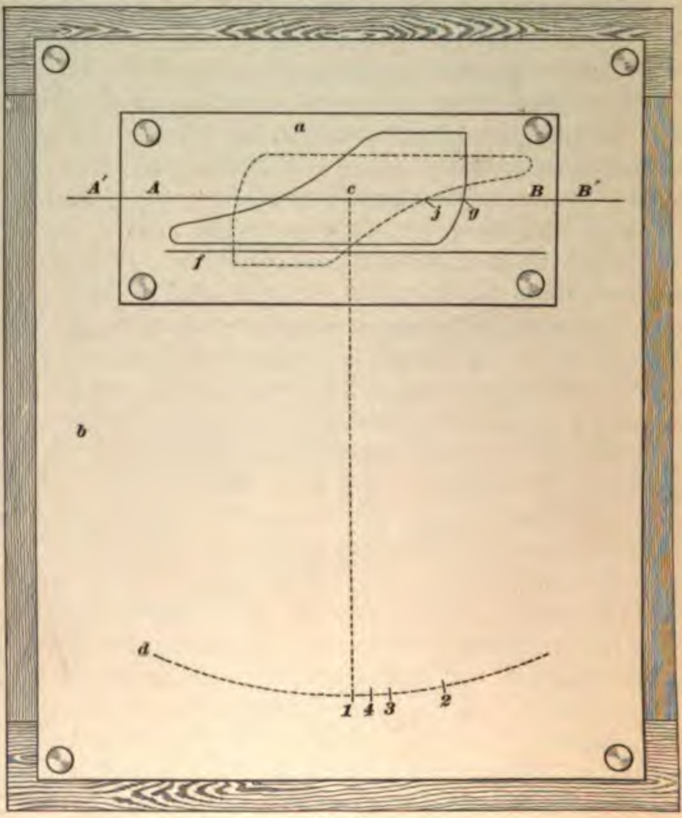


FIG. 13

mid-position *4* between *1* and *3* and measure the distance from *4* to *2*, using an accurately graduated scale. A scale graduated to fiftieths or hundredths of an inch is most convenient. The area of the diagram, in square inches, will then equal the distance *4–2* multiplied by the length of the planimeter.

In order that the measurement may be accurate, it is necessary that the tracing point and the arc forming the edge of the hatchet lie in the same plane, and that the distance between the points *1* and *2* and the length of the planimeter be correctly measured. It is best to locate the actual center of gravity of the diagram, although a small error in this respect will not cause serious inaccuracy, provided that the planimeter is set approximately at right angles to the atmospheric line when starting.

The alinement of the hatchet with the point may be tested by drawing a straight line on a horizontal drawing board, and then placing both tracing point and hatchet on the line and moving the tracing point along it. If the plane of the hatchet is true, the hatchet will follow the line; if not, it will run either to one side or the other.

14. Where a planimeter is not available, the following method of finding the M. E. P. is fairly rapid and accurate: Draw a tangent to each end of the diagram perpendicular to the atmospheric line. Divide the horizontal distance between the tangents accurately into ten or more equal parts; ten or twenty parts are the most convenient, but any other number may be used. Indicate by a dot on the card the center of each division, and draw lines through these dots parallel to the tangents from the upper line to the lower line of the card. On a strip of paper, mark off successively, and with care, the lengths of these lines, the total length thus representing the sum of all the lines. Measure this total length, divide by the number of measurements made, and multiply the quotient by the scale of the spring; the result will be the M. E. P.

EXAMPLE.—The projection of the head-end diagram of Fig. 7 on the atmospheric line is the distance *A Z*, Fig. 14, and it is divided in this case into fourteen equal spaces. The lengths of the perpendicular lines drawn across the diagram through the centers of these spaces are marked on the lines themselves, and the sum of these lengths is 18.11 inches. The scale of the spring used in obtaining the diagram was 40; therefore, $\frac{18.11}{14} \times 40 = 51.74$ pounds per square inch. which is the M. E. P. of the head-end diagram.

A convenient method of dividing the length AZ into fourteen equal parts is to draw any other line from A, as AB, Fig. 14, at a small angle to AZ, and then to lay off any convenient distance AC fourteen times successively, along AB. Connect the last point B with Z, and from the other points, D, E, etc., draw lines parallel to BZ until they intersect AZ. These points of intersection will divide the line AZ into fourteen equal spaces. The middle points of these spaces can then be located by direct measurement and the ordinates erected at these middle points.

A shorter method is shown in Fig. 15. The line ZB is drawn so as to make a small angle with ZA. Then any

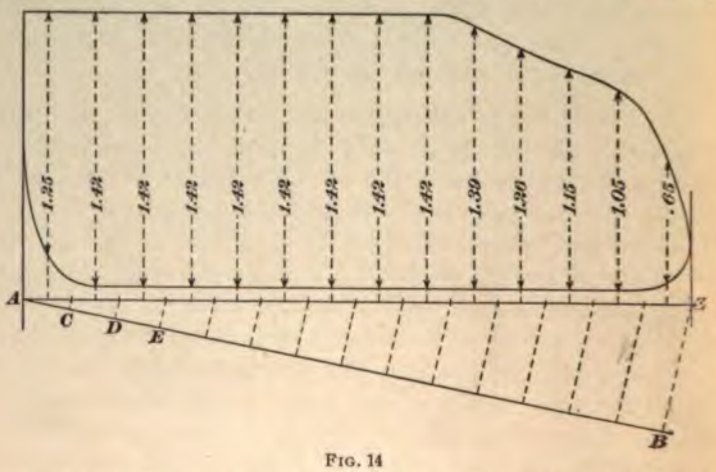


Fig. 14

convenient distance ZC is laid off, as shown. Next, this distance ZC is doubled, and laid off thirteen times in succession from the point C to the point G, making CD, DE, EF, etc., each equal to twice ZC. Finally, GB is laid off equal to ZC. BA is then drawn, and from the fourteen points C, D, E, etc. along ZB lines are drawn parallel to BA and continued until they intersect ZA. At the points where these lines intersect ZA, the ordinates are erected.

EXAMPLE.—The projection of the crank-end diagram, Fig. 8, on the atmospheric line is the distance AZ, Fig. 15, and it is divided in this case into fourteen equal spaces by the last method described

above. The lengths of the perpendicular lines are marked on the lines themselves, and the sum of these lengths is 17.78 inches. The scale of the spring is 40 pounds; therefore, $\frac{17.78}{14} \times 40 = 50.8$ pounds per square inch, which is the M. E. P. of the crank-end diagram. Therefore, the average M. E. P. in the cylinder during a complete revolution of the crank is $\frac{51.74 + 50.8}{2} = 51.27$ pounds per square inch.

It is preferable to divide the diagram into ten equal parts, instead of some other number, to shorten the work of calculation. Thus, in the two examples just given, if the number

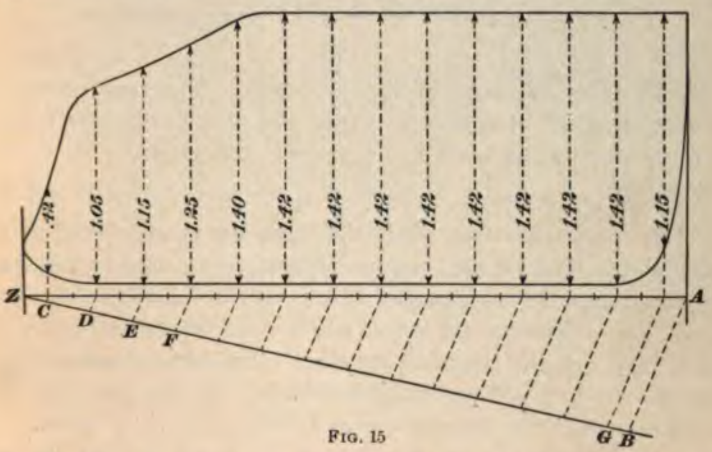


FIG. 15

of divisors had been ten instead of fourteen, and the sum of the ordinates had been 12.94 inches, the mean ordinate would have been $\frac{12.94}{10} = 1.294$ inches, and the M. E. P., $1.294 \times 40 = 51.76$ pounds per square inch. All that is necessary, when the diagram is divided into ten equal parts, is to add the ordinates and shift the decimal point one place to the left to obtain the mean ordinate. This method saves the time required to divide by some inconvenient number, as fourteen.

15. Sometimes the expansion line of the card will fall below the back-pressure line, as shown in Fig. 16. In such a case, the area of the loop *a c* must be subtracted from the

remainder of the card bcd. When the planimeter is used, the subtraction is made automatically by the instrument; but when the card is divided into parts by the method of ordinates, the sum of the ordinates of ac must be sub-

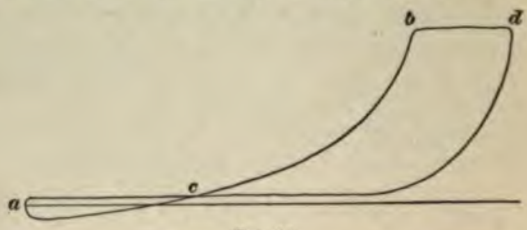


FIG. 16

tracted from the sum of those of bcd. The result divided by the number of spaces will give the mean ordinate; multiplying this by the scale of the spring will give the M. E. P., as before.

16. Horsepower.—All the data necessary for finding the work done in the engine cylinder, expressed in horsepower, are now at hand. Work is the product of force and the distance through which the force moves. In the case of the engine cylinder, the total force is the M. E. P. per square inch multiplied by the area of the piston; and the distance moved through in a minute is the number of strokes in a minute multiplied by the length of stroke.

Let P = M. E. P., in pounds per square inch;
A = area of piston, in square inches;
L = length of stroke, in feet;
N = number of strokes, per minute.

Then, the work done per minute is $PLAN$ foot-pounds. One horsepower = 33,000 foot-pounds per minute. Therefore, the indicated horsepower, frequently written I. H. P., of the engine is found by means of the formula,

$$\text{I. H. P} = \frac{PLAN}{33{,}000}$$

EXAMPLE.—The diameter f the piston of an engine is 10 inches, and the length of stroke 15 inches; it makes 250 revolutions per

minute, with an M. E. P. of 40 pounds per square inch; what is the horsepower?

SOLUTION.—As it is not stated whether the engine is single-acting or double-acting, it is assumed that it is double-acting; then, the number of strokes is $250 \times 2 = 500$ per min. Substituting in the formula,

$$\frac{PLAN}{33{,}000} = \frac{40 \times \frac{15}{12} \times (10^2 \times .7854) \times 500}{33{,}000} = 59.5 \text{ H. P. Ans.}$$

17. When the real cut-off and the steam pressure at the beginning of the stroke are known, the M. E. P. may be found approximately by the following formula:

$$\text{M. E. P.} = \frac{.9\,P(1 + 2.3 \log e)}{e} - .9\,p$$

in which P = absolute steam pressure, that is, the gauge pressure + 14.7 pounds;
e = ratio of expansion;
p = absolute back pressure.

p is usually taken as about 3 pounds for condensing engines, and 17 pounds for non-condensing engines.

EXAMPLE.—A non-condensing engine cuts off at two-thirds stroke; the clearance is 5 per cent.; the gauge pressure is 59.3 pounds. What is the approximate M. E. P.?

SOLUTION.—From *Steam-Engine Mechanism*, $k_1 = \frac{2}{3}$.

$$k = \frac{k_1 + i}{1 + i} = \frac{.66\frac{2}{3} + .05}{1.00 + .05} = \frac{.71\frac{2}{3}}{1.05} = \frac{215}{315} = \frac{43}{63}.$$

$$e = \frac{1}{k} = \frac{63}{43}, \log \frac{63}{43} = .16587.$$

$$P = 59.3 + 14.7 = 74 \text{ lb.}$$

$$\text{M. E. P.} = \frac{.9\,P(1 + 2.3 \log e)}{e} - .9\,p$$

$$= \frac{.9 \times 74 \times (1 + 2.3 \times .16587)}{\frac{63}{43}} - .9 \times 17 = 47.5 \text{ lb. Ans.}$$

18. The product LN of the formula of Art. **16** gives the total distance, in feet, traveled by the piston per minute. This is called the **piston speed.** If the length of the stroke L be given in inches, the piston speed will be $\frac{LN}{12}$. If R is the number of revolutions per minute,

$$\frac{LN}{12} = \frac{L \times 2R}{12} = \frac{LR}{6}.$$

Letting S represent the piston speed in feet per minute,

$$S = \frac{LR}{6}$$

$$L = \frac{6S}{R}$$

$$R = \frac{6S}{L}$$

The piston speeds used in modern practice are about as follows:

	Feet per Minute
Small stationary engines	300 to 600
Large stationary engines	600 to 1,000
Corliss engines	400 to 750
Locomotive engines	600 to 1,200

19. Having given the I. H. P. of the engine and knowing the available M. E. P., there are two methods of calculating the length of stroke and diameter of piston: (1) The number of revolutions and the ratio of the length of stroke to the diameter of cylinder may be assumed. (2) A suitable piston speed may be assumed, and the number of revolutions and length of stroke chosen to correspond. An example will serve to illustrate the above methods.

Given an engine that is to develop 250 I. H. P. with an M. E. P. of 50 pounds per square inch; find the diameter of piston and length of stroke.

First, assume that the engine makes a certain number of revolutions per minute, as 75, and that the length of stroke in inches is say twice the diameter of the piston. Substituting in the formula of Art. **16,**

$$250 = \frac{50 \times L \times A \times (75 \times 2)}{33{,}000}$$

or

$$LA = \frac{33{,}000 \times 250}{50 \times 75 \times 2} = 1{,}100$$

In this expression, L is taken in feet. It is more convenient to use inches, so both sides of the equation will be multiplied by 12. $12\,L_{ft.} \times A = L_{in.} \times A = 13{,}200$. But $A = .7854\,D^2$ and $L = 2\,D$, according to the above assumption.

Substituting, $LA = 2D \times .7854\,D^2 = 1.5708\,D^3 = 13{,}200$.

$$D^3 = 8{,}403$$

$$D = \sqrt[3]{8{,}403} = 20.33 \text{ inches}$$

$$L = 20.33 \times 2 = 40.66 \text{ inches}$$

Since it is customary to avoid fractions of an inch in the stroke of an engine, this engine would be given a stroke of 40 inches. As this is less than 40.66 inches, the diameter of the cylinder would be taken as $20\frac{1}{2}$ inches, which is slightly greater than 20.33 inches. The diameter of the cylinder is usually given to the nearest $\frac{1}{4}$ inch.

Second method. Assume a certain piston speed, say 500 feet per minute; then, as above, $250 = \dfrac{50 \times LAN}{33{,}000}$.

But the piston speed $= 500$ feet $= LN$.

Therefore, $$250 = \frac{50 \times A \times 500}{33{,}000}$$

or $$A = \frac{250 \times 33{,}000}{50 \times 500} = 330 \text{ square inches}$$

$$.7854\,D^2 = 330$$

$$D^2 = 420$$

$$D = 20\tfrac{1}{2} \text{ inches}$$

If, as before, 75 revolutions per minute is assumed, the length of the stroke, from the formula in Art. **18**, is

$$\frac{500 \times 6}{75} = 40 \text{ inches}$$

In calculations in which considerable accuracy is required, allowance must be made for the area of the piston rod, which reduces the effective area of the piston on the side toward the crosshead. To the area of piston obtained must therefore be added one-half the area of the piston rod.

Let A = average effective area of two sides of piston;
A_1 = actual area of piston;
a = area of piston rod.

Then the effective area on the head-end side of the piston is A_1 and the area of the crank-end side $A_1 - a$. The average effective area is therefore

$$\frac{A_1 + (A_1 - a)}{2} = \frac{2A_1 - a}{2} = A_1 - \frac{a}{2}.$$

That is, $$A = A_1 - \frac{a}{2} \qquad (1)$$

or, $$A_1 = A + \frac{a}{2} \qquad (2)$$

Let D = diameter of circle equal in area to average effective area of piston;
D_1 = actual diameter of piston;
d = diameter of piston rod.

Then, since $A = .7854\,D^2$, $A_1 = .7854\,D_1^2$, and $a = .7854\,d^2$, formula **2** may be written $.7854\,D_1^2 = .7854\;D^2 + \frac{.7854\,d^2}{2}$

or $$D_1 = \sqrt{D^2 + \frac{d^2}{2}} \qquad (3)$$

EXAMPLES FOR PRACTICE

1. The mean ordinates of two diagrams taken from the two ends of the cylinder of an 18″ × 20″ non-condensing engine running at 200 revolutions per minute, are, respectively, .72 inch and .76 inch long; the scale of spring being 80, what is the horsepower of the engine?
Ans. 304.335 H. P.

2. In the above engine, assume the initial pressure to be 118 pounds per square inch, gauge, the apparent cut-off as one-fourth, and the clearance as 8 per cent. Find: (*a*) the theoretical M. E. P.; (*b*) the horsepower.
Ans. (*a*) 64.41 lb. per sq. in.; (*b*) 331.12 H. P.

3. An engine running at 165 revolutions per minute has a stroke of 28 inches; what is the piston speed? Ans. 770 ft. per min.

4. If an engine has a piston speed of 960 feet per minute, and runs at 72 revolutions per minute, what is the length of the stroke?
Ans. 80 in.

5. Initial pressure, 82 pounds, gauge; number of expansions, 1.83; back pressure, 4.2 pounds, absolute; what is the theoretical M. E. P.?
Ans. 72.48 lb. per sq. in.

6. The I. H. P. of an engine is 536.42; piston speed, 480 feet per minute; M. E. P., 61.15 pounds per square inch. Find diameter of cylinder to the nearest $\frac{1}{8}$ inch. Ans. $27\frac{3}{4}$ in.

7. A 16″ × 20″ engine develops 138 I. H. P. with 35 pounds M. E. P.; how many revolutions per minute does it make?
Ans. 194.14 rev. per min.

8. A 54″ × 66″ non-condensing engine develops 1,382.4 I. H. P., with an initial pressure of 63 pounds, gauge, when cutting off at one-fifth stroke and running at 82 revolutions per minute. (*a*) What is the actual M. E. P.? (*b*) With a back pressure of 1 pound above the atmosphere and a clearance of 3 per cent., what would be the theoretical I. H. P. calculated by the formulas in Arts. **16** and **17**?

Ans. { (*a*) 22.083 lb. per sq. in.
(*b*) 1,557 I. H. P., about

9. A 16″ × 14″ engine runs at 240 revolutions per minute; what is the piston speed? Ans. 560 ft. per mi.

10. If the average M. E. P. of the engine in the last example is 41.73 pounds per square inch and the diameter of the piston rod is 4 inches, what is the I. H. P., taking the piston rod into consideration? Ans. 137.93 H. P.

---

20. From the measurement of the indicator diagrams has been obtained what has been termed the indicated horsepower, that is, the horsepower developed in the engine cylinder. A portion of the I. H. P. is absorbed in overcoming the friction of the engine itself. The remainder is available for doing the required work.

The power absorbed by the engine itself is termed **friction horsepower.**

The power available for doing useful work is termed the **net,** or **actual, horsepower.**

21. The actual horsepower of any engine is found by first computing its I. H. P. from a set of indicator diagrams taken when the engine is running under full load, and then subtracting from this the I. H. P. computed from a set of indicator diagrams taken when the engine is running under no load, but making the same number of revolutions per minute as before. The horsepower developed by the engine in this last case will only be sufficient to keep the working parts of the engine in motion at the same speed. To produce this result, some means will have to be resorted to of checking the steam supply. These will be discussed later.

EXAMPLE.—Indicator diagrams taken from an engine when running under full load, and having a piston speed of 498 feet per minute, showed an indicated horsepower of 242.7. With the same piston speed, and running under no load, the indicator diagrams showed an

indicated horsepower of 29.2. Therefore, $242.7 - 29.2 = 213.5$, which is the actual horsepower of the engine.

It has been found that the frictional losses remain very nearly the same for all loads at a constant engine speed. The friction of the eccentric and the valve motion, of the piston in the cylinder, and of the piston rod and valve stem in their stuffingboxes is practically constant. The variable losses are those due to the change in pressure on the crosshead guides, the crosshead, and crankpins, and the main bearings. If these parts are properly lubricated, the variation in lost work for the extreme range of load is slight. For this reason, and because it is more easily obtained, the friction at no load is taken for the friction at full load in calculating engine horsepower.

22. The **mechanical efficiency** of an engine is the ratio of the actual horsepower to the indicated horsepower; or it is the percentage of the mechanical energy developed in the cylinder that is utilized in doing useful work.

To find the efficiency of an engine, when the indicated and actual horsepowers are known:

Rule.—*Divide the actual horsepower by the indicated horsepower.*

EXAMPLE.—The indicated horsepower of an engine is 242.7, and the actual horsepower is 197.5. Therefore, $\frac{197.5}{242.7} = 81.38$ per cent. efficiency.

The mechanical efficiency of engines in good order varies from 75 to 92 per cent.

23. The **efficiency** of the ideal steam engine is the same as that of any other heat engine. This was shown, in *Heat*, Part 2, to be

$$\frac{T_1 - T_2}{T_1}$$

where T_1 = absolute temperature of the entering steam;
T_2 = absolute temperature of the exhaust steam.

EXAMPLE.—The pressure of the entering steam is 100 pounds above vacuum, and at exhaust it is 16 pounds above vacuum; what is the efficiency of the ideal engine?

SOLUTION.—Temperature of incoming steam, from Steam Table, 327.58°. Temperature of exhaust steam, from Steam Table, 216.32°.

Absolute $T_1 = 327.58 + 460 = 787.58$. Absolute $T_2 = 216.32 + 460 = 676.32$.

$$\text{Efficiency} = \frac{T_1 - T_2}{T_1} = \frac{787.58 - 676.32}{787.58} = .1413 = 14.13 \text{ per cent. Ans.}$$

READING INDICATOR DIAGRAMS

24. The determination of the indicated horsepower is not the only or the most important function of the indicator. By its use defects in steam distribution may be detected, the correction of which may result in largely increased economy in the working of the engine.

The form of a good diagram depends largely on the type

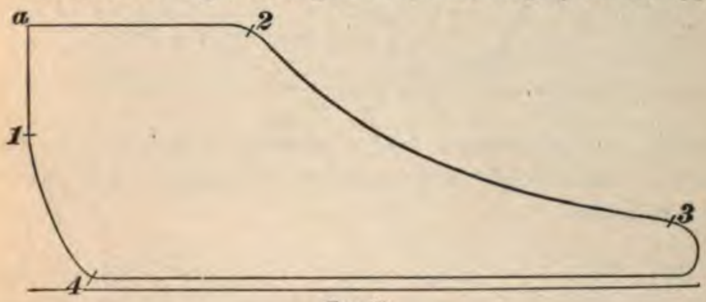


FIG. 17

of engine, the style of valve, and the speed. The same style of diagram from all engines is not possible or desirable.

In Fig. 17, *1* is the point of admission;
2 is the point of cut-off;
3 is the point of release;
4 is the point of compression.

Some of the most common faults in steam distribution are given below:

I. Admission may be too early;
II. Admission may be too late;
III. Cut-off may be too early;
IV. Cut-off may be too late;
V. Release may be too early;
VI. Release may be too late;
VII. Compression may be too early;
VIII. Compression may be too late.

25. Case I.—The effect on the diagram of a too early admission is shown in Fig. 18. The admission line *1 a* instead of being straight and perpendicular to the atmospheric line, as in Fig. 17, curves backwards. With a single slide valve, like the one already described, all the other

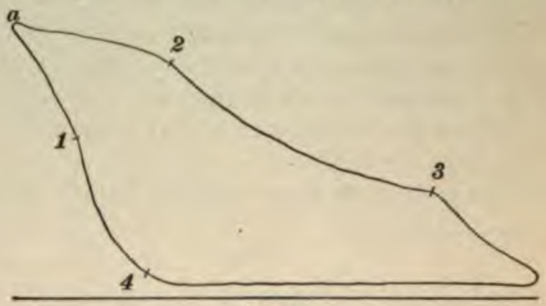


FIG. 18

events, cut-off, release, and compression, will also be too early. The remedy is to decrease the angular advance of the eccentric.

26. Case II.—In this case, the admission is too late, and the admission line *1 a* on the diagram will curve forwards, as

FIG. 19

shown in Fig. 19. The remedy is to increase the angular advance until the admission line *1 a* becomes perpendicular to the atmospheric line. It will be noticed that, in the case of a too late admission, the other events at *2*, and particularly at *3* and *4*, are also too late.

27. Case III.—Cut-off too early. In this case, the steam expands below the back-pressure line and forms a loop, as shown at *a c*, Fig. 16. This may be remedied by decreasing the outside lap, which will make the cut-off later. This, however, will make admission earlier, but further inspection of the diagram shows that the release and compression are slightly early, which will allow the too early admission to be remedied by decreasing the angular advance. It will be seen that this aids in making the cut-off still later.

28. Case IV.—Cut-off too late; see Fig. 20. Here it will be noticed that the terminal pressure is very high.

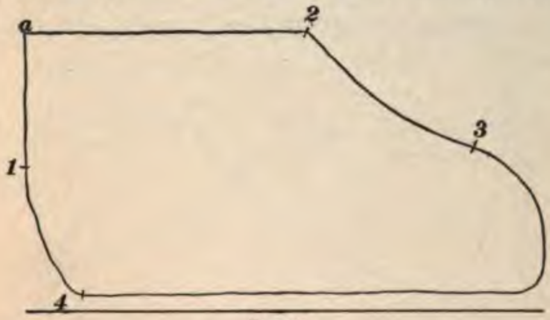


FIG. 20

When this is the case, a great deal of the benefit of expansion is lost, with a consequent waste of steam.

For Cases III and IV, make the cards alike for both ends of the cylinder. For a too early cut-off in engines in which the point of cut-off is regulated by the governing mechanism, as in the Corliss engine, lower the boiler pressure or decrease the number of revolutions per minute. For a too late cut-off, raise the boiler pressure or increase the number of revolutions per minute.

In engines with throttling governors, the cut-off does not change, but the initial steam pressure in the cylinder is controlled by the governing mechanism. To make the cut-off earlier or later, the outside lap must be increased or decreased, or the angle of advance must be changed, if this can be done without detriment to the other events.

The cut-off is most correctly equalized by making the terminal pressure at both ends of the cylinder the same.

Case V.—Release too early; see Fig. 18.

Case VI.—Release too late; see Fig. 19.

For Cases V and VI, adjust the valve so that one-half of the fall of pressure from the point of release to the back-pressure line occurs before the piston starts on the return stroke.

29. Case VII.—Compression too early. Fig. 21 shows the effect of too early compression. A loop is formed, and the area of this loop must be subtracted from the larger area in computing the M. E. P. With the same cut-off and the proper amount of compression, the area gained would be *a b 4 a*, included between the line *4 a* and the dotted

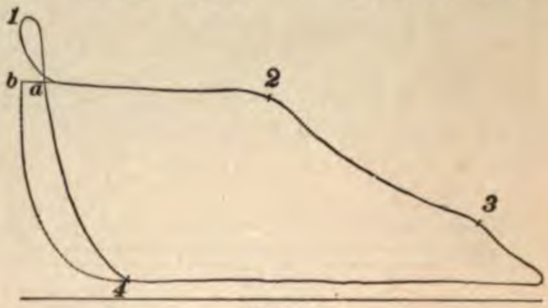


FIG. 21

line *a b 4*, plus the area of the loop. The remedy in this case is to decrease the amount of inside lap.

The required amount of compression depends on the speed of the engine, slow-running engines not requiring so much compression as high-speed engines. In any case, the compression should not extend above the initial or boiler pressure.

It is good practice to compress to about nine-tenths the initial pressure with high-speed engines, five-tenths with medium-speed engines, and from two-tenths to three-tenths with slow-speed engines.

Case VIII.—Compression too late. When there is insufficient compression, the engine is liable to pound in passing the dead centers. The remedy is to increase the inside lap until the pounding disappears, provided, of course, that the

noise is due to insufficient cushioning of the reciprocating parts. An engine should have no more compression than is required to enable it to run smoothly and quietly.

All the above faults can be detected as soon as the indicator is applied.

30. With a plain slide valve, it will be found that if one of the events of the stroke is early or late, the others are liable to be so also; for example, an early admission usually produces an early release and compression.

When the steam line falls abruptly, as shown in Fig. 18, it may be inferred that the steam is throttled; that is, either the steam pipe or the port is too small for the required duty. A very high piston speed would also produce this effect.

The diagram shown in Fig. 22 indicates that the back

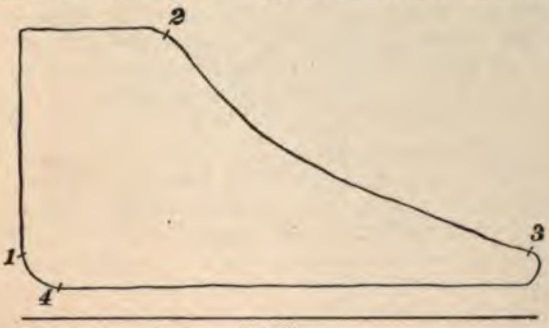


FIG. 22

pressure is excessive. This may be the case when the exhaust port is too small or when the exhaust steam is used for heating purposes, and, in consequence, has to be pushed through coils of pipe.

STEAM CONSUMPTION

31. The indicator diagram makes it possible also to find approximately the amount of steam consumed by the engine. It is customary to express the steam consumption in *pounds of steam consumed per horsepower per hour.*

Take a point *a* on the expansion line before the release, as shown in Fig. 23; measure the pressure from the vacuum

line, and from the Steam Table find the weight of a cubic foot at that pressure. The cubic contents of the cylinder, including the clearance, up to the point *a*, multiplied by the weight per cubic foot, will give the weight of steam in the cylinder at this instant. Were it not for compression and cylinder condensation, the above weight would represent the steam consumed per stroke. On account of compression, some steam is saved by the early closure of the exhaust port. To find its weight, take a point *b* on the compression curve, measure its pressure from vacuum, as before, and compute the weight of the steam in the cylinder up to *b*. Subtract this from the weight first obtained, and the difference will be

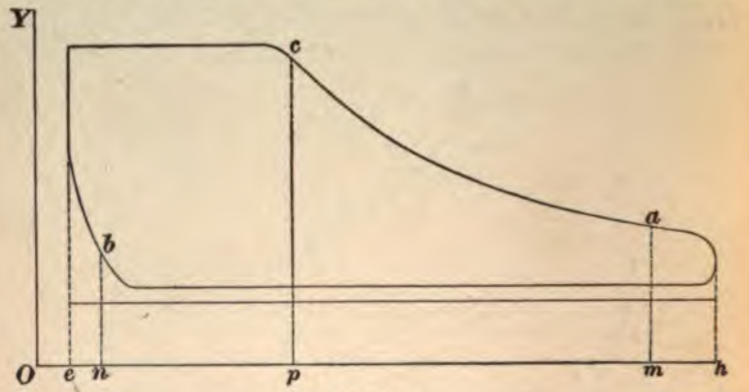


FIG. 23

the weight of steam per stroke, accounted for by the indicator. Multiply this weight per stroke by the number of strokes per hour, and divide by the I. H. P. of the engine. The result will be the steam used per I. H. P. per hour.

EXAMPLE.—Fig. 23 represents an indicator diagram taken from an engine with an 18″ × 24″ cylinder, running at 120 revolutions per minute and developing 130 horsepower; the clearance is 5 per cent. Find the steam consumption per I. H. P. per hour.

SOLUTION.—Project the two ends of the diagram perpendicularly on the vacuum line, as at *e* and *h*; *e h* is then the length of the diagram. Lay off *e O* equal to the clearance—that is, equal to 5 per cent. of *e h*. Draw *O Y* perpendicular to *O h*. Take the point *a*, near the point of release, and measure the distances *a m* and *O m*. Take the

point b, somewhere on the compression line, and measure the distances $b\,n$ and $O\,n$. The measurements are found to be: $a\,m$ = .71 in.; $O\,m$ = 3.17 in.; $b\,n$ = .6 in.; $O\,n$ = $\frac{1}{3}$ in. The length of the diagram = $e\,h$ = $3\frac{1}{3}$ in.; the length of the stroke is 2 ft. Hence, each inch of length of the card equals $2 \div 3\frac{1}{3}$ = .6 ft. of stroke. The scale of the indicator spring is 45. Hence, the above measurements reduced to pressures in pounds per square inch and feet of stroke become: $a\,m$ = $.71 \times 45$ = 31.95 lb.; $b\,n$ = $.6 \times 45$ = 27 lb.; $O\,m$ = $3.17 \times .6$ = 1.9 ft.; $O\,n$ = $\frac{1}{3} \times .6$ = .2 ft.

The area of the piston is $18^2 \times .7854$ = 254.47 sq. in. = $\frac{254.47}{144}$ = 1.767 sq. ft. Consequently, the volume of steam in the cylinder, when the piston is at the point represented by a, is 1.9×1.767 = 3.3573 cu. ft. The volume, when the piston is at b, is $.2 \times 1.767$ = .3534 cu. ft. The weight of a cubic foot of steam at an absolute pressure of 31.95 lb. per sq. in. is found from the Steam Table to be .07809 lb.; and at a pressure of 27 lb. the weight is .06666 lb. Hence, the weight of the steam in the cylinder is $.07809 \times 3.3573$ = .26217 lb.; while the weight of steam saved by compression is $.06666 \times .3534$ = .02356 lb. The steam used per stroke is, therefore, .26217 − .02356 = .23861 lb., and the amount used per I. H. P. per hour is

$$\frac{.23861 \times 120 \times 2 \times 60}{130} = 26.43 \text{ lb.}$$

Suppose the weight of the steam in the cylinder to be calculated by taking the point c, near the point of cut-off. $c\,p$ = 1.59 in., or 1.59×45 = 71.55 lb.; $O\,p$ = $1\frac{1}{3}$ in., or $\frac{4}{3} \times .6$ = .8 ft. of stroke. The volume of steam in the cylinder when the piston is at c is, therefore, $.8 \times 1.767$ = 1.4136 cu. ft. One cubic foot of steam at the pressure of 71.55 lb., absolute, weighs .1661 lb. The weight of the steam in the cylinder at c is, therefore, $.1661 \times 1.4136$ = .2348 lb. Subtracting the steam saved by compression, the steam used per stroke is .2348 − .02356 = .21124 lb., and the steam per I. H. P. per hour is

$$\frac{.21124 \times 120 \times 2 \times 60}{130} = 23.4 \text{ lb.}$$

Now, unless the valve leaks, the weight of the steam when the piston is at a can be no greater than when it is at c, since no fresh steam has been allowed to enter; but the calculation shows that there is .26217 lb. in the cylinder when the piston is at a, and only .2348 lb. when the piston is at c. This shows that .26217 − .2348 = .02737 lb. has been condensed to water by the time the piston has arrived at c, but has been reevaporated before the piston arrives at a. Hence, by calculating the steam consumption at cut-off, and then at release, a good idea of the amount of cylinder condensation may be obtained. If the steam used by the engine be actually caught and weighed and then compared with the weight as calculated from release an idea may

be obtained of the amount of condensation at release. The computed consumption is always less than the actual consumption.

32. Where there is a sufficient amount of compression, the work may be simplified by taking the two points a and b at the same height above the vacuum line, as shown in Fig. 24. Since the absolute pressure at a and b is the same, the clearance may be left entirely out of account. Then the volume of steam in the cylinder at the pressure a is represented by the line Om. During exhaust, this volume is swept out of the cylinder, except the portion remaining when the exhaust closes, and when compressed to the pressure b it is represented by the volume On. The volume swept out, at the

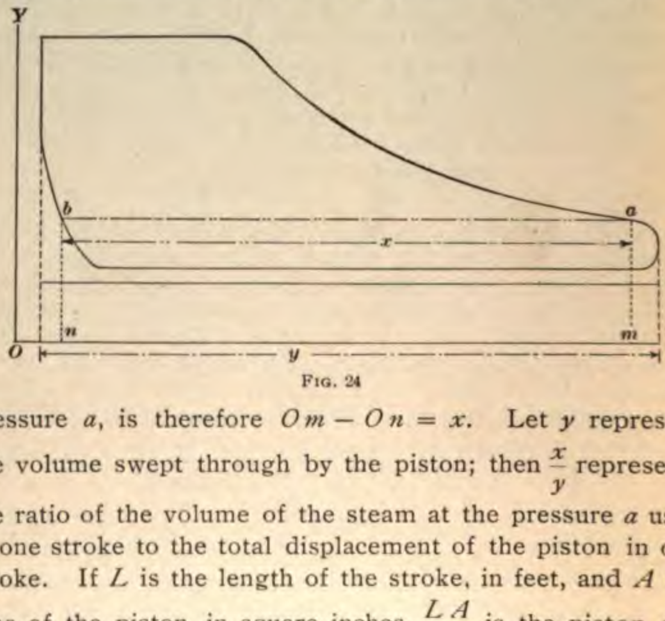


FIG. 24

pressure a, is therefore $Om - On = x$. Let y represent the volume swept through by the piston; then $\frac{x}{y}$ represents the ratio of the volume of the steam at the pressure a used in one stroke to the total displacement of the piston in one stroke. If L is the length of the stroke, in feet, and A the area of the piston, in square inches, $\frac{LA}{144}$ is the piston displacement in cubic feet. Hence, $\frac{x}{y} \times \frac{LA}{144}$ equals the volume, in cubic feet, used in one stroke. This volume multiplied by N, the number of strokes per minute, by 60, the minutes in an hour, and by w, the weight of 1 cubic foot of steam

at the pressure a, gives the total weight of steam used in 1 hour by the engine. The total weight of steam is then $\frac{x}{y} \times \frac{LA}{144} \times N \times 60 \times w = \frac{60\, x\, L\, A\, N\, w}{144\, y}$. This weight divided by the indicated horsepower of the engine as represented by the formula, H. P. $= \frac{PLAN}{33,000}$, will give the pounds of steam per I. H. P. per hour. Let Q be this quantity; then $Q = \frac{60\, x\, L\, A\, N\, w}{144\, y} \times \frac{33,000}{PLAN}$, or $Q = \frac{13,750\, x\, w}{P\, y}$, in which P is the M. E. P. of the diagram from which the steam consumption is being computed. The steam consumption should be calculated for both the head-end and crank-end diagrams, and the average of these two results will then represent the approximate steam consumption of the engine.

EXAMPLE.—From a diagram taken from an $18\frac{1}{2}'' \times 30''$ engine, the following measurements were obtained (see Fig. 24): $a\,m$ = .667 inch; x = 3.08 inches; y = 3.5 inches; M. E. P. = 35 pounds. What is the steam consumption per I. H. P. per hour?

SOLUTION.—The indicator diagram being taken with a 45 spring, the pressure at a is $45 \times .667 = 30$ lb., absolute. The weight of a cubic foot of steam at this pressure is .0736 lb. Substituting in the formula,

$$Q = \frac{13,750\, x\, w}{P\, y} = \frac{13,750 \times 3.08 \times .0736}{35 \times 3.5} = 25.44 \text{ lb. Ans.}$$

EXAMPLES FOR PRACTICE

1. Size of engine, 12 in. $\times$ 20 in.; length y of diagram, 3.4 inches; length x, $2\frac{1}{4}$ inches; height $a\,m$, $\frac{3}{8}$ inch; spring, 30; M. E. P., 18 pounds per square inch. What is the steam consumption per I. H. P. per hour?
Ans. 25.39 lb. per I. H. P. per hr.

2. Size of engine, 12 in. $\times$ 12 in.; M. E. P., 51.1 pounds; length y of diagram, 2.6 inches; length x, 1.8 inches; height $a\,m$, .7 inch; spring, 70. What is the steam consumption per I. H. P. per hour?
Ans. 21.72 lb. per I. H. P. per hr.

3. If, in the engine in example 2, the pressure at cut-off is 110 pounds, absolute; the clearance is 8 per cent.; the length of the

diagram to the point of cut-off is .7 inch; the pressure at a point on the compression curve is 49 pounds, absolute, and the distance of this point from the end of the diagram is .14 inch, what is the steam consumption per I. H. P. per hour at cut-off?

Ans. 19.14 lb. per I. H. P. per hr.

NOTE.—The weight of steam should be calculated at the point of cut-off and during compression separately, in example 3. The formula in Art. 32 cannot be applied directly.

SIMPLE NON-CONDENSING STEAM ENGINES

TYPES OF SIMPLE STEAM ENGINES

1. Principal Types of Slide-Valve Engines.—There are a number of forms of **slide-valve engines,** varying principally in the kind of valve and the method of governing used. The principal types are: those with throttling governors and those with variable cut-off mechanisms, the latter being usually regulated by shaft or fly-wheel governors. A throttling governor is one that regulates the speed of the engine by varying the amount of opening of a throttling valve through which the steam must pass on its way to the engine cylinder. The steam is thus throttled, or wire-drawn, to a greater or less extent, and the mean pressure in the cylinder is thus varied according to the load on the engine.

In engines using throttling governors, the point of cut-off is fixed. In engines having a variable cut-off, the point of cut-off is changed by a shaft governor according to the demands for power made on the engine. A **shaft governor** is a mechanism contained in the flywheel, or in an auxiliary wheel, the purpose of which is to regulate the speed of the engine. This type of governor acts directly on the eccentric, changing the valve travel and thus regulating the amount of steam admitted to the cylinder. In this respect it differs from the throttling governor, as the latter does not affect either the valve travel or the eccentric position.

PLAIN SLIDE-VALVE ENGINES

2. Principal Parts of a Plain Slide-Valve Engine. In Fig. 1 (*a*) is shown a plain slide-valve engine, and in Fig. 1 (*b*) a sectional view through the cylinder and valve. The plain solid piston *a* is fastened to the piston rod *b* by having the rod riveted and peened over to prevent the piston

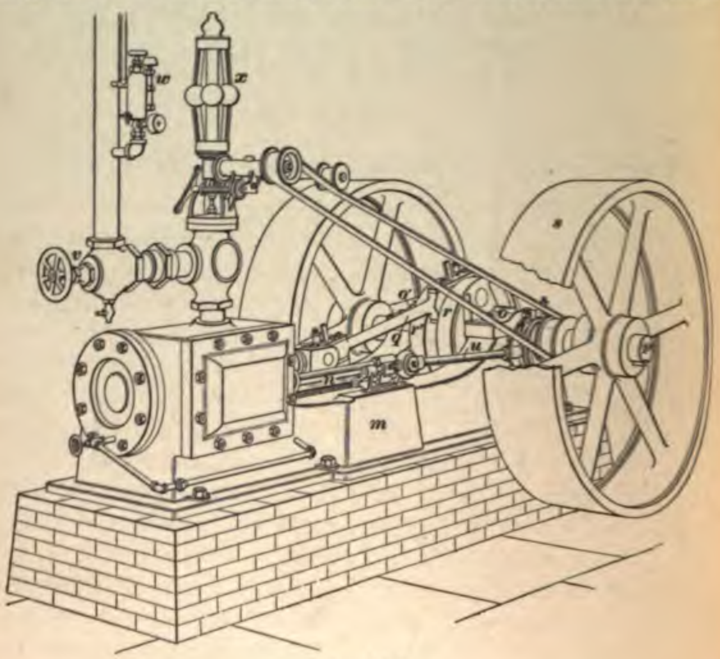


FIG. 1 (*a*)

from coming loose. The cylinder *c* has a front head *d* and a back head *e*; a stuffingbox *f* is placed in the front head for the purpose of holding packing around the piston rod to prevent the leakage of steam from the cylinder to the atmosphere. At *g* and *g'* are the steam ports through which the steam passes on its way from the steam chest *h* to either end of the steam cylinder as the slide valve *i* moves to and fro. The exhaust also passes out of the ports *g* and *g'* into the valve cavity and out through the exhaust port *j* to the atmosphere

through a pipe not shown. The valve stem *k* is threaded and screwed into the valve, being held by the jam nut, as shown. It passes out of the steam chest through a stuffingbox similar to the one used on the piston rod. The engine frame *m*, Fig. 1 (*a*), carries the crosshead guides *n*, and also the main bearings *o*, *o'* in which the shaft *p* turns. The connecting-rod *q* is connected to the crosshead at one end and to the crank-disks *r*, *r'* at the other end. The flywheel *s* is attached directly to the shaft, as is also the eccentric *t*. The eccentric rod *u* connects the eccentric with the valve stem and gives the valve its to-and-fro motion. At *v* is shown the

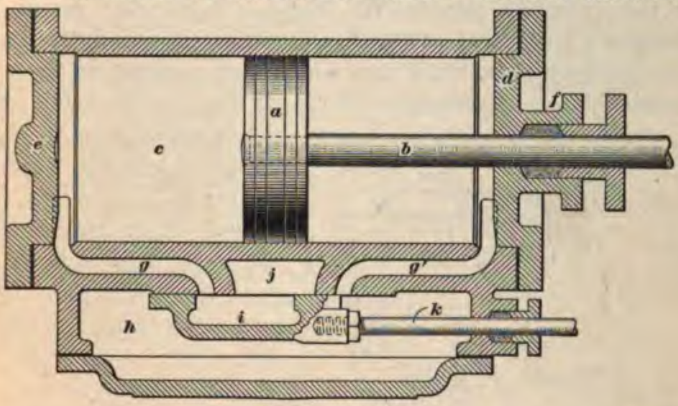


FIG. 1 (*b*)

throttle valve, which admits or shuts off the steam from the boiler to the cylinder. The cylinder lubricator is shown at *w* and the governor at *x*, the latter forming a part of the steam-pipe connections.

3. The Valve and Its Motion.—The action of the eccentric, as already explained, is simply that of a crank, which moves the slide valve back and forth on its seat. The eccentric, being fixed to the shaft, gives an unvarying valve motion, so that the admission and cut-off of the steam and the opening and closing of the valve to exhaust are always the same. This type of valve is wasteful of steam because the point of cut-off is fixed and the steam is admitted for the same portion of the stroke of the engine for all loads. It will

be seen, also, that the valve is subjected to steam pressure on the entire outer surface, while on the inside, it is subjected to pressure on only the very small area over the steam ports. The unbalanced steam pressure on the valve, therefore, forces it down on its seat, and causes an amount of friction and wear that is largely unnecessary.

THE GOVERNOR

4. Purpose of the Governor.—When a steam engine is running at a uniform speed, the work done by the steam in the cylinder, neglecting friction, must just equal the resistance overcome at the flywheel rim. Should the resistance become less than the work, the amount of work in excess of that necessary to overcome the resistance will cause the moving parts to move faster and faster, and the engine will "race" or "run away." If, on the contrary, the resistance should exceed the work, the engine will slow down, and finally stop. The work required of the engine cannot, of course, remain always constant; hence, it is necessary to have some means of automatically adjusting the steam supply to the variations in the resistance. This is accomplished by the **governor**.

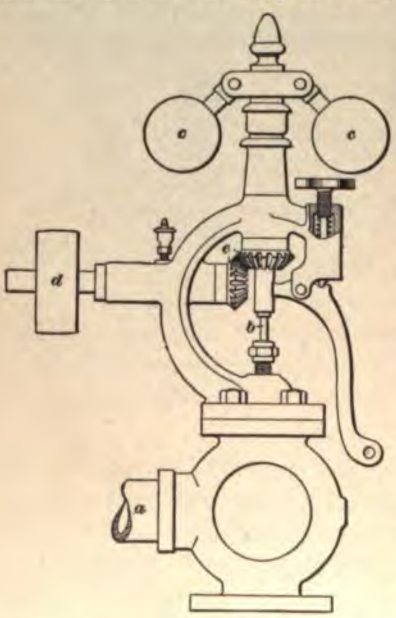


FIG. 2

5. Throttling Governor.—The ordinary throttling governor, shown in Fig. 2, consists of a balanced throttle valve placed on the steam pipe *a*; this valve is attached to the spindle *b*, at the upper end of which are the two **flyballs** *c*, *c*, each arm and

flyball forming what is known as a **revolving pendulum.** The spindle and balls are driven from the main shaft of the engine by a belt on the pulley *d*, and by the bevel gears *e*. If the engine moves faster than the desired speed, the flyballs are forced to revolve at a higher speed, and will, consequently, move outwards and upwards through the action of centrifugal force. This forces the spindle *b* downwards and partly closes the throttle valve. The engine thus takes less steam and the speed falls to the desired point, the governor balls in the meantime returning to their original position. Should the resistance become greater than the power of the engine, it slows down slightly and the balls drop and open the valve wider. More steam is admitted and the engine immediately regains its original speed. The chief objection to the throttling governor is that the steam is wiredrawn. The term *wiredrawn* is applied to any case in which the steam pressure is reduced owing to the insufficiency of valve opening; the term *throttled* is also applied to such cases. Steam is more or less wiredrawn in all engines fitted with plain slide valves; the rounded corners on the indicator diagram prove this; this is because the movement of the valve is comparatively slow when opening and closing the ports. With Corliss and other releasing gear engines, the valve movement at cut-off and release is very rapid, and the wiredrawing very slight.

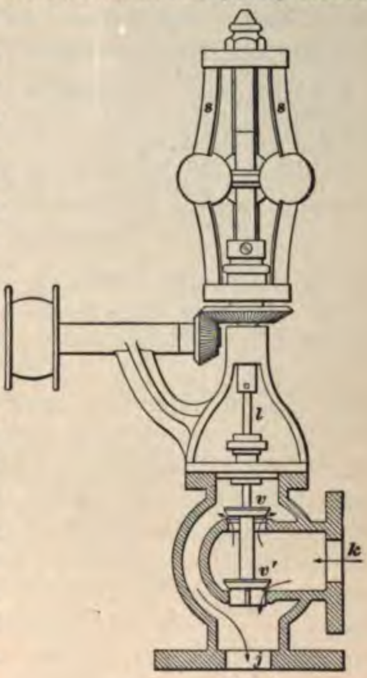


FIG. 3

Another form of throttling governor, which is similar to the one shown on the engine in Fig. 1, is shown in Fig. 3;

this is known as the **Pickering governor.** There are three balls, and they move outwards against the resistance of gravity and the three flat springs *s*. In so doing, they lower the valves *v* and *v'*. The steam enters at *k*, flows in the direction of the arrows, and then through *j* into the steam chest. Since the steam presses against these two valves in opposite directions and with equal intensity, they are *balanced*. The object of using two valves instead of one is to afford a large opening with a small lift of the valve.

6. Indicator Diagrams From Engine with Throttling Governor.—Fig. 4 shows indicator diagrams taken

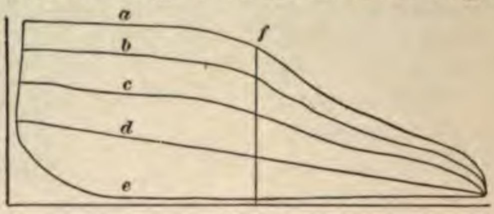


Fig. 4

from a plain slide-valve engine with a throttling governor working under a variable load. In this type of engine, the steam line of the indicator diagram always inclines toward the atmospheric line instead of being nearly parallel to it, as in the diagrams previously given; in consequence of this, the mean effective pressure is less. This inclination of the steam line is shown by the lines *a*, *b*, *c*, and *d*, Fig. 4, which shows the diagrams for four loads on the engine. It will be noticed that the back-pressure line *e* is constant for all loads, and that the point of cut-off *f* comes at the same place in the stroke in each case.

CUT-OFF OR EXPANSION VALVES

7. Purpose of Cut-Off Valves.—In order to extend the range of cut-off beyond that which may be obtained with the plain slide valve driven by a fixed eccentric, and at the same time to leave unaffected the events of admission, release, and compression, various types of auxiliary valves, generally known as **expansion valves**, or **cut-off valves**,

have been designed for use in conjunction with the plain slide valve. These auxiliary valves are generally driven by separate eccentrics. Their sole purpose is to stop the flow of steam to the cylinder at the desired point in the stroke, leaving the other features of steam distribution to be controlled by the main valve in the usual manner. Since the point of cut-off is the only event controlled by the auxiliary valve, any change in the proportions of this valve or in the position of its eccentric that will secure the desired change in cut-off may be made without affecting the other events in the steam distribution. Consequently, the point at which the auxiliary valve cuts off the supply of steam may be varied in any convenient way; for example, the lap of the valve or the angle of advance or the eccentricity of the auxiliary eccentric may be changed as may be most convenient.

8. Gonzenbach Valve.—A simple application of the cut-off valve, known as the **Gonzenbach valve,** is shown in Fig. 5. The main valve V is a plain slide valve driven by

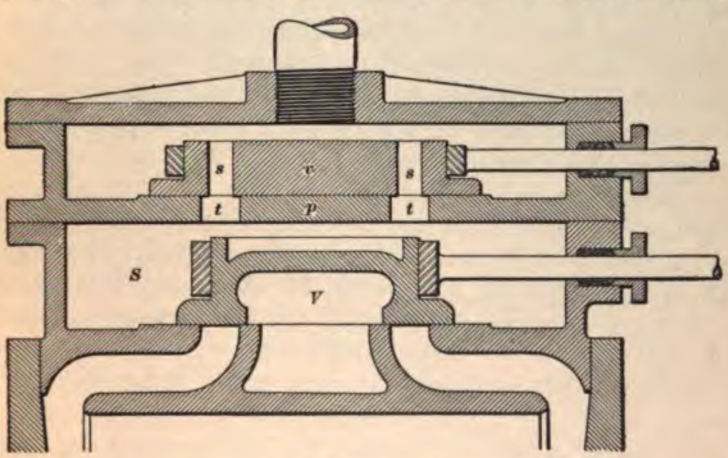


FIG. 5

a fixed eccentric in exactly the same manner as in the plain slide-valve engine. This valve is given such proportions as will secure the desired points of admission, release, and compression, care being taken that the lap is such that it

will not cut off steam earlier in the stroke than the latest point at which it may be desired to have cut-off occur, usually about three-fourths stroke.

The main valve is located in a section S of the steam chest that is separated from the remainder by a plate or partition p. This partition has ports or openings t, t that form passages connecting the two chambers, and it also forms the valve seat of the auxiliary valve v. The auxiliary valve has ports s, s. It is driven from the main shaft by an auxiliary eccentric. When it is in mid-position, as shown in the figure, the ports t, t are open and allow steam to fill the chamber around the main valve. As it moves either way from the mid-position, it covers the ports t, t, and so shuts off the supply of steam from the main valve.

The main valve is set in exactly the same manner as any plain slide valve. The auxiliary valve must be so proportioned and set that it will open the ports t, t as soon as or before the main valve opens the main port, but they must not be reopened before the cut-off occurs with the main valve, otherwise steam will be readmitted to the cylinder and the effect of cut-off will be destroyed. The point of cut-off may be varied by changing either the eccentricity or the angle of advance of the auxiliary eccentric, the latter being the more common method.

A disadvantage of this type of expansion valve is that when cut-off takes place, the space in the main-valve chamber is filled with steam. This has the effect of increasing the clearance during the period of expansion, and thus destroys some of the benefits derived from the earlier cut-off. On account of these limitations and disadvantages, this simple form of expansion valve is but little used.

9. Meyer Cut-Off Valve.—An improvement on the preceding type and one that is free from most of its disadvantages is the **Meyer cut-off valve.** The main valve consists of a flat plate with a chamber e on the under side. It acts in exactly the same manner as a plain slide valve, from which it differs only in that the ends of the valve are

extended so as to form passages, or ports, *a*, *a* leading to the edges that control the admission of steam to the cylinder. The cut-off valve consists of two plates *b*, *b* that slide on the top of the main valve so as to close the passages *a*, *a* at the point at which it is desired to have cut-off take place.

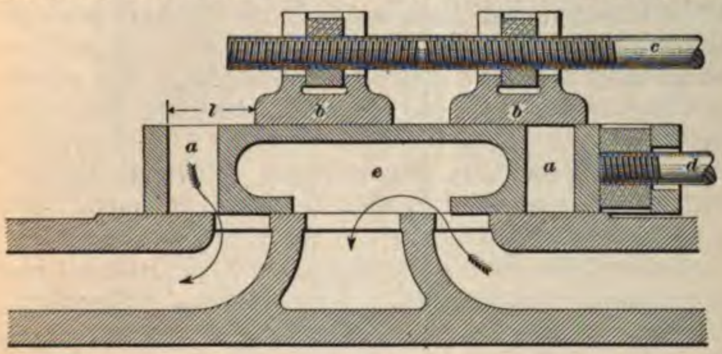


FIG. 6

The main valve is driven from its eccentric by the valve stem *d*, and the cut-off valve is operated from a separate eccentric by the valve stem *c*. The main eccentric is set in the same manner as with the plain slide valve. The auxiliary eccentric is so set that, when cut-off takes place, the cut-off valve has a motion opposite in direction to that of the main valve. The cut-off valve thus closes the passage *a* through which steam is being admitted to the cylinder. The action may be studied by considering Fig. 7 in connection with Fig. 6.

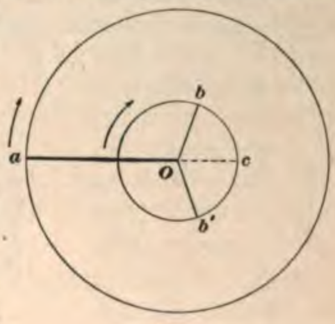


FIG. 7

10. In Fig. 7, Oa represents the crank in the dead-center position, corresponding to the valve position shown in Fig. 6, while Ob and Oc represent the corresponding positions of the main and auxiliary eccentrics. It will be noticed that the main eccentric Ob is set in its usual position of 90° + angle of advance ahead of the

crank, thus giving the main valve the amount of lead shown in Fig. 6. The auxiliary eccentric *Oc* is set in a position nearly opposite to that of the crank. By inspecting the two figures, it is seen that as the crank moves in the direction shown by the arrows, the main valve, driven by the eccentric *Ob*, will move to the right and open the left-hand port; at the same time, the cut-off valve, driven by the eccentric *Oc*, will move to the left and close the passage *a* as soon as the combined motion of the two valves is equal to the distance *l*, Fig. 6.

The valve stem *c*, Fig. 6, is provided with right- and left-hand threads that work in threaded plates or nuts in the valve plates *b*, *b*. By turning *c* in one direction or the other, the distance between the outside edges of the plates *b*, *b* can be increased or diminished. If the distance between the plates is increased, the distance *l* becomes shorter and cut-off takes place earlier; the opposite adjustment makes cut-off take place later. As usually arranged, the rod *c* may be turned by a hand wheel placed outside the steam chest, so that cut-off may be changed while the engine is running.

11. A modification of the Meyer valve is shown in Fig. 8. Here the cut-off valve consists of a single plate *a* with

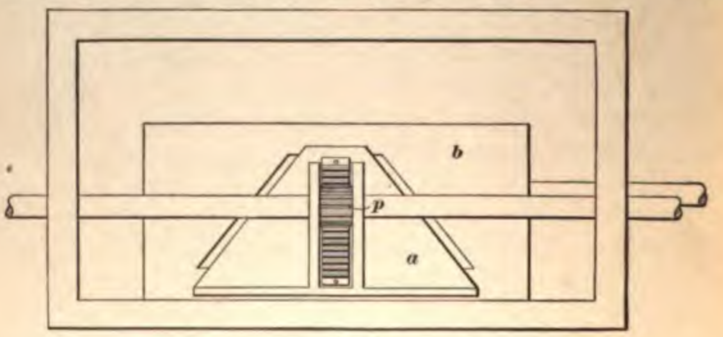


FIG. 8

inclined edges working over a main valve *b* having passages similarly inclined. The cut-off valve stem, in addition to the endwise motion given to it by its eccentric, may be rotated. It carries a pinion *p* that works in a rack on the cut-off

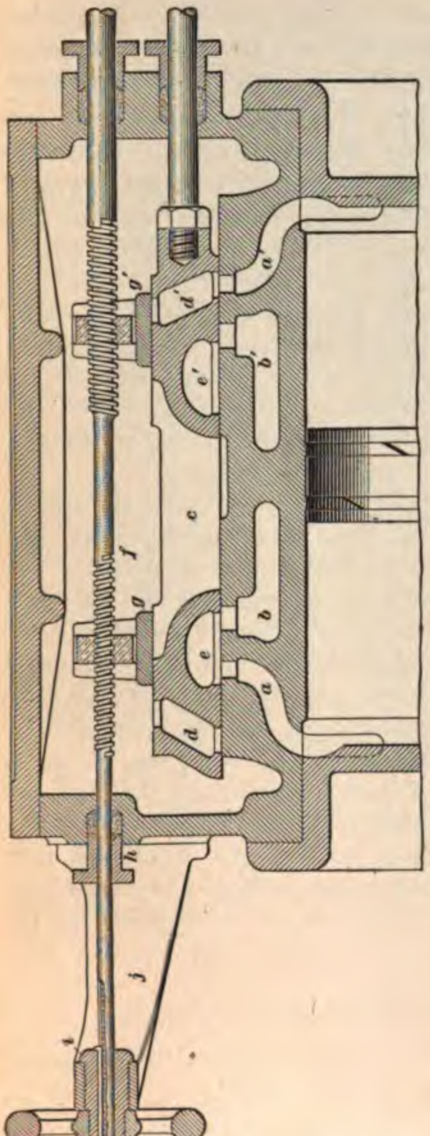


Fig. 9

valve. By this means, the cut-off valve may be moved across its seat on the main valve; this will have the same effect on the distance between the edges of the cut-off valve and the edges of the ports through the main valve as the changes made by the right- and left-hand screws on the distance between the plates of the Meyer valve.

Another somewhat similar modification is made by giving the cut-off valve and its seat a cylindrical form; in this case, the ends of the cut-off valve and the openings through the main valve are helical, their shape being the same as though the valve and seat of Fig. 8 were wrapped around a cylinder. By rotating the cut-off valve around its center by means of the valve stem, the same effect is produced as by sliding the cut-off

valve in Fig. 8 across its seat. The turning motion of the valve stem in the two types of cut-off valves here described is often accomplished by a governor, thus producing an engine with an automatic cut-off.

12. An improved Meyer cut-off valve that is used very largely in connection with air compressors is shown in Fig. 9. The steam ports of the cylinder are shown at a, a' and the exhaust ports at b, b'; there are two exhaust ports in this case, instead of one, in order to bring them nearer the ends of the cylinder and shorten the steam ports. The main valve c has two steam passages d, d' and two exhaust passages e, e'; it will be noticed that the exhaust passage e of the valve is long enough to cover both the steam port a and exhaust port b when the left-hand end of the cylinder is exhausting.

On the cut-off valve stem f are right- and left-hand square threads turning in nuts to move the cut-off valves g, g' toward each other or apart as the cut-off is regulated. The valve stem f extends through the stuffingbox h and a sleeve i held in the bracket j. The sleeve i has a hand wheel fixed on its outer end; the sleeve is free to rotate in the bracket and is splined to the valve stem, so that the stem must turn with it and the hand wheel, but is free to move longitudinally through them. By turning the hand wheel, the cut-off is easily changed while the engine is in motion. In the position shown, the left-hand end of the cylinder is exhausting and the steam has been cut off from the right-hand end. One objection to the Meyer valve is that it is unbalanced, and hence has excessive friction, as in the case of the **D** slide valve.

AUTOMATIC HIGH-SPEED ENGINES

13. Principal Features of Automatic Engines. Automatic high-speed engines belong to a class of engines brought into use by the demands of electric lighting. They are called automatic because the point of cut-off is automatically changed by a shaft governor, which acts

directly on the eccentric and affects the cut-off by changing either the throw of the eccentric or the angle of advance, or sometimes both. These engines usually run at 200 or more revolutions per minute. The governors must be sensitive and respond readily to great changes of load, so that the total variation of speed shall not be more than about 2 per cent. This means exceptionally close regulation, for which flyball governors are inadequate.

So far as the construction of the frame, cylinder, and reciprocating parts are concerned, the automatic high-speed engine generally resembles the plain slide-valve engine illustrated in Fig. 1. The great difference lies in the valve construction and the method of governing.

THE STRAIGHT-LINE ENGINE

14. There is, however, one type of automatic high-speed

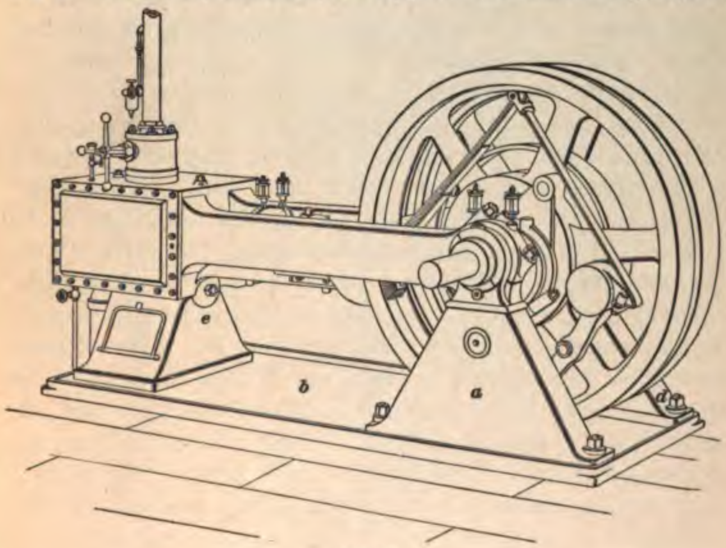


FIG. 10

engine that differs quite radically in its general construction from other high-speed engines; namely, the **straight-line engine.**

This engine was designed with two main ideas in view; one being that any structure having considerable length and breadth, in proportion to depth, must be supported on three points to avoid twisting when on an unstable foundation, or when subject to expansion or contraction; and the other that all stresses act in straight lines.

Fig. 10 shows this engine set on its foundation. On the bedplate *b* are three supports *a*, *d*, and *e* on which rests the engine proper and which are so constructed that the engine will not be thrown out of alinement if the foundation should settle unevenly.

The horizontal section, Fig. 11, illustrates much of the mechanism of the engine. Instead of one large flywheel, two smaller ones *g*, *g'* are used. They are fastened together by a large crankpin *p*; the rims are rectangular in section and hollow. The following are the principal parts: *r* is the connecting-rod, *s* the shaft, *h* the crosshead, *k* the piston rod, *l* the piston, *m* the stuffingbox, *c* the cylinder, *x* the valve chest, *z* the balance plate, *v* the valve, *t* the valve seat, *y* the valve stem, *e* the eccentric, and *w* the eccentric rod. The crosshead pin *f* is hollow, and is fast to the connecting-rod *r*, and turns in babbitt-lined boxes, which may be adjusted by means of the wedges *a* and *b* and the bolt *d*.

Fig. 12 shows, in section, a valve of the form used in this engine. It is of the pressure-plate type. The valve *v* works between the pressure plate *a* and the seat *b*. The plate *a* relieves the pressure on the back of the valve and thus prevents excessive friction, which would not only consume power but would also prevent the proper working of the governor. This arrangement also permits the use of a larger valve with a greater travel and a greater port opening. Distance pieces at the sides prevent the pressure plate *a* from resting on the valve. There are two openings through the valve for steam and two for exhaust. The manner in which the steam passages *c*, *c* through the valve double the port openings is shown by the arrows. Live steam enters at *c* and *d*, and the exhaust from the other end of the cylinder passes beneath the valve at *e* and through

15

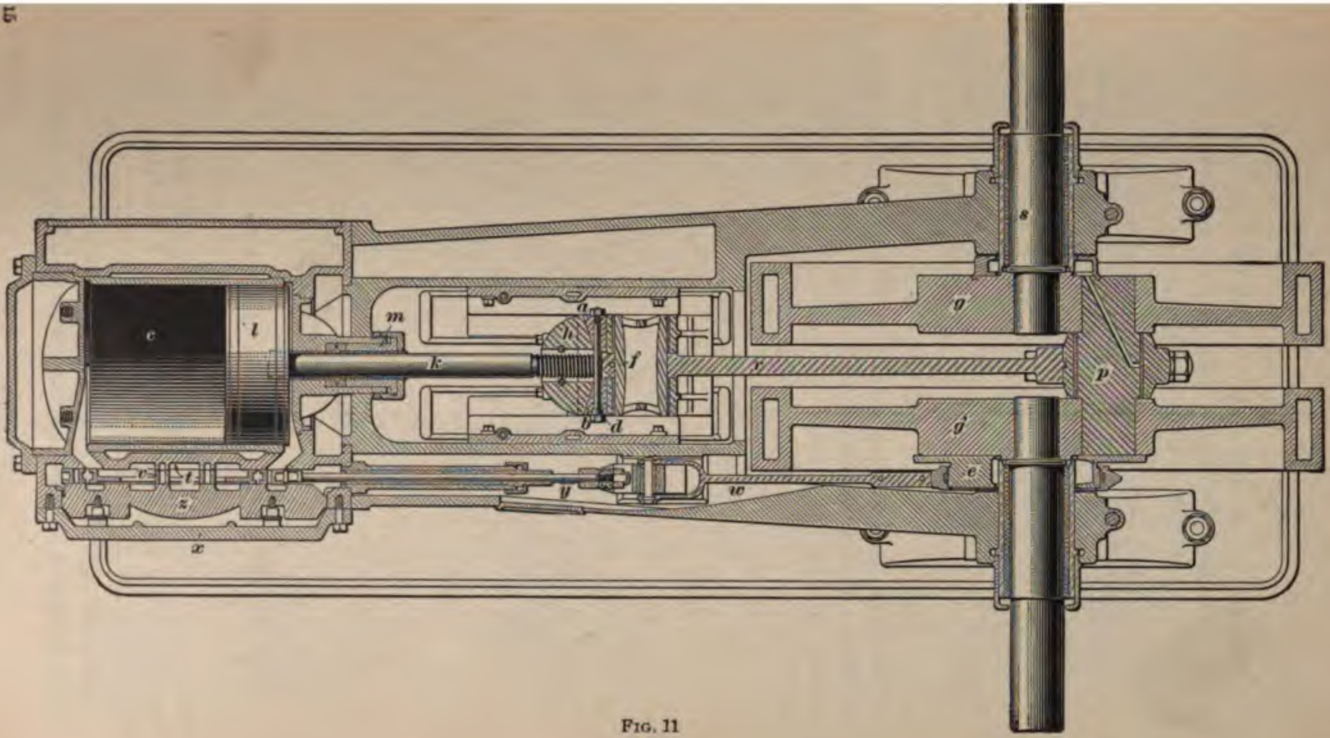


Fig. 11

the passage *f* into the exhaust cavity *h*. To prevent the cutting action of the wet steam on the valve and its seat the ledges *g*, *g* are cast on the valve.

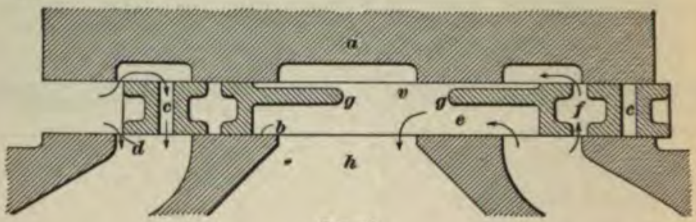


FIG. 12

15. The governor and valve motion of the straight-line engine are illustrated in Fig. 13. The governor consists of

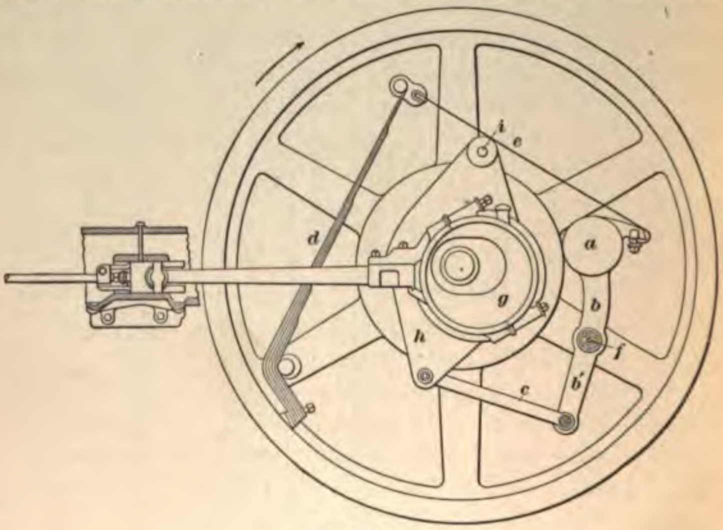


FIG. 13

a weight *a*, on a lever arm *b b'* connected to the eccentric by a link *c*, and to a spring *d* by a metal strap *e*. The location of the ball is so chosen that it is a counterbalance to the eccentric and its attachments. The eccentric *g* is carried by the plate *h* pivoted at *i*.

When the speed reaches a point where the centrifugal force of the weight *a* is greater than the resistance of the spring *d*,

it moves outwards from the center of the shaft, causing the arm *b b′* to turn on the pin *f*, and the link *c* to move inwards, shifting the eccentric *g* to the left. This change of position of the eccentric decreases its throw and the travel of the valve, and controls the amount of steam admitted to the engine cylinder. When a sudden load is thrown on the engine, the speed begins to decrease. The centrifugal force is lessened and the spring, by means of the metal strap, pulls the ball nearer to the shaft. This shifts the eccentric to the right by means of the link, as shown in the figure, thus decreasing the angular advance, and cut-off takes place later in the stroke. As the steam at initial pressure is admitted up to a later point in the stroke, the mean effective pressure and the work done are increased and the engine speeds up to its correct number of revolutions per minute. Since this governor depends for its action on the centrifugal force of the weight, it is known as a *centrifugal governor*.

INDICATOR DIAGRAMS OF AUTOMATIC ENGINES

16. The automatic engine is liable to be subjected to sudden changes of load. Fig. 14 shows a series of diagrams

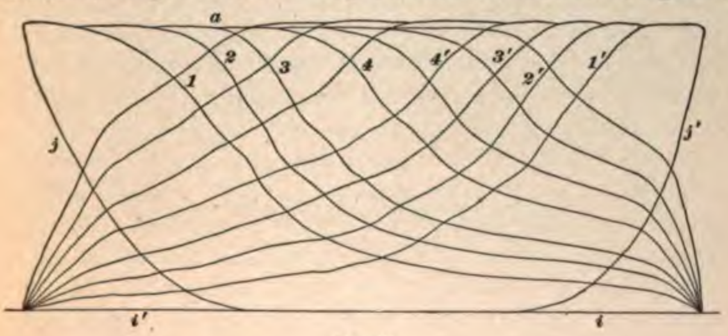


FIG. 14

from an automatic engine, taken at a time when the load was changing. The effects of the variable load and the resulting changes in the cut-off are shown by the successive expansion curves *1*, *2*, *3*, etc., and *1′*, *2′*, *3′*, etc. The admission lines *a* are practically horizontal for all loads. Taken in the order

1, *2*, *3*, the expansion lines show increasing loads on the engine; if taken in the reverse order, they show decreasing loads. The back-pressure lines i, i' from opposite ends of the cylinder coincide for part of their length. The back-pressure line i and the compression curve j are the same at all loads. The same is true of i' and j'.

VALVES FOR AUTOMATIC ENGINES

17. It is well known that the action of a shaft governor is materially affected by the force required to drive the valve; a plain **D** slide valve, requiring as it does a great driving force, not only absorbs a considerable proportion of the power of the engine, but has a disturbing effect on the action of the governor that, especially with large engines, becomes so serious as to make the use of this type of valve with shaft governors impracticable. To relieve the governor as much as possible from the effects of the frictional resistances incident to the **D** slide valve, a great number of balanced valves, of which the piston valve is one type, have been invented.

A **balanced valve** is one that is acted on in opposite directions by equal steam pressures. The frictional resistance is thus lessened and the wear is greatly decreased.

When used in conjunction with a shaft governor that varies the throw of the eccentric, the plain slide valve has another defect that is not overcome by the mere process of balancing. This defect is that the reduced travel of the valve consequent on the short throw of the eccentric at early cut-offs has the effect of greatly restricting the port opening. At high speeds, such a reduction in port opening causes an imperfect filling of the cylinder with live steam; that is, the steam is wiredrawn in its passage through the partly opened ports. This difficulty can be largely overcome by increasing the travel of the valve so as to give it a considerable amount of **overtravel** for late cut-offs. By overtravel is meant that the valve is given a travel greater than is required to give a full opening of the steam ports. Such a remedy, however, involves other undesirable conditions. To provide for the

overtravel, the central chamber of the valve and the bridges between the steam and exhaust ports must be made wider, thus making a larger and heavier valve. The attempt to secure a liberal port opening with early cut-offs by giving the valve overtravel thus involves an increase in the travel of the valve and in its weight, both of which add to the power required to move it.

Several designs for shaft-governor engines secure an increased port opening by the use of multiple-ported valves, in which steam enters the cylinder through two or more passages. While the travel of the valve may be reduced by this means, its size and weight are generally increased, and this neutralizes the advantage of short travel to a considerable degree.

18. Piston Valve.—In Fig. 15 is shown a **piston valve,** which consists of a hollow cylinder sliding in a cylindrical valve seat. The ports p, p extend clear around the valve. The steam is admitted into the central chamber a,

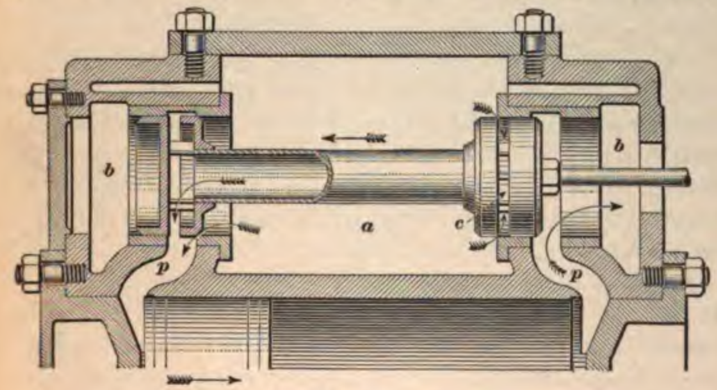


FIG. 15

and the exhaust steam escapes at the two ends b, b. As shown in the figure, the piston is just about to start toward the right and the valve is moving to the left, thereby uncovering the left steam port and allowing the steam to enter past its inside edge. To give a larger admission, steam also passes into the center of the valve through the channel c

and thence into the left port. The exhaust steam meanwhile escapes directly through the right steam port into the chamber *b*.

The piston valve is one of the lightest and most perfectly balanced types of valve that has yet been devised, and it is readily made double-ported. It has, however, several features that have restricted its use. Unless it is provided with some form of packing there is no means of adjustment to provide for wear; consequently, it soon leaks and wastes steam. In vertical engines, where the weight of the valve is not supported by the seat, the wear is not so great, and there are many cases of such engines in which piston valves with no packing except grooves in which water collects have given the best of service. In a horizontal engine, however, the weight of the valve invariably wears the lower part of the seat and destroys its circular form. This type of valve as generally constructed offers no relief to water that may be caught in the compression space when the exhaust port closes.

19. Pressure-Plate Valve.—A type of valve that has many of the advantages of the piston valve and at the same time overcomes some of its faults is the **pressure-plate valve.** Fig. 16 shows a form of this valve that, with several minor modifications, is used in a large number of high-speed engines. The valve *a* consists of a thin rectangular casting with openings or ports through it for the passage of live and exhaust steam to and from the cylinder. It works between the face of the valve seat and a similar face formed by a heavy pressure plate *b*. The pressure plate is separated from the valve seat by strips *c*, *c'* that are made just enough thicker than the valve to permit the valve to slide freely between the pressure plate and the valve seat. When the engine is working, the pressure plate is held in place against the strips *c*, *c'* by the steam pressure; when steam is shut off, the spring *d* prevents the plate from falling away from its position against the strips. This construction permits the valve and pressure plate to act as a relief valve for

the escape of water from the cylinder; an excessive pressure in the cylinder will lift the valve and plate and compress the spring. The danger of breakage from water in the cylinder is thus somewhat reduced. It is not, however, a perfect protection; with high speeds, the pressure often becomes great enough to seriously damage the engine before the water can be forced out through the ports and overcome the inertia of the valve and pressure plate.

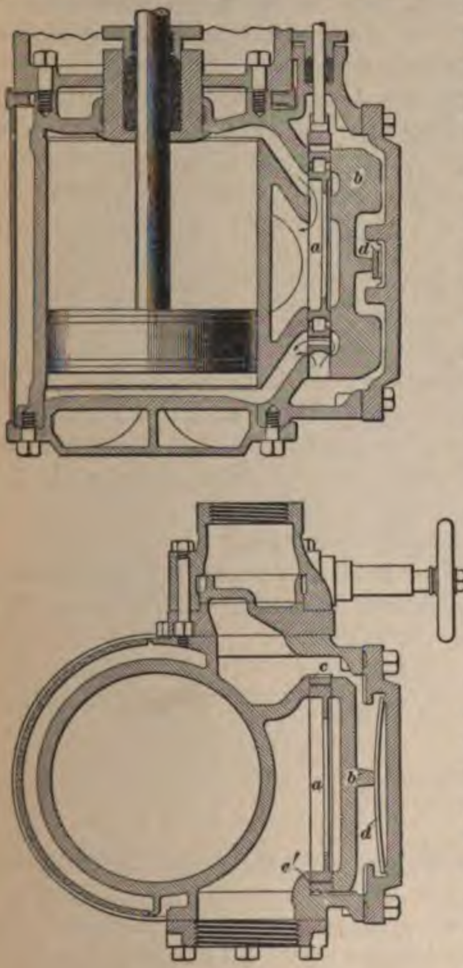


FIG. 16

20. An advantage of the pressure-plate valve over the piston valve is that in horizontal engines the wear due to the weight of the valve, coming as it does on the lower edge and the strip *c'*, does not affect the tightness of the valve. Another advantage is that when the valve wears so as to allow steam to leak between it and the faces of the valve seat and pressure plate, the strips *c*, *c'* can be planed or scraped so as to overcome this leakage; this operation, however, requires considerable care and skill. If the valve or

the faces between which it works are unevenly worn, it will be necessary to scrape them to a new surface.

Besides decreased wear, another advantage is that the governor parts, eccentrics, etc. can be made much lighter. A pressure plate should not be used unless some provision has been made for allowing the valve to be raised from its seat in case water should collect in the cylinder owing to the condensation of steam. In the case of Fig. 16 this is provided by the spring *d*, which will allow the pressure plate and valve to be raised together should water collect in the cylinder.

SHAFT GOVERNORS

21. The Buckeye Governor.—One of the first shaft governors to be introduced in the United States is the centrifugal governor, used on the Buckeye engine. The engine is provided with two valves and two eccentrics. One of the latter is keyed to the shaft and drives the main valve, controlling the admission, release, and exhaust closure. The other eccentric, which is movable, is attached to the cut-off valve, which rides on the main valve.

In Fig. 17 is shown a diagram of a Buckeye valve motion. About the center of the shaft *o* rotates the crank *o a*, the

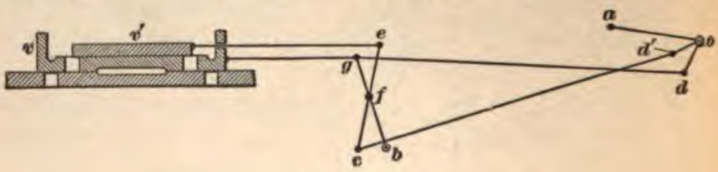


FIG. 17

fixed eccentric *o d*, and the eccentric *o d'*, which turns about the shaft under the action of the governor. From the fixed eccentric *o d* extends the eccentric rod *d g* to the end of a rocker-arm *g b*, which is pivoted at *b*. From *g*, a valve stem extends to the main valve *v*. From the eccentric *o d'*, which is under the control of the governor, extends the eccentric rod *d' c*, connected to the cut-off valve *v'* through the rocker-arm *e c*, pivoted at its center *f* on *g b*. The motion of the

cut-off valve is reversed because its valve stem and eccentric rod are connected to the opposite ends of the rocker-arm *ec*.

The governor is shown in Fig. 18, and its direction of rotation is shown by the arrow. The weights a, a' of the governor are bolted to arms b, b', which swing on pivots c, c'. The free ends of the weights are connected through links d, d'

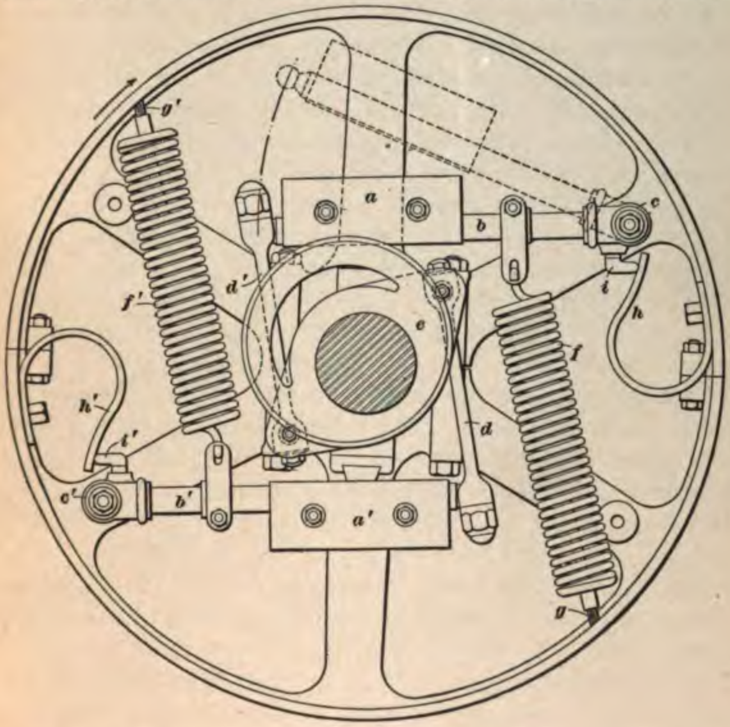


Fig. 18

to ears on the eccentric e. The dotted lines show the position of the weight a when the eccentric has been rotated around the shaft one-fourth of a revolution in the direction of rotation of the wheel. The outward motion of the weights, which is caused by centrifugal force, is resisted by the springs f, f'. The tension in these springs is adjusted by means of the screws g, g'. The springs h, h' assist the springs f, f' in producing suitable regulation. The forces

that they exert tend to throw the arms outwards during the first half of the total movement, after which they cease to be in contact with the fingers *i*, *i'*, against which they have been pressing.

When steam is admitted to the engine cylinder, the rotation of the flywheel causes the weights to leave the stops, after a certain speed has been reached, and to move outwards under the effects of centrifugal force, putting additional tension in the springs. The eccentric, as it turns forwards on the shaft, under the action of the force exerted by the weights, increases the angular advance. When the proper point of cut-off for the given load has been reached, the speed remains constant. Should the load become less, the revolutions tend to increase, the centrifugal force becomes greater, and the arms move outwards, causing an increased angular advance and an earlier cut-off. If the load increases, the reverse action occurs and the normal speed is restored.

The two extreme positions of the governing mechanism are shown in the figure. The solid lines represent the position of the parts when the engine is at rest, and the dotted lines represent them when cut-off is earliest and the number of expansions greatest.

22. The Fitchburg Governor.—Fig. 19 shows a Fitchburg governor, which operates by shifting the eccentric across the shaft without rotating it relative to the shaft. The eccentric *a* has an elliptical opening through which the shaft passes, and this opening is large enough to allow the eccentric center to be shifted from *b* to *c* and returned. The eccentric is not fastened directly to the flywheel, but is held in place at the points *d*, *d'* by the governor parts. The arms *e*, *e'* pivoted at *f*, *f'* and carrying the weights *g*, *g'*, which balance the weight of the eccentric, are fastened to the eccentric at *d*, *d'*. The large weights *h*, *h'*, pivoted at *i*, *i'*, are attached to the arms *e*, *e'* by the links *j*, *j'* at the points *k*, *k'*. The spiral springs *m*, *m'* are attached to the ends of the large weights, and by their tension hold the

weights away from the flywheel rim. Because the links j, j' are attached to the arms e, e' on opposite sides of the fulcrums f, f' relative to the weights, the outward motion of each weight has an equal influence in moving the eccentric in one direction across the shaft.

The action of this governor is as follows: So long as the engine runs below the normal speed, the eccentric is kept in

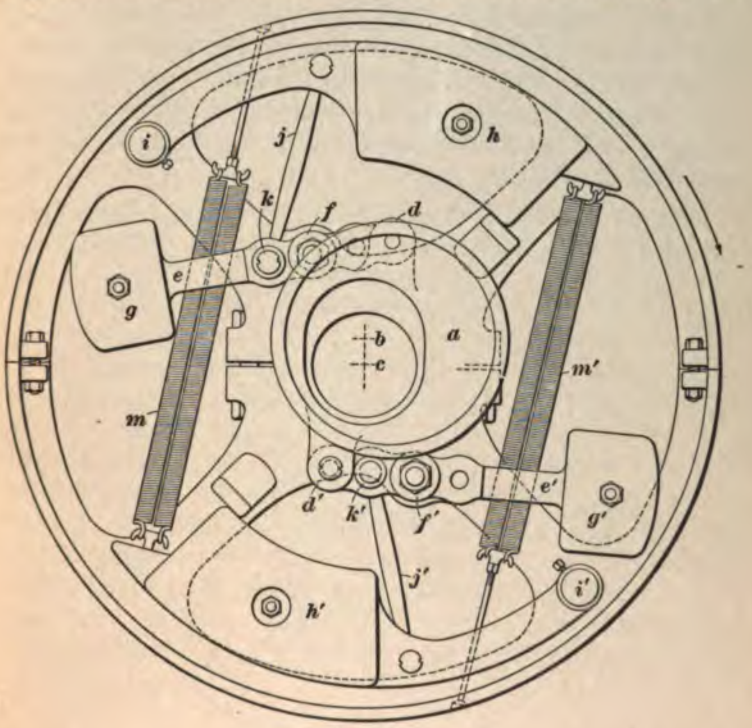


FIG. 19

the position of its greatest throw by the tension of the springs, and steam is cut off at about three-fourths stroke; but as soon as the proper speed is reached, centrifugal action causes the weights h, h' to overcome the tension of the springs and to move outwards; this outward motion of the weights h, h' turns the weights g, g' on their fulcrums f, f', and the eccentric is carried across the shaft from b toward c,

and as the arcs described by the pins d, d' are equal and curve in opposite directions, they compensate each other, and the center b of the eccentric follows a straight line in its movement, preserving a constant lead. This manifestly decreases the eccentricity and increases the angle of advance, giving an earlier cut-off to the valve, until, when the eccentric is swung squarely back of the crank, the valve opens only the amount of the lead. On the least diminution of speed, the springs have more power than the centrifugal force of the weights, and the motion of the parts is arrested and turned in the opposite direction, giving a later cut-off as more work is demanded of the engine. This is called a **shifting eccentric.**

23. The Rites Governor.—The **Rites governor** belongs to a class known as *inertia governors*, because the

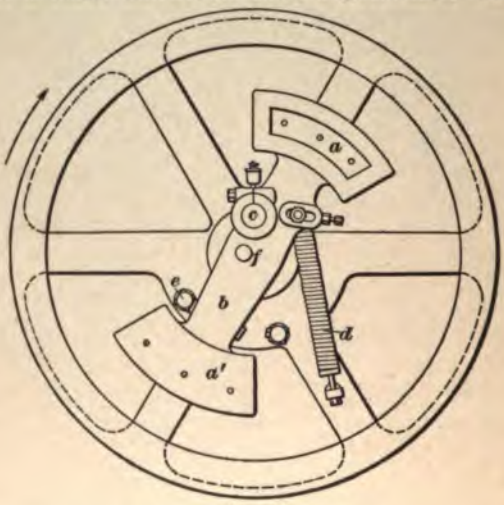


FIG. 20

inertia of the weights aids the centrifugal force in producing good regulation. This governor, shown in Fig. 20, consists of two weights a, a' connected by a bar b that is pivoted on the pin c. A spring d tends to hold the weights and arm in the starting position, against the stop e. The pin f, which is fixed on the arm b, takes the place of an eccentric, the

eccentric rod being connected to it. As the speed of the engine rises, the centrifugal force of the weights increases until it overcomes the initial tension of the spring *d*, and swings the bar *b* about its pivot *c*, moving the pin *f* until the point of cut-off is suited to the load.

If now there should be a sudden change of load, it would require some time for the weights to find their new positions, but owing to the tendency of the weights to continue at the same speed at which the wheel traveled before the change of load, they will lag behind when the speed suddenly increases, or run ahead when the speed decreases. This action, which takes place immediately, is utilized to change the position of the eccentric pin with reference to the wheel, so as to check the change of speed while the centrifugal force is moving the weights to meet the requirements of the new load. By this arrangement a closer regulation can be obtained than with a governor in which centrifugal force alone is employed.

This governor differs from the two just described in that it controls the valve by swinging the eccentric about the pivot *c*. When the eccentric center travels toward the shaft, the angular advance increases, the throw decreases, and the lead changes. In the straight-line and Buckeye governors, the speed is controlled by centrifugal force. In the Fitchburg governor, since the weights are heavy, inertia assists somewhat in the regulation. In the Rites governor, the inertia of the heavy arm and weights supplements the centrifugal force very materially and greatly increases the sensitiveness.

CORLISS ENGINES

24. The **Corliss engine** was brought out about the year 1850 by its inventor, George H. Corliss, and is one of the most radical improvements in engine construction since the time of James Watt. It has two steam and two exhaust valves, set at right angles to the center line of the cylinder. These are caused to oscillate by a wristplate and connecting mechanism, which in turn is made to oscillate by an eccentric

on the main shaft. The ports are very short and of small volume, so that the clearance is much reduced, and as the steam does not enter through passages previously cooled by the exhaust and the surfaces of the ports are small, the cylinder condensation is materially lessened.

The valve moves rapidly when opening or closing the ports, and very slowly between these two events. By remaining wide open during admission, the valve allows the steam to enter the cylinder without being wiredrawn, and the temperature of the working steam in the cylinder is approximately the same as in the valve chest. The steam line, as shown by the indicator diagram, is nearly horizontal, and the point of cut-off is well defined. The exhaust valves in horizontal engines are so placed that the water of condensation will drain to them and be carried out by the exhaust steam. All the valves are capable of independent adjustment, and work with very little friction.

25. Types of Corliss Engines.—Corliss engines may be classified in a number of ways. There are *horizontal* and *vertical*, *single-eccentric* and *double-eccentric*, and *box frame* and *girder frame* Corliss engines. Besides these divisions there is another, based on the direction of motion of the steam valves; they may move *inwards*, that is, toward each other, in opening; or *outwards*, that is, away from each other, in opening.

The single-eccentric Corliss engine uses but one eccentric and all four of the valves are moved by this eccentric. In the double-eccentric Corliss engine, the steam valves are operated by one eccentric and the exhaust valves by another. The terms *box frame* and *girder frame* refer to the resemblance of the frames to a box or to a girder. These distinctions all refer to certain subordinate features, however, and do not indicate separate or distinct classes of engines.

26. The Single-Eccentric Corliss Engine.—Fig. 21 shows the general appearance of a common type of horizontal Corliss engine. This is a single-eccentric engine with outward-moving valve motion. The steam valves a, a'

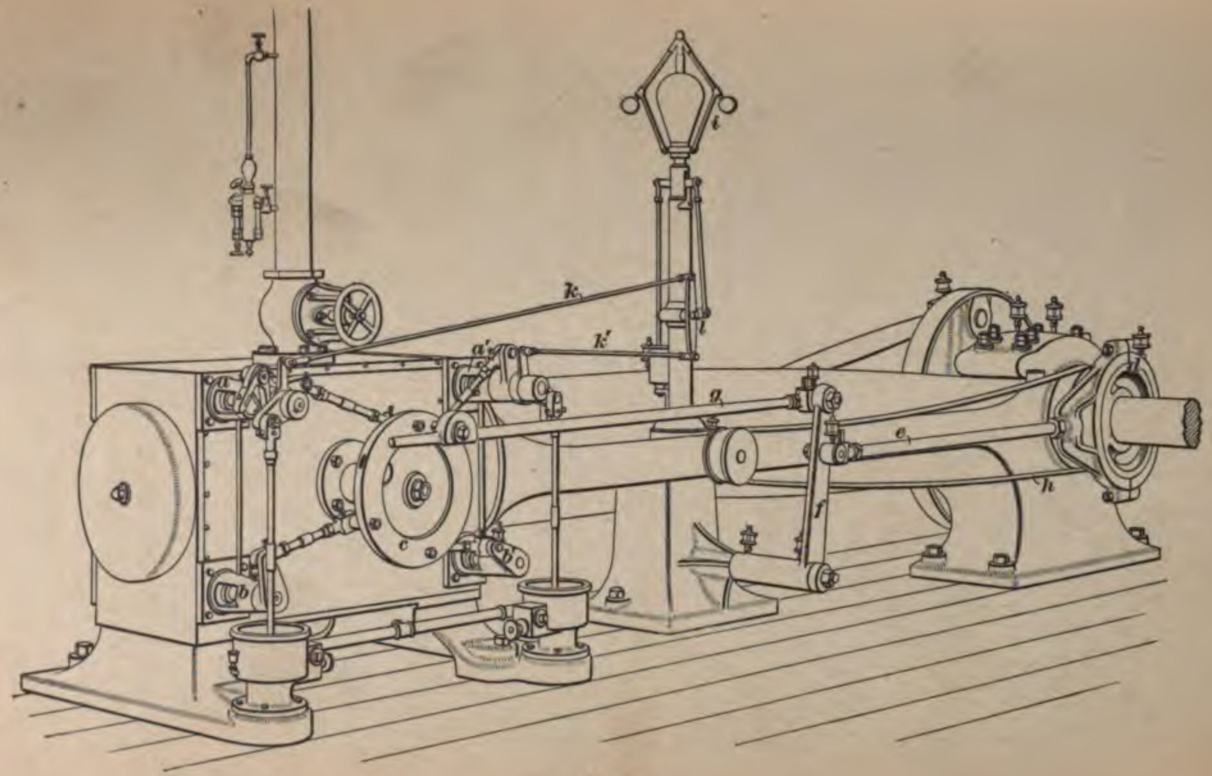


FIG. 21

and the exhaust valves *b*, *b'* are all connected to the wristplate *c*, which derives its motion from the eccentric *d* through the eccentric rod *e*, rocker-arm *f*, and hook rod *g*. At *h* is shown the belt that operates the governor *i*, deriving its motion directly from the shaft of the engine. In the Corliss engine, the governor changes the point of cut-off of the steam through the reach rods *k*, *k'*, which it moves by means of a rod connected to the bell-crank *l*.

27. Double-Eccentric Corliss Engine.—Fig. 22 shows a box-frame type of double-eccentric Corliss engine with inward-moving valves. The eccentric *a*, through its rod *b* and hook rod *c*, operates the back wristplate *d* to which the steam-valve rods *e*, *e'* are attached. These valve rods move the steam valves in the same manner as in the single-eccentric engine.

The eccentric shown at *f* has a different angle of advance from that of eccentric *a*, and may have a different throw. It is attached, through its eccentric rod *g* and hook rod *h*, to the front wristplate *i*, which moves the exhaust valve rods *j*, *j'*, thus operating the exhaust valves. The latches *k*, *k'*, when raised, disconnect the hook rods *c* and *h* from the wristplate pins, permitting the rods to move without imparting motion to the wristplates. Starting bars may be inserted in the sockets *l*, *l*, and the wristplates turned by hand, to admit steam to the cylinder for the purpose of warming it before starting the engine. The principle of operation, other than that involved in having separate eccentrics for the steam and exhaust valves, is the same as in the single-eccentric engine. The advantage of using two eccentrics is that the steam valves and exhaust valves may be set independently of each other.

28. Corliss Cylinder and Valves.—A section through the cylinder and valves of a Corliss engine is shown in Fig. 23. The arrows indicate the path of the steam as it flows from the steam pipe into the valve chest *a*, through the upper left-hand steam port *b*, and into the cylinder *c*. From the crank end, the exhaust passes out through the lower right-hand exhaust valve *d*, and thence to the exhaust connection at the

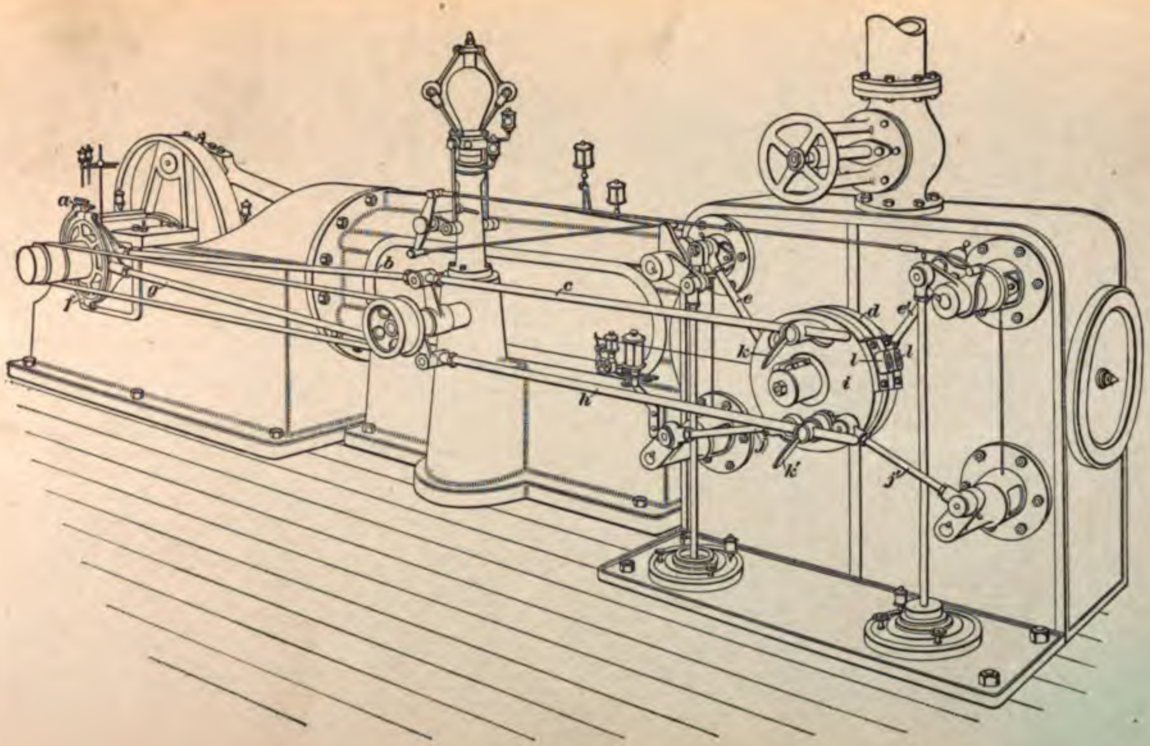


FIG. 22

bottom. This cylinder is cast in one piece, bored and

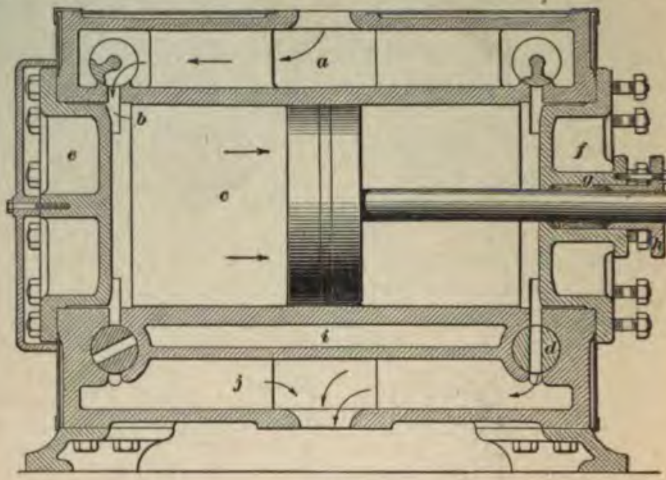


FIG. 23

counterbored, and the two heads *e* and *f* are inserted. The head *f* for the crank end contains the stuffingbox *g* and the gland *h* for the piston rod. The space *i* between the cylinder *c* and the exhaust chamber *j* contains dead air, or a non-conducting substance if the engine be unjacketed, to reduce the radiation of heat from the cylinder walls to the exhaust steam.

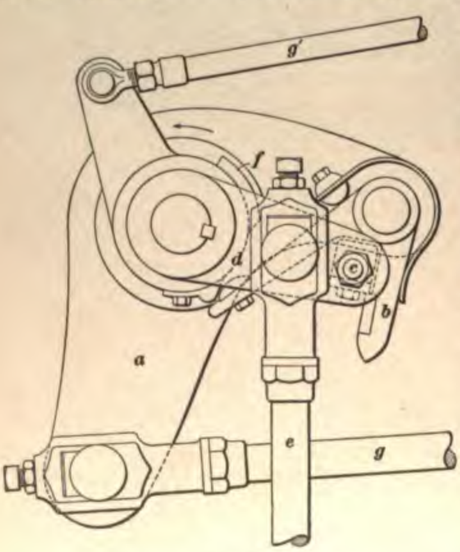


FIG. 24

29. Corliss Releasing Gears. There are a number of forms of Corliss releasing gears

having the same general principle, and differing only in details. They may all be included, however, according to the direction of rotation of the valves, in the two classes already mentioned. One class of releasing gear that rotates the valves inwardly is shown in Fig. 24. In admitting the steam, the valve rod *g* moves to the right and pulls with it the lifting arm *a*, causing the hook *b*, which has caught the stud *c* of the valve arm *d*, to lift the dashpot rod *e* and turn the valve in the direction shown by the arrow. At the moment of cut-off, the knock-off cam *f* causes the latch *b* to move outwards and release *c*, thus permitting the dashpot to close the valve by pulling down the valve arm *d*. The position of

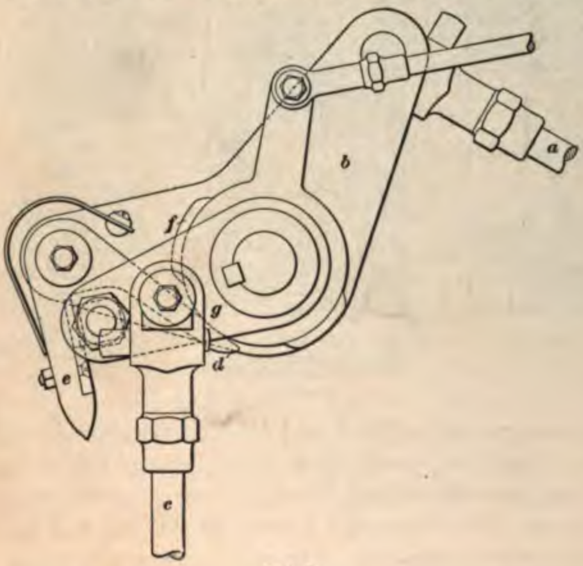


Fig. 25

the knock-off cam is controlled by the governor through the reach rod g'.

In the second type of releasing gear, shown in Fig. 25, the valves turn outwards to admit steam. The valve rod *a*, by pulling the lifting arm *b* to the right, opens the steam valve and lifts the dashpot rod *c*. When the valve has been open the correct length of time, the arm *d* of the hook *e*

strikes the cam *f*, releasing the hook, and the dashpot rod pulls the valve arm *g* around and closes the valve. It should be noticed that as the valve moves outwards in opening, the steam travels directly from the steam chest into the cylinder without passing over the valve, thus making a direct steam passage. When the valve moves in the opposite direction, the steam must pass over the valve in order to enter the steam port.

30. Corliss Valves.—The cross-sections of several forms of Corliss valves are illustrated in Fig. 26, *a*, *c*, and *e*

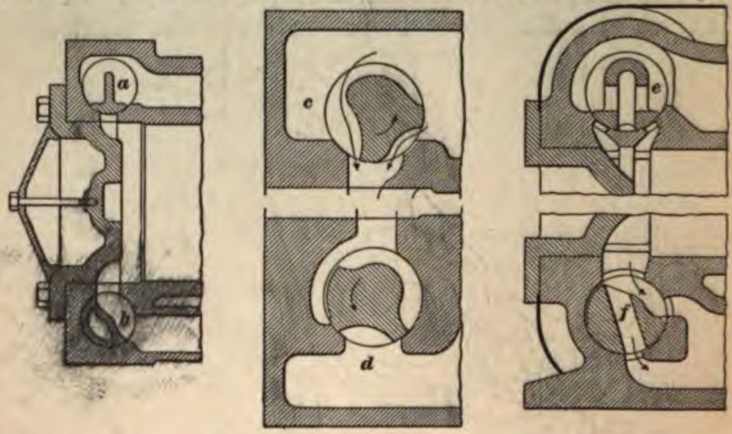


FIG. 26

being steam valves and *b*, *d*, and *f* the corresponding exhaust valves. The latter, it will be noticed, are so shaped that the steam pressure within the cylinder forces them to their seats, and thus prevents the steam from lifting them and escaping before the exhaust period. The valves shown at *a* and *b* are said to be single-ported because the steam passes through but one opening, while *c*, *d*, *e*, and *f* are double-ported. The steam valves are so made and set that when in the mid-position there is sufficient lap to prevent leakage. Since the eccentric must cause the valve to move a distance equal to the lap to clear the edge of the port and admit the steam, it is evident that the range of cut-off is restricted by adding lap. To obviate this difficulty, when it is advisable to do so,

separate eccentrics are used for the steam and exhaust valves, as shown in Fig. 22.

Fig. 27 (*a*) and (*b*) shows in perspective the simple valves shown at *a* and *b*, Fig. 26. The slots in the ends of these valves, shown at *a*, receive the T-shaped heads of the valve stems, which transmit the motion to the valves. Double-ported valves require only half the travel of single-ported valves in order to give the same amount of port opening. The movement of the complete valve gear and its eccentric is thus reduced one-half.

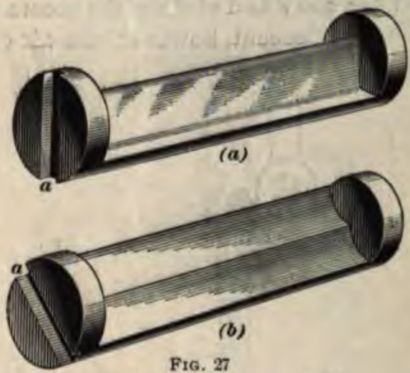


FIG. 27

31. Dashpots.—Dashpots are used for the purpose of quickly closing the steam valves after the knock-off cams have released the arms attached to the valve stems. They may be placed either in a vertical or in an inclined position, and they are made in a large variety of forms, although the principle is practically the same in all. One form of dashpot is shown in section in Fig. 28. It consists of a stationary base *a*, which is fixed either to the engine frame or to the floor, and a movable plunger *b*, which is connected by the rod *c* to the arm of the steam valve. The plunger *b* is accurately turned and bored so as to fit the base *a*, and packing rings are used to make air-tight joints. Fig. 28 (*a*) shows the position of the plunger when it is at rest, in its lowest position.

When the hook block is picked up by the disengaging hook, the rod *c* is lifted, drawing the plunger *b* upwards to the position indicated in Fig. 28 (*b*). An annular chamber *d* is thus formed beneath the lower end of the plunger, air being drawn into this annular chamber through the partly opened valve *e*. At the same time, a partial vacuum

is formed in the chamber *f* beneath the upper end of the plunger. Consequently, as soon as the knock-off cam releases the steam-valve arm, the atmospheric pressure on the top of the plunger *b* forces it down very rapidly, thus pulling down on the rod *c* and closing the steam valve quickly.

In its descent, however, the air caught in the chamber *d* is

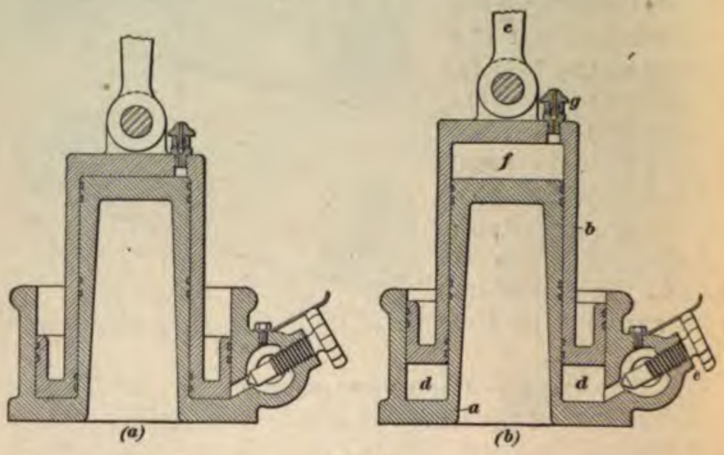


FIG. 28

compressed, so that it forms a cushion that brings the plunger to rest easily and prevents it from striking the bottom of the annular space. The amount of cushioning effect is regulated by the valve *e*. The valve *g* is a small relief valve opening outwards from the chamber *f*. It permits the escape of any air in the chamber *f* when the plunger falls.

CORLISS-ENGINE GOVERNORS

32. Types of Corliss-Engine Governors.—The governor of the Corliss engine is usually of the standard flyball type, driven from the engine shaft by a belt. It keeps the speed within assigned limits by moving the knock-off cams so as to change the point of cut-off, thus varying the amount of steam admitted to the cylinder at each stroke. There are several of these governors in use; the ordinary type, loaded

or without a weight; the inverted type, weighted or not weighted; and the spring-loaded type.

Fig. 29 shows the ordinary form of loaded governor. The arms *a*, *a* carry the balls *b*, *b*, and are pivoted to the top of the spindle *c*. The links *d*, *d* form the connections between the arms *a*, *a* and the loose collar *e* that supports the weight *f*. When the spindle rotates, it carries the arms, balls, links, and collar around with it; and when the centrifugal force of the revolving balls becomes great enough they fly outwards and raise the weight *f* and collar *e*.

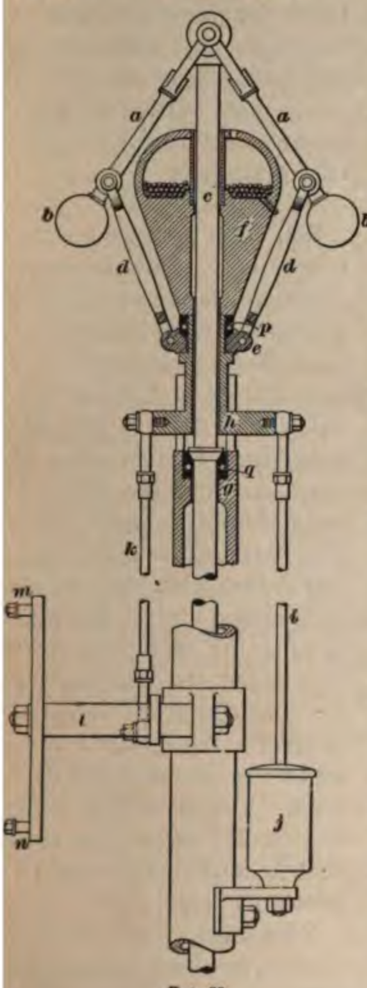


FIG. 29

The governor stand *g* rests on the engine frame, as shown in Figs. 21 and 22. The yoke sleeve *h* rests on a collar when the governor is in its lowest position, but it is raised with the collar *e* when the governor speed is great enough. The rod *i* is connected to the yoke at one end and to the piston of the dashpot *j* at the other end. The dashpot has a steadying influence and prevents sudden changes in the position of the governor. The rod *k*, attached to the rocker-arm *l*, causes the latter to move the reach rods that connect to the arm *l* at *m* and *n*, thus changing the positions of the knock-off cams and regulating the point of cut-off. By using ball bearings at *p* and *q*,

the power required to drive the governor is made comparatively small. When the speed of the engine increases, the governor revolves faster and the balls fly outwards. This raises the sleeve *h* and causes cut-off to take place earlier. When the engine slows down, the weights fall and the sleeve lowers, causing the cut-off to take place later.

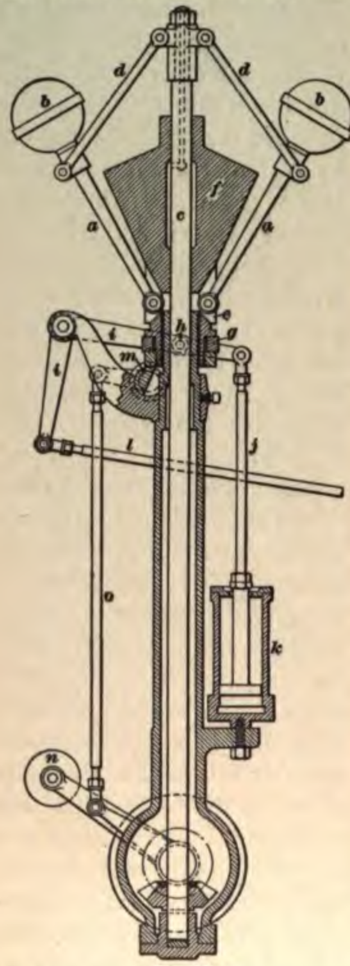


FIG. 30

33. The Inverted Governor.—The **inverted governor** is shown in Fig. 30. The similarity of the arms *a*, *a*, the balls *b*, *b*, the spindle *c*, the links *d*, *d*, the collar *e*, and the weight *f* to those in Fig. 29 is apparent. In this case, the arms *a*, *a* and the links *d*, *d* have simply changed places. The loose collar *g* moves up and down with the weight, and carries the pin *h* with it, causing the bell-crank *i*, *i* to turn about its fulcrum. On the long arm of the bell-crank is pivoted the dashpot rod *j*, which runs down to the dashpot *k*. The short arm of the bell-crank *i* carries the reach rod *l*, which moves the knock-off cams.

The safety stop *m* is a journal, partially cut away, as shown, so that if the governor belt should break the idler *n* will drop, pulling down the rod *o* and turning the safety stop. This would permit the weight and collar *g* to fall down and turn the bell-crank *i*, drawing the reach rod *l* back until the

knock-off cams are brought into positions that prevent the valves from opening and no steam is admitted to the cylinder.

34. The Spring Governor.—Fig. 31 shows a simple and powerful **spring governor** sometimes used on Corliss engines. An enclosed spring *a* takes the place of a weight and presses down on the inner arms of the two bell-cranks *b*, *b*. The balls *c*, *c* on the upper arms of these bell-cranks act in a manner similar to that of the flyballs on the other governors just described. The spindle, which is inside the standard, is made to revolve by a belt and gears, as shown in Fig. 30. The outward motion of the balls is resisted by the spring *a*. The reach rods that control the knock-off cams are connected to the lever *d*, which is moved by the link *e* connected to the collar *f*. By adjusting the spring, the engine may be made to run steadily at the desired speed. This governor usually runs at about 200 revolutions per minute and does not require a dashpot or other device for preventing unsteadiness.

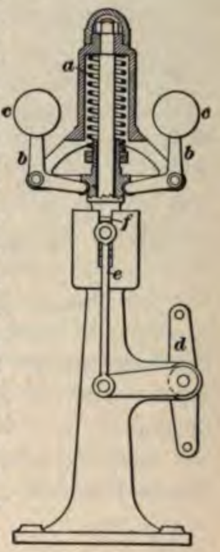


FIG. 31

REVERSING ENGINES

35. Many engines, as marine, railroad, traction, and rolling-mill engines, require valve gears that will not only enable them to be reversed, but will control the distribution of the steam in their cylinders as well. One of the first types of reversing gears consisted of a loose eccentric, which was compelled by a lug to revolve with the shaft. When rotation in the opposite direction was necessary, the engine was stopped and the eccentric shifted to another position, where it was under the control of another lug. The second position differed from the former position by 180° minus twice the angle of advance.

In another form, used on large marine engines, the eccentric was moved across the shaft, as shown in Fig. 32, the crank being shown at *a* and the eccentric center at *b*. The angle by which the eccentric is ahead of the crank is *aob*. Now, if the eccentric *e* be moved in the direction of the arrow, in a line perpendicular to the center line of the crank, until its center is at *b'*, the engine will turn in the opposite direction. The eccentric is driven by the plate *p*, which is keyed to the shaft, the construction being such that the eccentric may be shifted by loosening the nut *d* and tightening it in the new position.

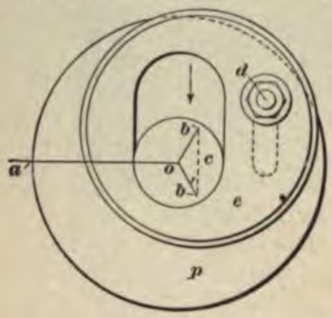


FIG. 32

It will be seen that the throw of the eccentric *ob* continually decreases as the angle between the crank and the eccentric approaches 180°; after the position *c* is passed, the angle decreases and the throw becomes greater. This device has been superseded by two other types of gears, namely, the *link motion* and *radial gears*.

LINK MOTIONS

36. The Link Motion.—A **link motion** consists of two eccentrics attached, usually, by rods to the ends of a slotted link, a block attached to the stem of a slide valve moving in the slot. One of the eccentrics is set to operate the valve when it is desired to run the engine forwards, and the other when it is desired to run it backwards. When the block in the end of the valve stem is held at one end of the link, the eccentric attached to that end operates the valve. Usually, some arrangement is provided for shifting either the link or the block in the link, so that the engine may be operated either way at pleasure.

There are three types of link motion: the *Stephenson*, having a shifting link concave toward the shaft; the *Gooch*,

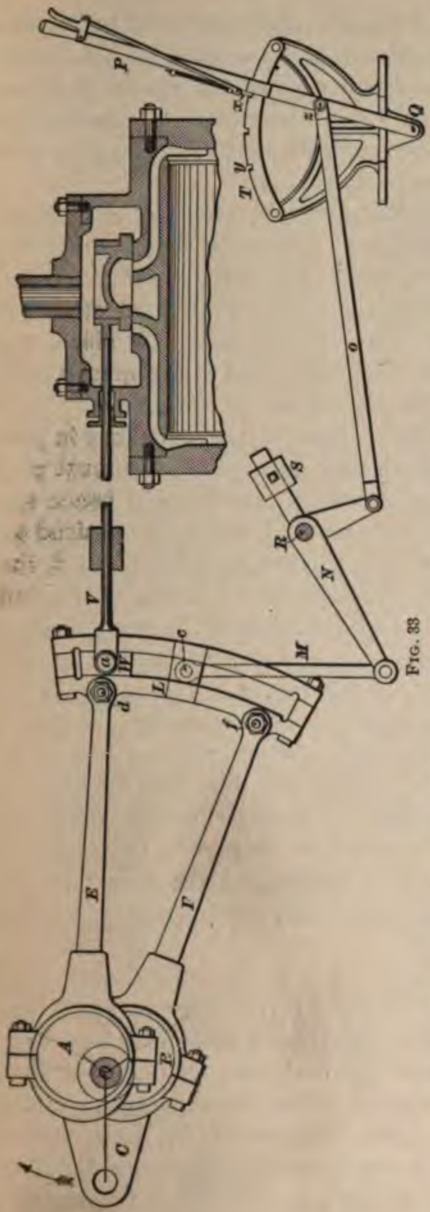


FIG. 33

having a stationary link, convex toward the shaft; the *Allen*, having a straight link.

37. The Stephenson Link Motion.—In many engines, it is necessary that the ratio of expansion be often varied and the direction of rotation changed. Many mechanisms have been devised to accomplish this result. The earliest, as well as one of the most efficient, is the **Stephenson link motion**, shown in Fig. 33.

Two eccentrics *A* and *B* are keyed to the shaft *O*. The two eccentric rods *E* and *F* are fastened to a link *L*. The valve stem *V* is bolted to a block *W*, which fits into *L*. *O A* and *O B* represent the lines from the center of the shaft to the centers of the eccentrics *A* and *B*, respectively, and *O C* the center line of the crank *C*.

When the mechanism is in the position shown in the figure, the

eccentric rod E is in line with the valve stem, and the action is precisely similar to the action of an engine with a single eccentric. The eccentric A will govern the steam distribution, and B will have no effect whatever. The angular advance is $A\,O\,C$ minus 90°; consequently, the engine will run in the direction shown by the arrow.

By means of the reversing lever P, and the links o, N, and M, pivoted at R, the link L can be raised until the eccentric rod F is in line with the valve stem V. The eccentric B will then impart motion to the valve; $B\,O\,C$ minus 90° will be the angular advance, and since the eccentric must be ahead of the crank, the latter must move in the direction opposite to that shown by the arrow.

If the link is raised until the point of suspension c is just opposite the point a of the valve stem, the valve must partake equally of the motion of both eccentrics, and hence the engine will run in neither direction. If the link is raised so that a occupies a position somewhere between c and d, the engine will still run in the direction shown in the figure, but the motion of the point a will be influenced in some degree by the motion of F, and the travel of the valve will be less than when in the position of the figure. The result is an earlier cut-off and compression. In locomotive engines, the sector T has several notches. When starting, the lever is thrown in the last notch, thus giving the valve the longest possible travel and the engine the greatest power. After the train is started and less power is required, the lever is thrown nearer the center, thus decreasing the travel of the valve and, in consequence, the power developed by the engine.

38. The Gooch Link Motion.—The arrangement shown in Fig. 34, known as the **Gooch link motion,** consists of a link a that oscillates back and forth and is supported by an inverted hanger b. The two eccentrics c and d are connected to the link at the points e and f by the eccentric rods. The travel of the valve rod g and the steam distribution are changed by the radius bar h attached to the link

block, which is moved upwards or downwards by means of the reverse lever rod *i*, the tumbling shaft *j*, and connecting link *k*. The dotted lines represent the full-gear positions,

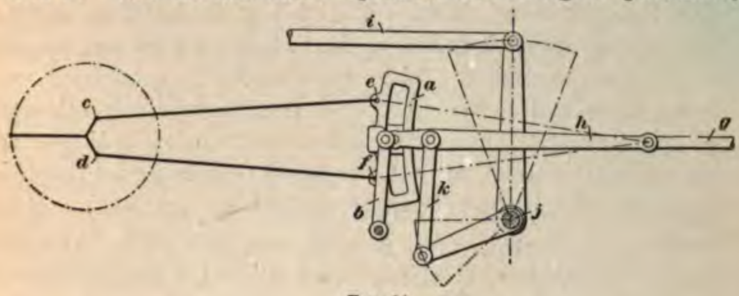


FIG. 34

forwards and backwards. This motion differs from the Stephenson link motion in that the block is movable up or down in the slot of the link, while the link itself is suspended from a fixed point and cannot be raised or lowered. It has the property of giving constant lead for any position of the block.

When, in any link motion, the link only is moved, the concave side must stand toward the shaft, as in the Stephenson motion; when the height of the link is fixed, and the link block is shifted, the convex side stands toward the shaft, as in the Gooch motion just described.

39. The Allen Link Motion.—In the **Allen link**

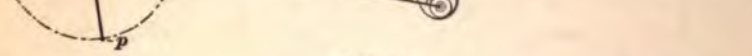


FIG. 35

motion, as shown in Fig. 35, the link *a* is straight, and the link and radius rod *b* are both shifted, one moving up and

the other moving down as the reverse lever rod *c* turns the tumbling shaft *d*. The rod *e* is attached to the lower end of the link, raising and lowering it as the rocker-arm *f* turns with the tumbling shaft. The rod *g* is fastened to the radius rod *b*, which it raises as the link is lowered and lowers as the link is raised. The centers of the eccentrics are shown at *m* and *n*, while the center of the shaft is at *o* and the crank at *op*. The action of this link motion is similar to those already shown, in that the valve is operated by either eccentric or by both, as may be desired.

40. Regulating the Steam Supply.—The original purpose of the link motion was to reverse the direction of rotation of the engine; it was soon discovered, however, that by its use the motion of the valve could be so regulated that the point of cut-off would be under the control of the operator. Hence, the work done in the cylinder could be varied to meet the demand on the engine. When the block is at either end of the link, the valve will have a range of motion that is determined by the throw of the eccentric connected to that end.

RADIAL VALVE GEARS

41. The Joy Valve Gear.—In the so-called **radial valve gears** no eccentric is used. The **Joy valve gear,**

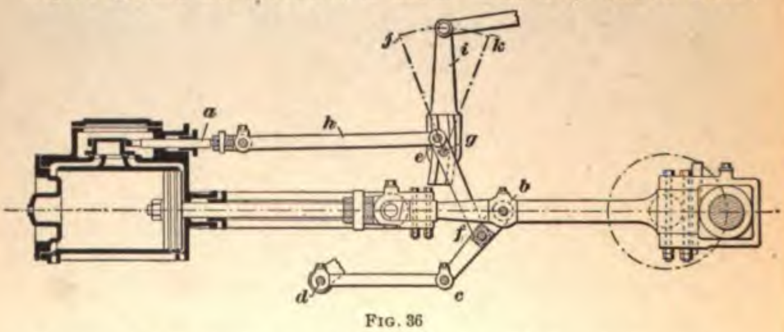


FIG. 36

shown in Fig. 36, belongs to this class; it is much used on marine engines and on European locomotives. The motion of the valve stem *a* is obtained from the point *b* on the

connecting-rod. Attached to this point is a vibrating lever *b c*, that is compelled to move back and forth in a plane by the link *c d* moving about the pin *d*. The lever *e f* oscillates back and forth on the pin of a block *g* as a fulcrum. This block, as the engine rotates, slides in a curved guide while it transmits motion to the valve stem *a* through the rod *h*. The direction of motion of the engine is changed by shifting the slotted guide about a stationary pin as a center, as indicated by the dotted radii *j* and *k*, the guide being moved to either side of the vertical by means of the lever *i*. When *i* is in the position *j*, the engine will run ahead; and when *i* is in the position *k*, it will rotate in the opposite direction. When the guide block is in its mid-position, as shown in the figure, no rotation will take place. Instead of the curved guide, a vibrating link is frequently used to guide the point *g* of the lever *e f*.

COMPOUND AND CONDENSING ENGINES

SIMPLE CONDENSING ENGINES

1. Condensation of Exhaust Steam.—If the exhaust from a steam engine is led into a cool drum or closed vessel, the steam will condense until the temperature and pressure in the drum are practically equal to that of the steam at release. When provision is made to keep the temperature low enough, so that the process is continuous, the vessel in which the steam is condensed is called a **condenser.**

When an engine is working with a condenser, provision must be made for the removal of the water condensed from the steam, and also for the removal of any air that may enter with the steam. This air and water are ordinarily removed by means of an air pump, which serves the double purpose of maintaining a low pressure in the condenser and of getting rid of the water. This reduction of pressure in the condenser makes it possible to obtain a more perfect expansion in the cylinder and to operate the engine with a lower back pressure. By the use of a condenser and air pump, a vacuum of from 24 to 28 inches can be maintained. Water col.
This means a back pressure of from 1 to 3, instead of from 15 to 17, pounds per square inch absolute, the latter being the case when exhausting into the atmosphere.

The latent heat of the steam is taken up by allowing cold water to flow through the condenser, or by spraying it into the space into which the exhaust steam passes. The temperature of the steam and of the water due to condensation

will also be lowered during this process, thus making it possible for the pump to maintain the pressure in the condenser. The exhaust steam will flow from the engine into the steam space of a condenser so long as the cooling water condenses the steam and the air pump removes the air and water.

When the engine exhausts into the atmosphere, it is said to be a **non-condensing engine**; when it exhausts into a condenser, operated as explained, it is said to be a **condensing engine**.

2. Increased Horsepower by Using a Condenser. Other conditions being the same, the horsepower of the

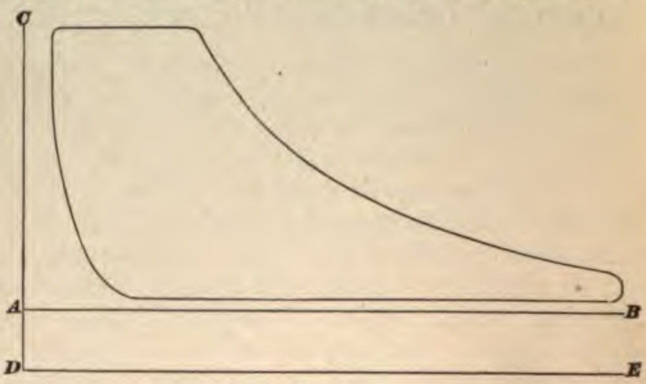


FIG. 1

engine depends on the difference in pressure on the two sides of the piston. Lowering the back pressure while the forward pressure remains unchanged has the same effect as increasing the forward pressure without changing the back pressure. The direct effect of using a condenser, therefore, is to lower the back pressure and increase the horsepower of the engine.

A study of the indicator diagrams of condensing and non-condensing engines will make clearer the reason for this increase of power. Figs. 1 and 2 represent diagrams from the same engine with steam at the same initial pressure and cutting off at the same point. In Fig. 1, the engine is non-condensing and exhausts at a pressure slightly above the

atmosphere; in Fig. 2, the engine has a condenser and the back pressure in consequence is less than atmospheric pressure. The line AB represents the atmospheric pressure in both cases, and the line DE, which represents zero pressure, is below AB a distance AD equal to 14.7 pounds per square inch. The line DC is the clearance line. The steam was cut off at one-fourth stroke in both these cases, and the diagrams show quite clearly, by the difference in area, the gain in work due to the use of a condenser. The expansion line,

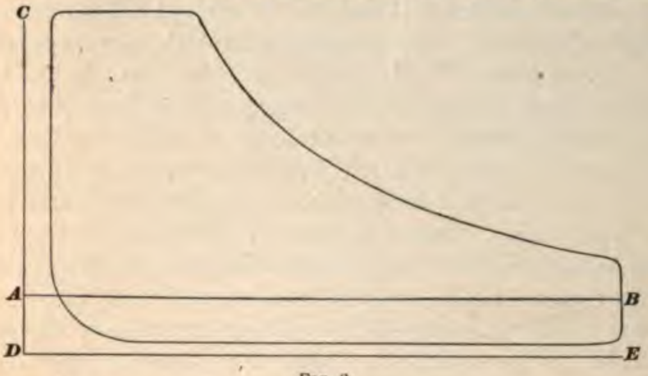


FIG. 2

in Fig. 2, would seem to indicate that there would be greater economy if the steam were expanded further; this means that, in order to use the steam to the best advantage, the ratio of expansion should be greater. It may not be possible to do this in one cylinder, but by expanding it in two or more cylinders a greater ratio can be obtained.

COMPOUND ENGINES

3. Compound Expansion.—When steam is expanded in one cylinder, then conducted to a second cylinder, and expanded still further, the expansion is said to be **compound.** Engines working with two cylinders in this way are called **compound engines.** The cylinder in which the first expansion takes place is known as the **high-pressure cylinder,** and the expansion in this cylinder is referred to as the **first stage.** The cylinder in which the second expansion takes place is known as the **low-pressure cylinder,** and the expansion is referred to as the **second stage.** Inasmuch as the steam volume is much larger in the second stage than in the first stage, the low-pressure cylinder is always larger than the high-pressure cylinder. It is customary to make both cylinders of the same length, so that the difference in size is obtained by the difference in the areas of the cross-sections of the cylinders. Since the cylinders are circular, this means simply a difference in diameter.

4. Cylinder Condensation.—The efficiency of the perfect heat engine has been shown to be $\frac{T_1 - T_2}{T_1}$, which is sometimes called the *thermodynamic efficiency* of the engine. This expression is not entirely true for steam, as it does not take into account the latent heat of steam, which affects its action in the engine cylinder. The expression does, however, show the limit of possible economy under ideal conditions.

T_2 cannot, in practice, be lower than the absolute temperature of the condenser. Hence, the only way of increasing the fraction $\frac{T_1 - T_2}{T_1}$ is to increase T_1, or, in other words, to increase the temperature and, consequently, the pressure of the entering steam. Following out this idea, steam pressures have steadily increased from 8 to 10 pounds, in the time

of Watt, to from 150 to 250 pounds per square inch, the pressures used in modern locomotives and marine engines. But here the evil of cylinder condensation again appears, for by increasing the range of temperature, $T_1 - T_2$, the loss by cylinder condensation is largely increased. To see this clearly, let the pressure of the steam passing into the condenser be 4 pounds above vacuum; its temperature is about 153°. Let the pressure of the entering steam, be, say, 60 pounds above vacuum; its temperature is about 293°. The fall in temperature is 293° − 153° = 140°, nearly.

Suppose that the entering steam has a pressure of 200 pounds absolute; its temperature will be approximately 382° and the fall of temperature during the stroke will be 382° − 153° = 229°. With the steam at 382°, the walls of the cylinder will be at a higher temperature, and the loss due to radiation will be greater than when steam at 153° is used. The heat thus lost by radiation comes from the steam, causing condensation, and the greater the radiation the greater is the amount of incoming steam condensed to maintain the temperature of the cylinder walls. Hence, increasing the range of temperature increases the loss due to cylinder condensation.

To obtain the advantages of a high pressure, and to avoid at the same time, as much as possible, the loss due to cylinder condensation, the steam may be allowed to expand successively in two or more cylinders. The fall of temperature is thus divided between the two or more cylinders, and consequently the loss from condensation in both or all of them is made considerably less than it would be if the same fall of temperature were allowed to take place in one cylinder. This is due to the fact that the walls of each cylinder of the compound engine vary through a much smaller range of temperature than the single cylinder of the simple engine.

It might seem that by thus increasing the number of cylinders the condensation would be increased because of the increased radiating surface. But this is not the case. The rapidity with which the heat of the steam is radiated depends on the difference of temperatures inside and outside the

cylinder, and on the area of the exposed surface. Now, in the simple engine, the entire expansion occurs in one large cylinder, exposing a large radiating surface when the temperature range is greatest; in the case of two cylinders, the steam expands partially in each. When its temperature is greatest, the steam is in the high-pressure cylinder, whose radiating surface is small; and when it reaches the low-pressure cylinder, where the radiating surface is greater, the difference of temperature is much reduced. The result is that there is less total condensation in the compound than in the simple engine.

In modern practice, it is frequently found advantageous to expand the steam in more than two cylinders. When the expansion takes place in three successive cylinders, the engine is said to be **triple expansion;** and when it takes place in four successive cylinders, **quadruple expansion.** All engines in which the expansion of the steam takes place in more than one cylinder are also called **multiple-expansion** engines.

5. Types of Compound Engines.—Compound engines are usually made in one of the two types shown in Fig. 3. In the type shown in Fig. 3 (*a*), the two cylinders are placed in line and the two pistons are attached to the same piston rod; when the cylinders are placed in this way, the engine is said to be a *tandem compound*. The cylinder *h* first receives steam from the boiler. After the steam has expanded in *h*, it passes to the larger cylinder *l*; from here the steam is exhausted into the atmosphere or into a condenser.

Fig. 3 (*b*) shows what is known as the *cross-compound engine*. The steam enters the high-pressure cylinder *h* from the boiler, exhausts into a separate vessel *r*, called the *receiver*, whence it passes to the low-pressure cylinder *l*, and finally exhausts into the atmosphere or into a condenser.

A cross-compound engine has two piston rods and two cranks; the cranks may be placed at any angle with each other. The compound engine without a receiver may have one piston rod and crank, as shown in the tandem type, or it

may have two piston rods and two cranks, the cylinders being placed side by side. In any compound engine, without a receiver, the two pistons must begin and end their strokes at the same time, and if the piston rods are not connected to the same crosshead, the two cranks must be either

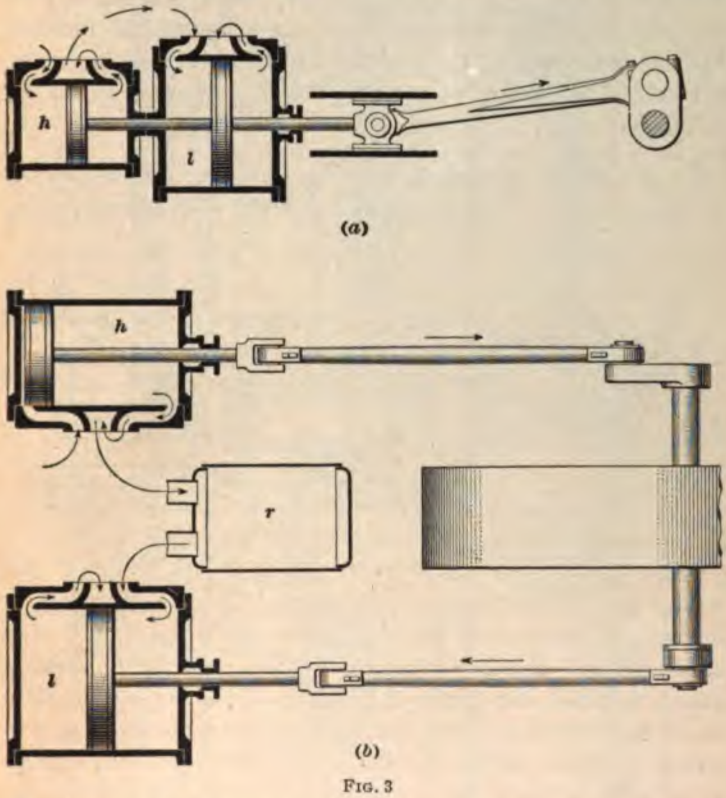


Fig. 3

together or 180° apart, so that the steam may pass directly from the high- to the low-pressure cylinder.

If the cylinders are placed side by side and both piston rods are attached to the same crosshead, the engine is called a *twin compound*. If any of these types of engines have condensers, they are called *tandem*, *cross*, or *twin*, *compound condensing engines*.

In giving the size of a multiple-expansion engine, as in the case of a simple engine, the stroke is always written last. Thus, a compound engine whose high-pressure cylinder is 11 inches in diameter, low-pressure cylinder 20 inches in diameter, and stroke 15 inches would be expressed as an 11″ and 20″ × 15″ compound. In the same manner, a 14″, 22″, and 34″ × 18″ triple-expansion engine would indicate that the diameters of the cylinders were 14 inches, 22 inches, and 34 inches, and that they had a common stroke of 18 inches.

6. Tandem Compound Engine.—Fig. 4 shows an elevation of a **tandem compound non-condensing**

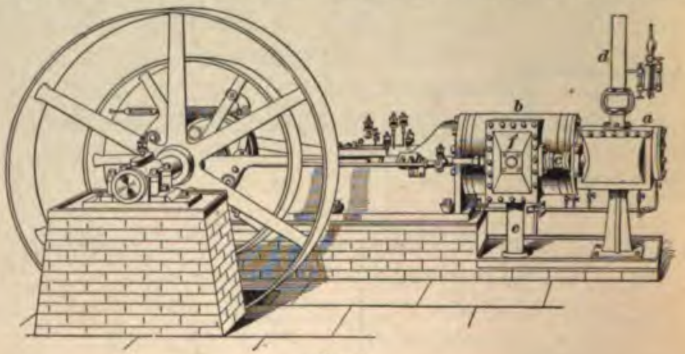


FIG. 4

engine, in which *a* is the high-pressure cylinder, which is placed behind the low-pressure cylinder *b*. Many makers prefer to place the large cylinder behind, since it is then easier to remove the pistons and examine the cylinders in case of repairs. When the small cylinder is behind, it must be entirely removed from the engine before the pistons can be taken out, while, if the large cylinder is behind, the small piston can be taken out. The steam pipe is shown at *d* and the exhaust pipe at *e*. After the steam has expanded in *a*, it is discharged through the connecting pipe *c* into the steam chest *f* of the low-pressure cylinder. After doing its work in *b*, it is exhausted into the condenser or atmosphere through the pipe *e*. As will be seen, a shaft governor is

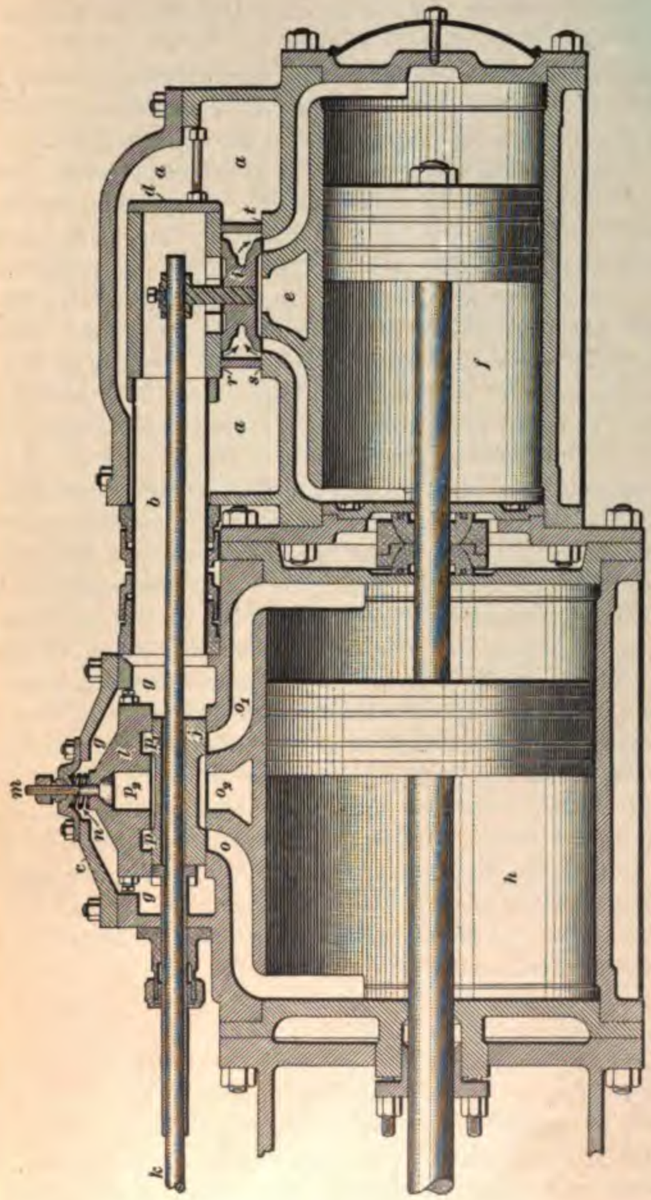


FIG. 5

used. Both pistons are, of course, attached to the same piston rod.

Fig. 5 shows a horizontal section taken through the center line of the cylinders of this engine. The live steam fills the space *a*, *a*, but has no communication through the pipe *b* that leads to the steam chest *c* of the low-pressure cylinder. The plate *d* closes one end of the pipe *b*. The exhaust port *e*, however, connects directly with *b*, so that the steam, after expanding in *f*, exhausts through *e* into *b* and fills the space *g*, *g*, *g*, whence it passes to the cylinder *h*. Both valves *i* and *j* are moved by the same valve stem *k*. These valves differ somewhat from those already described; hence a short description will be given here. It will be seen that they are of the pressure-plate type. The plates and valves are ground together so that they fit perfectly, and no steam can get behind the valves. For convenience, the valve *j* only will be described. The flat plate *l* is the **pressure plate**, and the steam pressure on the back of the plate is prevented by the bolt *m* from pressing the valve with great force to its seat. The pressure that keeps the valve to its seat is supplied by the spring *n*. The valve would still be unbalanced from beneath if the bottom of the pressure plate were a perfectly smooth surface, for, when in the position shown, the steam filling the ports *o* and o_1 presses upwards against the valve and forces it against the pressure plate. To counteract this, recesses *p* and p_1, having the same width and length as the steam ports *o* and o_1 are cut in the pressure plate and steam is allowed to enter each from the corresponding ports *o* and o_1, which are exactly opposite the recesses. For the same reason, the steam in the recess p_2 balances the exhaust steam entering the port o_2.

7. A peculiarity of the high-pressure cylinder valve is that it has two points or openings for the admission of steam to the cylinder. The two points of admission are secured by making the valve hollow and allowing the steam to enter at *r* and *s*, and flow through the valve and into the head-end steam port, as shown by the arrows, while at the

same time steam enters the port at t. This occurs when the valve has moved to the left from the position shown in the figure. The object of this is to allow a wider port opening without a corresponding increase in the travel of the valve.

8. The **ratio of expansion** of a compound or triple-expansion engine is the ratio between the volume of steam exhausted into the atmosphere or into the condenser per stroke, and the volume of steam in the high-pressure cylinder at the point of cut-off.

The **apparent cut-off**, as defined in *Steam-Engine Mechanism*, is the ratio of the portion of the stroke completed at cut-off to the total length of the stroke.

The **real cut-off** is the ratio between the volume of steam in the cylinder and clearance space at cut-off and the volume, including clearance, at the end of the stroke.

Let e = ratio of expansion in high-pressure cylinder;
E = total ratio of expansion in both cylinders;
v = volume of cylinder receiving steam from boiler;
V = volume of cylinder exhausting into atmosphere or condenser;
i = clearance, expressed as a per cent. of stroke;
k = real cut-off in high-pressure cylinder;
k_1 = apparent cut-off in high-pressure cylinder;
r = apparent ratio of expansion in high-pressure cylinder, or $\frac{1}{k_1}$.

Then,
$$k = \frac{k_1 + i}{1 + i} \qquad (1)$$
$$e = \frac{1}{k} = \frac{1 + i}{k_1 + i} \qquad (2)$$
and,
$$E = \frac{eV}{v} = \frac{(1 + i)V}{(k_1 + i)v} \qquad (3)$$
that is, the *total ratio of expansion*, or, as it is often expressed, the *number of expansions*, is equal to the ratio of expansion of the small cylinder multiplied by the ratio between the volumes of the low- and high-pressure cylinders. The total ratio of expansion in a compound engine depends only on the relative

volumes of the cylinders and the point of cut-off in the high-pressure cylinder; it does not depend at all on the point of cut-off in the low-pressure cylinder. The number of expansions in a compound engine generally varies from 6 to 12; in a triple-expansion, from 10 to 25; and in some cases higher.

EXAMPLE 1.—It is desired to have a total ratio of expansion of 9; the number of expansions in the high-pressure cylinder is 2.72; the volume of the high-pressure cylinder is 6 cubic feet. What must be the volume of the low-pressure cylinder?

SOLUTION.— $E = 9$; $e = 2.72$; $v = 6$. Substituting in formula **3**,

$$9 = \frac{2.72\,V}{6}, \text{ or } V = \frac{6 \times 9}{2.72} = 20 \text{ cu. ft., nearly.} \quad \text{Ans.}$$

EXAMPLE 2.—The low-pressure cylinder is four times as large as the high-pressure cylinder, the apparent cut-off of the latter is .4, and the clearance 5 per cent.; what is the total ratio of expansion?

SOLUTION.—From formula **3**,

$$E = \frac{(1 + .05) \times 4}{(.4 + .05) \times 1} = 9\tfrac{1}{3}. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. A compound engine has cylinders 15 inches and 25 inches in diameter and 20 inches stroke; that is, its size is 15″ and 25″ × 20″; the clearance in the high-pressure cylinder is 14 per cent., and the apparent cut-off is $\frac{1}{3}$. What is the number of expansions? Ans. 6.69

2. A 28″, 48″, and 74″ × 60″ triple-expansion engine cuts off at $\frac{2}{5}$ stroke in the high-pressure cylinder; clearance in the high-pressure cylinder, 2 per cent. Find the number of expansions.
Ans. 17, nearly

3. If a 26″, 40″, 60″, and 70″ × 72″ quadruple-expansion engine cuts off the steam in the high-pressure cylinder at $\frac{1}{4}$ stroke, and the clearance in that cylinder is 3 per cent., what is the total number of expansions? Ans. $26\frac{2}{3}$, nearly

---

9. Twin-Compound Engine With Receiver.—When the exhaust from the high-pressure cylinder passes into an intermediate chamber before entering the low-pressure cylinder, this chamber is called a **receiver**, and the engine is sometimes called a **receiver engine**. All the space between the exhaust valve on the high-pressure cylinder and the admission valve on the low-pressure cylinder,

through which the steam passes from the high-pressure cylinder, comprises the receiver volume. The same quantity of steam must be taken into the low-pressure cylinder as is expelled from the high-pressure cylinder at each stroke. The varying pressures in the receiver and in both cylinders are shown in the following illustrative example:

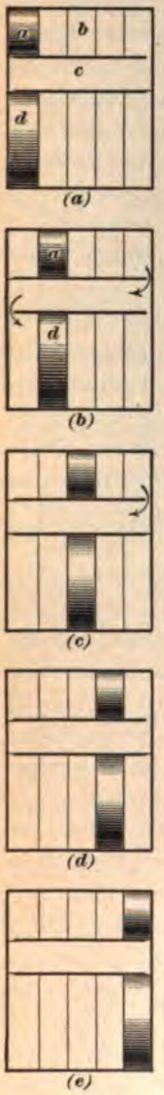


FIG. 6

Taking a twin-compound engine, with the diameter of the low-pressure cylinder twice that of the high-pressure cylinder, then, with the same length of stroke, the volume of the low-pressure cylinder will be four times that of the high-pressure cylinder. If the high-pressure cylinder contains 4 cubic feet of steam, the low-pressure cylinder contains 16 cubic feet. Let the receiver contain 2 cubic feet and the cylinders and receiver be located in the relation shown in Fig. 6. Assume, for convenience, that the steam expands according to the law $p v = p_1 v_1$, and that the 4 cubic feet of steam in the high-pressure cylinder is at a pressure of 36 pounds absolute when the piston is in the position shown at *a*, Fig. 6 (*a*), the steam port to the cylinder *b* is closed and the steam in the receiver *c* is at 90 pounds. Then, when the port opens between the receiver and the high-pressure cylinder, the pressures will equalize and there will be 6 cubic feet. The pressure at this point is obtained by the formula $VP = v_1 p_1 + v_2 p_2$. Then, $P = \frac{v_1 p_1 + v_2 p_2}{V} = \frac{4 \times 36 + 2 \times 90}{6} = 54$ pounds. The pistons *a* and *d* then move $\frac{1}{4}$ stroke to the position shown in Fig. 6 (*b*), and the steam expands to fill the space of 4 cubic feet in the low-pressure cylinder, 2 cubic feet in the receiver, and 3 cubic feet in the high-pressure cylinder, or a

total of 9 cubic feet with a pressure of $\frac{6 \times 54}{9} = 36$ pounds. In the position shown in Fig. 6 (*c*), the pistons have moved another $\frac{1}{4}$ stroke, but the valve into the low-pressure cylinder was closed before the movement began, so that 3 cubic feet in the high-pressure cylinder and 2 cubic feet in the receiver at 36 pounds have been compressed into 2 cubic feet in each, or 4 cubic feet at a pressure of $\frac{5 \times 36}{4} = 45$ pounds. In the low-pressure cylinder, the four volumes at 36 pounds have expanded to 8 cubic feet at $\frac{4 \times 36}{8} = 18$ pounds. In the position shown in Fig. 6 (*d*), the 4 cubic feet in the high-pressure cylinder and receiver has been compressed into 3 cubic feet, or to a pressure of $\frac{4 \times 45}{3} = 60$ pounds, and in the low-pressure cylinder the volume has increased to 12 cubic feet at a pressure of $\frac{8 \times 18}{12} = 12$ pounds. In Fig. 6 (*e*), there are only the two volumes of the receiver at a pressure of $\frac{3 \times 60}{2} = 90$ pounds, while in the low-pressure cylinder there are sixteen volumes at a pressure of $\frac{12 \times 12}{16} = 9$ pounds. Thus, the pressure in the receiver after passing through the cycle has returned to the original pressure. In the high-pressure cylinder, with the cut-off at $\frac{1}{4}$ and a final pressure of 36 pounds, the original pressure must have been $4 \times 36 = 144$ pounds; and in the low-pressure cylinder, with an initial pressure of 36 pounds and four expansions, the final pressure is $36 \div 4 = 9$ pounds. This is shown in the diagram in Fig. 7, in which the line AB represents the

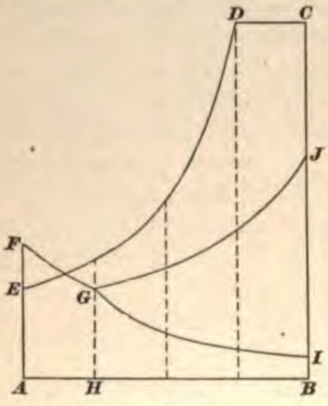


FIG. 7

length of stroke, and BC the initial pressure of 144 pounds per square inch. The cut-off takes place at D, or $\frac{1}{4}$ stroke, and the steam expands to E, or full stroke, having a pressure of $AE = 36$ pounds per square inch. At this point, communication to the receiver opens and the pressures are equalized at 54 pounds, shown by AF, and then the steam expands for $\frac{1}{4}$ stroke to $HG = 36$ pounds. At this point, the steam in the low-pressure cylinder is cut off, the low-pressure piston moves to the end of its stroke, and the steam expands along the line GI to the pressure $BI = 9$ pounds, while the steam in the receiver and high-pressure cylinder is compressed along GJ to $BJ = 90$ pounds per square inch.

10. Cross-Compound Engines With Receiver.—In a cross-compound engine, the variation of the pressures in the cylinders and receiver is different from the twin-compound because the cranks are at some other angle than 0° or 180°.

Fig. 8 illustrates the different positions of the pistons and cranks in a cross-compound engine with the cranks at 90°, the angularity of the connecting-rod being neglected. Assume the volume of the low-pressure cylinder to be 16 cubic feet, the volume of the high-pressure 4 cubic feet, and the volume of the receiver 2 cubic feet, as in the twin engine in Art. 9. Let the steam be cut off at $\frac{1}{4}$ stroke in both cylinders, and let the boiler pressure be 144 pounds per square inch, absolute. The beginning of the stroke in the high-pressure cylinder is shown in Fig. 8 (a), with the high-pressure piston at a, the low-pressure piston at b, high-pressure crank at c, and the low-pressure crank at d. The crank-circle has been drawn on the low-pressure cylinder for the purpose of comparison. The center of the crank-circle is at the middle point of the low-pressure cylinder, and the centers of the crankpins always lie on the vertical center lines of their respective pistons. The steam is admitted to the high-pressure cylinder for $\frac{1}{4}$ stroke at the full boiler pressure of 144 pounds absolute, and then expands to the end of the stroke, where it has a pressure of $\frac{144 \times 1}{4} = 36$ pounds.

Assume the pressure in the receiver to be 40.8 pounds. Then, when the exhaust valve from the high-pressure cylinder opens to the receiver, there will be 2 cubic feet at 40.8 pounds and 4 cubic feet at 36 pounds, to equalize, giving a receiver pressure of $\frac{4 \times 36 + 2 \times 40.8}{6} = 37.6$ pounds. The steam on the left-hand side of the piston *b* is at a pressure of about 18 pounds per square inch, and is expanding, while that on the right-hand side is exhausting into the air or condenser.

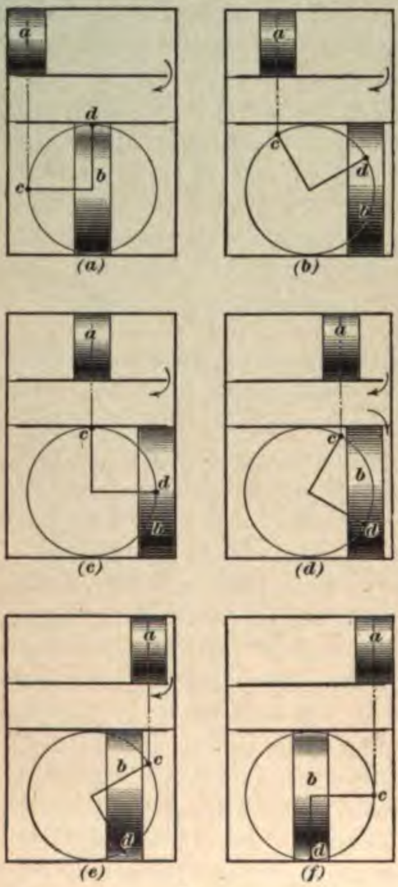


FIG. 8

When the high-pressure piston has moved to the position shown in Fig. 8 (*b*), cut-off takes place and there is in the cylinder 1 cubic foot of steam at 144 pounds pressure. The right-hand end of the high-pressure cylinder is still open to the receiver, but the volume has been reduced from 6 cubic feet to 5 cubic feet; and consequently, the pressure has increased to $\frac{37.6 \times 6}{5} = 45.1$ pounds. In the left-hand end of the low-pressure cylinder, the steam has expanded from a volume of 8 cubic feet to 14.92 cubic feet, nearly, and the pressure has been reduced from 18 pounds to $\frac{18 \times 8}{14.92} = 9.65$ pounds, nearly.

In the positions shown in Fig. 8 (*c*), the pressure at the left of *a* is $\frac{144 \times 1}{2} = 72$ pounds; that at the right and in the receiver is $\frac{45.1 \times 5}{4} = 56.4$ pounds, and in the low-pressure cylinder the pressure is $\frac{9.65 \times 14.92}{16} = 9$ pounds. At this point, the exhaust valve closes on the low-pressure cylinder and the valve to the receiver opens to admission on the right-hand side of the piston *b*.

In the position shown in Fig. 8 (*d*), the steam on the left-hand side of *a* has expanded to 3 cubic feet, and the pressure has been reduced to $\frac{72 \times 2}{3} = 48$ pounds. On the right-hand end, there is 1 cubic foot, the receiver has 2 cubic feet, and the right-hand end of the low-pressure cylinder contains 1.08 cubic feet, nearly; hence the volume of these three is 4.08 cubic feet and the receiver pressure is $\frac{4 \times 56.4}{4.08} = 55.3$ pounds, nearly. The left-hand side of the low-pressure cylinder is open to exhaust.

In the position shown in Fig. 8 (*e*), the volume at the left of *a* is 3.73 cubic feet, and the pressure is $\frac{48 \times 3}{3.73} = 38.6$ pounds, nearly. The volume on the right of *a* is .27 cubic foot, in the receiver 2 cubic feet, and on the right of *b*, 4 cubic feet, making 6.27 cubic feet and a receiver pressure of $\frac{55.3 \times 4.08}{6.27} = 36$ pounds, nearly. The left-hand side of the low-pressure cylinder is, of course, open to exhaust. The receiver is shut off at this point from the low-pressure cylinder, and the 2 cubic feet in the receiver and the .27 cubic foot in the high-pressure cylinder are compressed to 2 cubic feet at the position shown in Fig. 8 (*f*) at a pressure of $\frac{2.27 \times 36}{2} = 40.8$ pounds, nearly. On the left of *a*, the volume is now 4 cubic feet and the pressure is $\frac{38.6 \times 3.73}{4} = 36$ pounds, nearly. The volume on the right

of b has expanded from 4 cubic feet at 36 pounds to 8 cubic feet at 18 pounds. These pressures agree with those at the starting point, and the process will be repeated on the return stroke.

COMPOUND-ENGINE DIAGRAMS

11. In Fig. 9 are shown the ideal diagrams of a tandem compound engine that exhausts directly from one cylinder into the other; clearance and compression are not con-

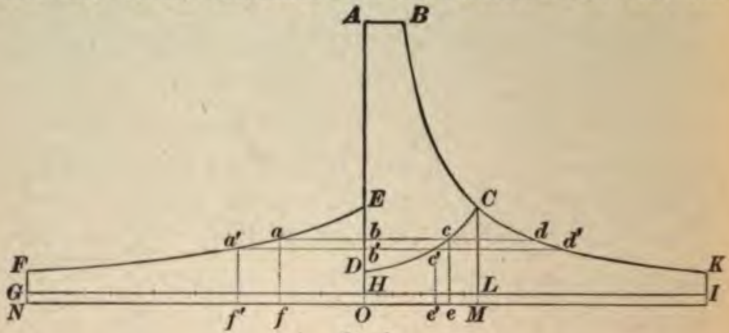


FIG. 9

sidered. $ABCD$ is the diagram from the high-pressure cylinder. Steam enters at the boiler pressure OA; at B cut-off occurs, and the steam expands along the line BC to the end of the stroke. The steam and expansion lines AB and BC are precisely like those of a simple engine. At C the exhaust opens. Instead, however, of exhausting into the atmosphere or a condenser, the steam exhausts into the low pressure cylinder. The low-pressure cylinder is always larger than the high-pressure cylinder; consequently, the volume of steam exhausting from the high-pressure into the low-pressure cylinder is constantly increasing. This is shown in Fig. 10; when the two pistons are at the end of the stroke, as shown at (*a*), the steam simply fills the small cylinder h; but at the middle of the stroke, as shown at (*b*), the steam fills half of h and also half of l. Hence, its volume grows greater as the two pistons move to the right.

Returning to Fig. 9, the steam, when the piston is at the end C of the stroke, just fills the small cylinder; on the

return stroke, however, as has just been shown, the volume increases, and hence the pressure falls, as shown by the back pressure line CD. At the end D of the return stroke, steam again enters the right end of the cylinder h, Fig. 10, from the boiler and raises the pressure to A, thus completing the cycle of operations.

$CDHL$ represents the card from the low-pressure cylinder l. Since the high-pressure cylinder exhausts directly into the low-pressure cylinder, the back pressure of the former must be the same as the forward pressure of the latter. Hence, CD is both the back-pressure line of h and the expansion line of l. At the end of the stroke, the pressure drops from D to H, OH being the pressure in the condenser. The remainder of the diagram is the same as that of a simple engine. In this description, the common length OM of the two diagrams has been made proportional to the length of stroke.

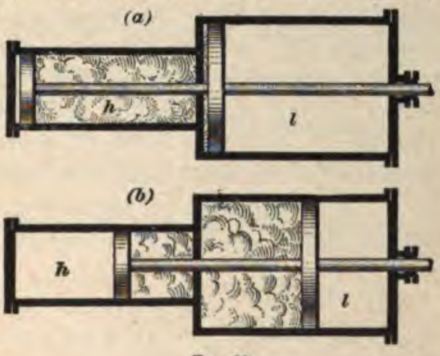


FIG. 10

The indicator diagram is so drawn that the vertical distances represent pressures in the cylinder in pounds per square inch, and the horizontal distances represent lengths of the stroke to some scale. As the area of cross-section of the cylinder is constant, the horizontal distances on the indicator diagram are often taken as representing volumes in the cylinder to a properly chosen scale. When indicator diagrams are taken from two cylinders of the same stroke but different diameters, the diagrams do not represent the volumes to the same scale and hence their areas do not represent the work of their respective cylinders to the same scale. In order to compare the two diagrams, they should be drawn to the same scale, which can readily be done in this case by choosing lengths of the diagram proportional to the volumes of the two cylinders.

12. Suppose, now, that the length of each diagram be taken to represent the volume of the cylinder to which it belongs. Let OM represent the volume of the high-pressure cylinder. Suppose that the volume of the low-pressure cylinder is three times the volume of the high-pressure cylinder. The length of the low-pressure diagram must then be three times that of the high-pressure diagram. From O lay off $ON = 3 \times OM$, and project the expansion line CD into the line EF. This is done by dividing the line OM into a convenient number of equal parts, in this case eight, and ON into the same number of equal parts. At the points of division e, e', etc., erect ordinates cutting CD in c, c', etc. At the points of division of ON erect ordinates fa, $f'a'$, etc. Through c, c', etc. draw lines parallel to MN, cutting af in a, $a'f'$ in a', etc.; a, a', etc. will be points on the required line EF. E and F are, of course, opposite C and D. $EFGH$ represents the low-pressure diagram to the same scales of pressure and volume used in $ABCD$, the high-pressure diagram. $EFGH$ has been laid off to the left of OA simply for convenience.

13. The two diagrams may be combined into one in the following manner: Draw a horizontal line, as ad, intersecting both diagrams. The volume of steam in the high-pressure cylinder at the pressure $ec = Ob$ is represented by the length bc; the volume of steam in the low-pressure cylinder at the same pressure is represented by ab. Hence, the total volume of steam at the pressure in question is $ab + bc = ac$. From c lay off $cd = ab$; then, $ab + bc = bc + cd = bd$ $=$ volume of steam at pressure Ob. In the same manner it is found that $b'd'$ equals the volume of steam in both cylinders when the pressure is represented by Ob'. By finding a sufficient number of these points, d, d', etc., the curve CK may be drawn. This curve represents the relation between the common pressure in the two cylinders and the total volume of steam in both cylinders, and it will be found that it is simply a continuation of the expansion curve BC of the high-pressure cylinder. It is seen that the combination of

these two diagrams forms one large diagram $ABCKIHA$, equal in area to the sum of the areas $ABCD$ and $EFGH$; for the mean ordinates of $EFGH$, $CDHL$, and $CKIL$ are equal from the nature of the construction; hence, representing the mean ordinate by m, the area of $EFGH = m \times GH$; of $CDHL = m \times HL$; and of $CKIL = m \times LI$. But $GH = 3\,HL$ and $LI = 2\,HL$; consequently, area $EFGH = m \times 3\,HL = m \times HL + m \times 2\,HL =$ area $CDHIK$. The length of the large diagram represents the volume of the low-pressure cylinder, while the initial pressure OA is the boiler pressure of the steam entering the high-pressure cylinder.

The ratio of expansion of the compound engine of Fig. 9 is $\frac{HI}{AB}$. If the large diagram $ABKIHA$ be considered as the diagram of a single engine, the ratio of expansion is also $\frac{HI}{AB}$. A single engine giving the large combined diagram $ABKIHA$ will do the same work as the compound engine giving the two diagrams $ABCD$ and $EFGH$; but in order that a single engine may give the diagram $ABKIHA$, the volume of its cylinder must be represented by HI—that is, it must be equal to the volume of the low-pressure cylinder of the compound engine, and, further, it must work with an initial pressure OA equal to the boiler pressure of the compound engine.

14. Approximate Horsepower of a Compound Engine.—From the foregoing statement, the following rule is apparent: *The horsepower of a compound engine is approximately equal to the horsepower of a single engine having a cylinder equal in volume to the low-pressure cylinder of the compound, and working with the same ratio of expansion and with the same boiler pressure.*

In general, it is customary to calculate the horsepower of a compound, triple, or quadruple-expansion engine as if the total expansion took place in the cylinder exhausting into the condenser. This will give a rough approximation to the

true horsepower, which will usually be less than the calculated value.

When the total number of expansions and the steam pressure during admission are known, the mean effective pressure, commonly written M. E. P., may be found approximately by the following formula:

$$\text{M. E. P.} = \frac{.9\,P\,(1 + 2.3 \log E)}{E} - .9p$$

in which P = absolute boiler pressure;
p = absolute back pressure;
E = total number of expansions.

In this formula, the constant 2.3 is used for convenience instead of the more accurate value 2.3026 used in other formulas. The value of M. E. P. from the above formula is to be substituted for P in the horsepower formula.

EXAMPLE.—The low-pressure cylinder of a compound condensing engine is 30 in. × 40 in.; the boiler pressure is 100 pounds (gauge) and the number of expansions 8. Find the approximate horsepower, assuming the number of revolutions per minute to be 60.

SOLUTION.—Absolute pressure $P = 100 + 14.7 = 114.7$ lb.; take back pressure p equal to 3 lb. for condensing engine; ratio of expansion $E = 8$. Substituting in the formula, $\text{M. E. P.} = \frac{.9\,P(1+2.3 \log E)}{E} - .9p$ $= \frac{.9 \times 114.7\,(1 + 2.3 \log 8)}{8} - .9 \times 3 = 37$ lb., nearly. Now, using the horsepower formula, $\text{I. H. P.} = \frac{PLAN}{33{,}000} = \frac{37 \times 40 \times 30^2 \times .7854 \times 60 \times 2}{33{,}000 \times 12}$ $= 317.02$ H. P. Ans.

15. Diagrams of a Tandem Compound Engine. Fig. 11 shows the ideal diagrams of a tandem compound engine, taking clearance and compression into account. The pressures are represented to the same scale, but the volumes are to different scales. The steam and expansion lines AB and BC of the high-pressure cylinder are similar to those of a single engine. At C, the pressure drops slightly as the steam is admitted to the low-pressure cylinder. The back-pressure line of the high-pressure diagram and the expansion line of the low-pressure diagram are parallel, the slight difference between them being due to the resistance of the

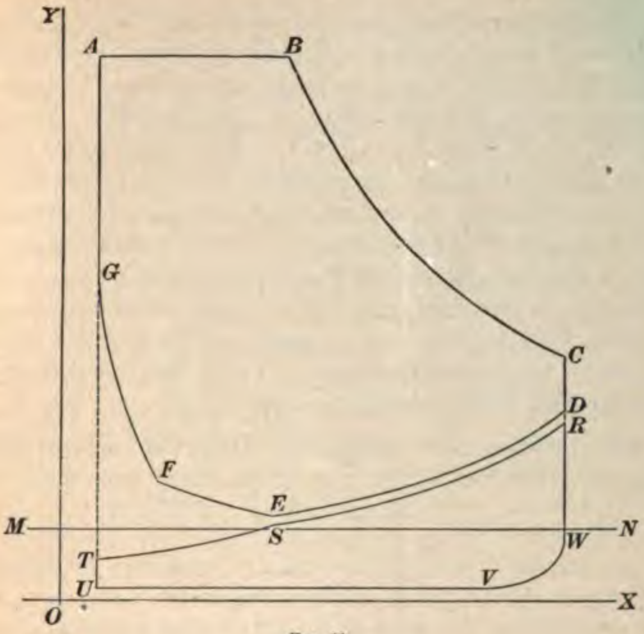


FIG. 11

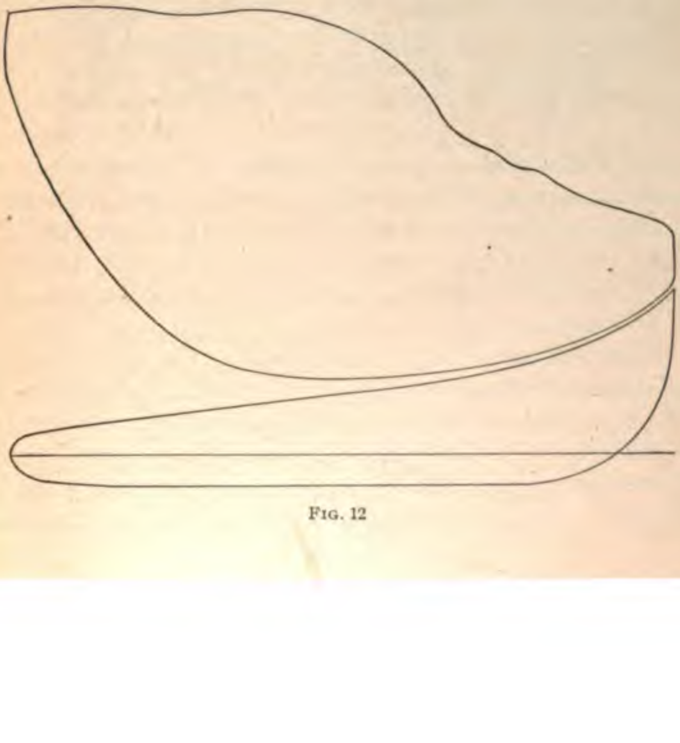


FIG. 12

pipe connecting the cylinders. At E, the steam is cut off from the large cylinder, and is compressed in the small cylinder and in the pipe connecting the two. At F, the exhaust closes and the steam is compressed in the small cylinder alone from F to G; at G, fresh steam again enters. When the cut-off of the low-pressure cylinder occurs, as at S, the steam already in the cylinder expands, following the ordinary equilateral hyperbola $S\,T$. At T, release takes place and the pressure falls to the pressure of the condenser. The remainder of the diagram is the same as for a simple engine.

In Fig. 12 is shown a diagram taken from an actual engine. The similarity between it and the theoretical diagram of Fig. 11 will readily be seen.

16. Action of Steam in the Cross-Compound Engine.—The action of the steam in a cross-compound

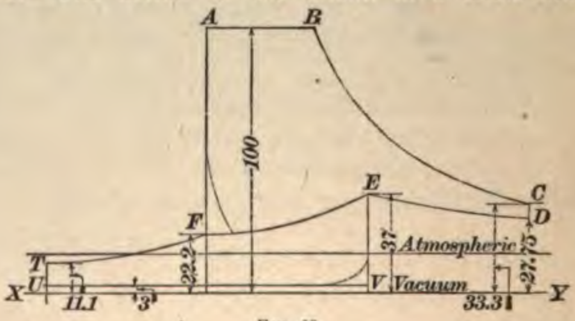


Fig. 13

engine with a receiver, may perhaps be better shown by assuming certain conditions to be fulfilled, and from them working out the theoretical diagram. Suppose that the volume of the low-pressure cylinder is 12 cubic feet; that the volume of the high-pressure cylinder is 4 cubic feet; that the volume of the receiver also is 4 cubic feet; that the steam is to be cut off in the high-pressure cylinder at $\frac{1}{3}$ stroke, and in the low-pressure cylinder at $\frac{1}{2}$ stroke; that the boiler pressure is 100 pounds absolute; and that the cranks make an angle of 90° with each other. Neglect clearance and compression. The stroke of the high-pressure cylinder begins at A, as shown in Fig. 13, the pressure at A being

100 pounds; at B, cut-off takes place and the steam expands along the equilateral hyperbola $B\,C$. The volume of steam at C is three times that of B, and, since $p_1 v_1 = p_2 v_2$, the pressure at C is $\frac{100 \times 1}{3} = 33.3$ pounds. The steam is released at C and passes into the receiver, where it mixes with steam at receiver pressure. In order to find the resulting pressure of the mixture of the steam in the high-pressure cylinder, and the steam in the receiver, it will first be necessary to find the pressure of the latter. The total ratio of expansion is found as follows: The volume of the high-pressure cylinder at cut-off is $\frac{4}{3}$ cubic feet; volume of low-pressure cylinder is 12 cubic feet; number of expansions $= 12 \div \frac{4}{3} = 9$. If clearance had been taken into account, the above would be modified to a certain extent. It can be proved that the size of the receiver exerts no effect whatever on the number of expansions or on the final terminal pressure. Hence, the terminal pressure in the low-pressure cylinder is $\frac{100}{9} = 11.1$ pounds. Since the low-pressure cylinder cuts off at $\frac{1}{2}$ stroke, the pressure at the point of cut-off is $11.1 \times 1 = p \times \frac{1}{2}$, or $p = 22.2$, the volume of steam at cut-off being half that at the end of the stroke. Now, just before cutting off, the low-pressure cylinder was receiving steam from the receiver; hence, the pressure in the receiver at the instant of cut-off in the low-pressure cylinder is 22.2 pounds. Since the cranks are at right angles to each other, the high-pressure piston is just at the end of its stroke when cut-off occurs in the low-pressure cylinder. Hence, the steam at 33.3 pounds pressure, on being released from the high-pressure cylinder, rushes into the receiver and mixes with the steam at 22.2 pounds pressure. The pressure of the mixture is found from the formula,

$$VP = v_1 p_1 + v_2 p_2, \text{ or } P = \frac{v_1 p_1 + v_2 p_2}{V}$$

The volume of steam in the high-pressure cylinder having a pressure of 33.3 pounds is 4 cubic feet, and that in the receiver having a pressure of 22.2 pounds is 4 cubic feet. The low-pressure cylinder being cut off, the total volume of the mixture is 8 cubic feet.

Substituting, $P = \frac{4 \times 22.2 + 4 \times 33.3}{8}$, or $P = 27.75$ pounds. The pressure thus drops from C to D. The steam is now compressed in the high-pressure cylinder and receiver by the return stroke of the high-pressure piston. The volume of steam at D is 8 cubic feet; when the piston has completed one-half of its return stroke, the volume is $4 + 2 = 6$ cubic feet; hence, the pressure is $27.75 \times \frac{8}{6} = 37$ pounds, as shown

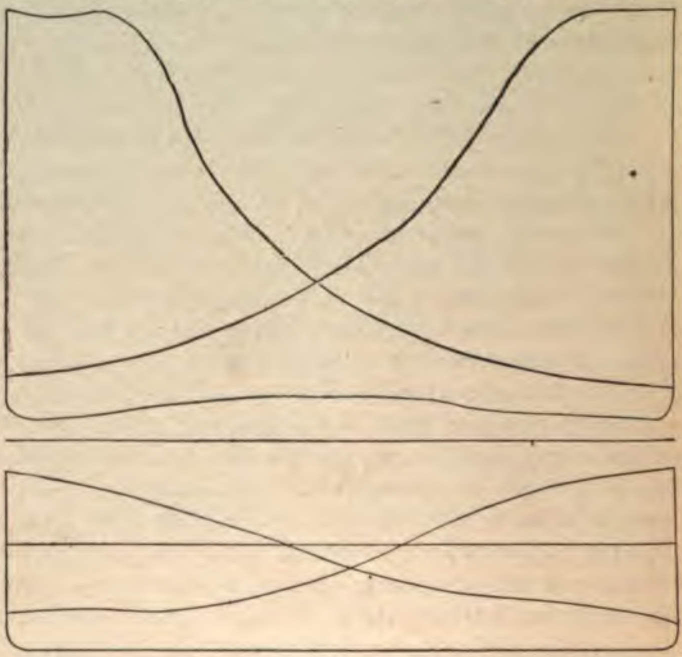


FIG. 14

at E. Since at E the high-pressure piston is at the middle of its return stroke, the low-pressure piston must be at the beginning of its return stroke. The steam in the receiver and high-pressure cylinder is now admitted to the other end of the low-pressure cylinder, its volume increases as shown in Fig. 10, and its pressure falls accordingly, as shown by the line EF, which is a part of the back-pressure line of the high-pressure diagram, and the steam line of the low-pressure

diagram. The pressure at F has already been found to be 22.2 pounds, and the terminal pressure at T to be 11.1 pounds. Hence, the expansion line of the low-pressure diagram is an equilateral hyperbola through F and T. At T, the pressure drops to that of the condenser, about 3 pounds; the remainder of the diagram is the same as for a simple engine.

In Fig. 14 is shown a diagram from a compound marine engine. It will be readily seen that it quite closely resembles the theoretical diagram.

USUAL METHOD OF COMBINING THE DIAGRAMS

17. The indicator diagrams from multiple-expansion engines may be combined so as to permit a careful study of the action of the steam during its passage through the engines. In Fig. 15, A and B are the diagrams from the high-pressure cylinder and C and D, the diagrams from the low-pressure cylinder of a tandem compound non-condensing engine. The diagrams A and B are taken with a 60 spring, and C and D with a 30 spring. That is, each inch of height on the high-pressure diagrams represents 60 pounds pressure; and the same height on the low-pressure diagrams represents 30 pounds pressure. The atmospheric line MN is shown beneath each set of diagrams. The diameters of the cylinders are 13 inches and 20 inches, respectively, and the clearance in each cylinder is 10 per cent. of the volume of the high-pressure cylinder.

To combine the indicator diagrams of any multiple-expansion engine, they must be reduced to the same scale of pressure and volume. In the diagrams in Fig. 15, the scales of pressure are not the same for the two sets of diagrams. Furthermore, the volume of the high-pressure cylinder is much smaller than that of the low-pressure, yet both are represented by the same length of diagram. It is evident, therefore, that the scales of volumes are likewise unequal. Since this is a tandem compound engine, the steam exhausted from one end of the high-pressure cylinder is admitted to the

opposite end of the low-pressure cylinder. Hence, diagrams *A* and *D* or *B* and *C* must be taken together. For the purpose of illustration, diagrams *B* and *C* will be combined. The diagrams may be combined in several ways. The scales of pressure and volume of both diagrams may be changed; the scale of volume of diagram *C* may be changed to correspond to that of *B* and the scale of pressure of *B* changed to that of *C*; or, the scale of volume of *B* may be changed to that

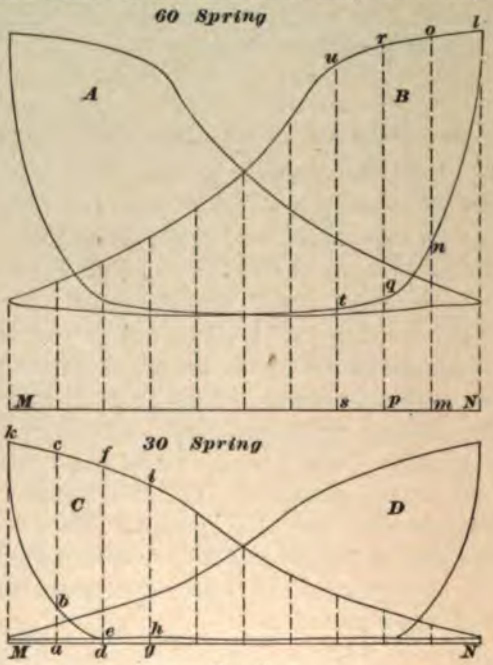


FIG. 15

of *C*, and the scale of pressure of *C* made the same as in *B*. The last-named method will be used in the following explanation.

18. The first step is to draw both diagrams as they would appear had each been taken with a 60 spring. Since *C* is taken with a 30 spring, it is evident that it would be only $\frac{30}{60}$, or one-half, as high when taken with a 60 spring. Consequently, divide the atmospheric line *M N* of each diagram

into ten equal parts, and from the points of division, as well as from the ends of the diagrams, draw vertical lines as shown in Fig. 15. Then draw a horizontal line MN, Fig. 16, equal in length to MN, Fig. 15, and divide it into ten equal parts, erecting perpendiculars at the points of division and at the ends. This will be the atmospheric line of the combined diagram. Now, since it is desired to reduce the diagram C to a scale of 60 pounds to the inch, lay off Mk, ab, ac, de, etc., Fig. 16, equal, respectively, to $\frac{1}{2}Mk$, $\frac{1}{2}ab$, $\frac{1}{2}ac$, $\frac{1}{2}de$, etc. of Fig. 15, measuring from the atmospheric line in each instance. This will give a series of points h, e, b, k, c, f, i, etc., Fig. 16, through which a smooth line may be traced, resulting in the reduced diagram. This diagram is now reduced to the proper scale of pressures, 60 pounds to the inch, and is just one-half as high as in Fig. 15.

19. The next step is to reduce the diagram B so that its scale of volume will agree with that of C. Its scale of pressure-

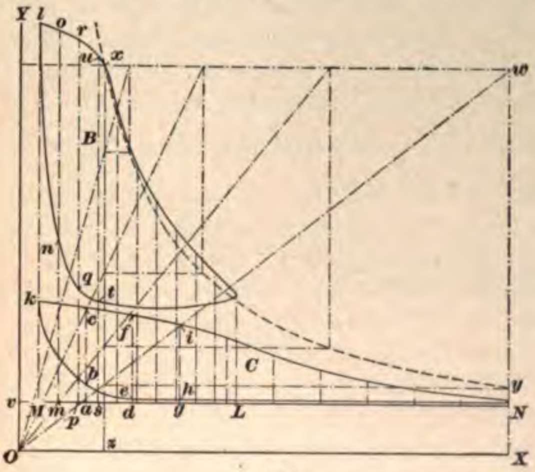


FIG. 16

sure is already 60; hence, that need not be changed. The two cylinders have equal strokes, and therefore their volumes are proportional to their areas. That is, $\frac{v}{V} = \frac{(13)^2 \times .7854}{(20)^2 \times .7854}$

$= \frac{169}{400} = \frac{.4225}{1}$. Hence, the volume of the high-pressure cylinder is only .4225 times that of the low-pressure cylinder. Now, the length MN of the low-pressure diagram, Fig. 16, may be taken to represent the volume of the low-pressure cylinder to a certain scale. It measures 2.43 inches in length. Hence, the volume of the high-pressure cylinder, to the same scale, will be represented by a length of $2.43 \times .4225 = 1.03$ inches. Consequently, lay off ML equal to 1.03 inches, as in Fig. 16, and divide this distance into ten equal parts, erecting perpendiculars at the points of division. Then, measuring from the atmospheric line, as before, lay off Ml, mn, mo, pq, pr, etc. of Fig. 16 equal to Nl, mn, mo, pq, pr, etc. of Fig. 15, which will give a series of points t, q, n, l, o, r, u, etc., Fig. 16, through which a smooth curve may be traced, thus outlining the diagram B reduced to the same scale of volume as C.

20. The final steps in the combination of the two diagrams are to locate the vacuum and clearance lines and to draw the theoretical expansion curve. Since both diagrams are drawn to a scale of 60 pounds to the inch, the vacuum line must lie $\frac{14.7}{60} = .245$ inch, say $\frac{1}{4}$ inch, below the atmospheric line. That is, OX, Fig. 16, is drawn $\frac{1}{4}$ inch below MN. The clearance in each cylinder is 10 per cent. of the volume of the high-pressure cylinder. Consequently, the clearance on the diagram must be represented by a line whose length is 10 per cent. of 1.03, since 1.03 represents the volume of the high-pressure cylinder. That is, $1.03 \times .10 = .103$ inch, say $\frac{1}{10}$ inch, is the distance representing the clearance to the assumed scale of volume. Therefore, lay off Mv equal to $\frac{1}{10}$ inch, and through v draw the perpendicular OY. Then OY is the clearance line, or line of zero volume. The point O is the point of zero volume and zero pressure. Now locate, by inspection of the diagram, the point of cut-off x on the diagram B. Through x draw the horizontal line xw and the vertical line xz. Then from O draw radial lines

intersecting the horizontal and vertical lines through x and construct the equilateral hyperbola.

If the clearance volume is not the same in both cylinders, the diagrams must be so located that their clearance lines will coincide, which means that the diagrams will lie at unequal distances from the common clearance line. In such a case, it will be necessary to draw the clearance line before reducing the diagrams. If there were no losses, the area

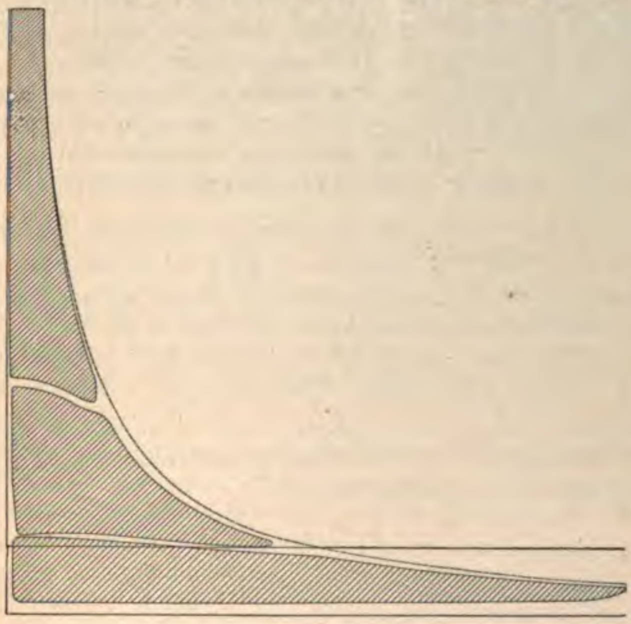


FIG. 17

$lxyNMl$ would represent the work done by the steam, and the difference between this area and the combined area of the two diagrams represents, approximately, the various losses in the cylinders and connecting passages. As will be seen, this loss is comparatively small, and could be made still smaller by making the cut-off in the low-pressure cylinder a trifle later.

In a manner entirely similar to that just described, the diagrams of a triple- or quadruple-expansion engine are

combined. Fig. 17 shows the combined diagrams taken from a Corliss triple-expansion condensing pumping engine. The diameters of the cylinders were 28, 48, and 74 inches, and the stroke was 60 inches; the clearance was 1.4 per cent., 1.5 per cent., and .77 per cent., respectively.

21. Ratio of Cylinders.—The ratio between the volumes of the two cylinders of a compound engine is so chosen that: (1) the power is divided as equally as possible between them; (2) the initial strains in the two cylinders are the same; (3) the drop of pressure between the high-pressure cylinder and the receiver is as small as possible. Numerous rules and formulas are used for this purpose. One rule is to make the number of expansions in the high-pressure cylinder 2.72. Substituting this in formula **3**, Art. **8**,

$$E = 2.72\frac{V}{v}, \text{ or } \frac{V}{v} = \frac{E}{2.72} \qquad (1)$$

EXAMPLE.—What should be the ratio between the volumes of the two cylinders if the total number of expansions be 10?

SOLUTION.— $\frac{V}{v} = \frac{E}{2.72} = \frac{10}{2.72} = 3.68$. Ans.

Another rule is to make the ratio of the volume of the low- and high-pressure cylinders equal to the square root of total ratio of expansion; that is,

$$\frac{V}{v} = \sqrt{E} \qquad (2)$$

Using this formula in the last example, $\frac{V}{v} = \sqrt{E} = \sqrt{10} = 3.16$.

EXAMPLES FOR PRACTICE

1. A compound engine is to be designed; the high-pressure cylinder is to be 17 in. × 20 in., with an apparent cut-off of .4, and a clearance of 10 per cent.; the total number of expansions is to be 7. What must be the size of the low-pressure cylinder having the same stroke?
Ans. 30.324 in. × 20 in., say $30\frac{1}{4}$ in. × 20 in.

2. Calculate the above by formula **1**. Ans. $27\frac{1}{4}$ in. × 20 in.

3. The low-pressure cylinder of a compound engine is 44 in. × 36 in. What must be the diameter of the high-pressure cylinder in order that

there may be 8 expansions, with .38 cut-off and 12 per cent. clearance in the high-pressure cylinder? Ans. 23.28 in., say $23\frac{1}{4}$ in.

4. An 11″ and $22\frac{1}{2}$″ × 18″ compound engine has 13 per cent. clearance in the high-pressure cylinder; find the point of apparent cut-off so that there may be 9 expansions. Ans. 39.5 per cent. of the stroke

---

HORSEPOWER OF COMPOUND ENGINES

22. The Indicated Horsepower.—The actual indicated horsepower, commonly written I. H. P., of a compound or triple-expansion engine may be obtained from the indicator diagrams. The method used is best shown by an example. A triple-expansion engine has the volumes of its cylinders in the ratio $1 : 2\frac{1}{2} : 6\frac{1}{4}$; that is, the low-pressure cylinder is $6\frac{1}{4}$ times as large as the high-pressure cylinder, and $\frac{6\frac{1}{4}}{2\frac{1}{2}} = 2\frac{1}{2}$ times as large as the intermediate cylinder. The low-pressure cylinder is 40 in. × 40 in. The engine makes 120 revolutions per minute. On measuring the diagrams, it is found that the M. E. P. of the high-pressure cylinder is 80.5 pounds; of the intermediate cylinder, 37.5 pounds; and of the low-pressure cylinder, 16.12 pounds. What is the I. H. P. of the engine?

It would be possible to calculate the I. H. P. by finding the work exerted by each cylinder separately, as if it were the cylinder of a simple engine, and then taking their sum. It is, however, easier to reduce all the pressures to the area of the low-pressure cylinder. This is done by dividing the M. E. P. of each cylinder by the ratio between the volume of the low-pressure cylinder and the volume of the cylinder considered. In the present case, the M. E. P. of the high-pressure cylinder is 80.5. The volume of the low-pressure cylinder is 6.25 times that of the high-pressure cylinder, or, what is the same thing, the area of the low-pressure piston is $6\frac{1}{4}$ times that of the high-pressure piston. Therefore, to produce the same work, the M. E. P., when acting in the low-pressure cylinder, must be $\frac{1}{6.25}$ of what it was in the high-pressure cylinder. The M. E. P. of the small cylinder, reduced to the area of the

low-pressure cylinder, is, therefore, $\frac{80.5}{6.25} = 12.88$ pounds. Likewise, the M. E. P. of the intermediate, reduced to the low-pressure cylinder, is $\frac{37.5}{2.5} = 15$ pounds. The M. E. P. of the low-pressure cylinder, of course, remains the same. The total M. E. P., reduced to the low-pressure cylinder, is, therefore, $12.88 + 15 + 16.12 = 44$ pounds. Now, substituting this M. E. P., the area of the low-pressure cylinder, the length of stroke and revolutions per minute, in the horsepower formula,

$$\text{I. H. P.} = \frac{PLAN}{33{,}000} = \frac{44 \times 40 \times 40^2 \times .7854 \times 120 \times 2}{12 \times 33{,}000}$$
$$= 1{,}340.4 \text{ H. P.}$$

23. Steam Consumption from Compound Engine Diagrams.—When it is desired to compute the approximate steam consumption of a multiple-expansion engine from its indicator diagrams, it is necessary to use only the diagram from the low-pressure cylinder, for all the steam used by the engine must pass through the low-pressure cylinder. And since the greatest amount of moisture is reevaporated into steam near the point of release in the low-pressure cylinder, the most reliable results are obtained by using the low-pressure diagram. The method to be used is that explained in *Steam-Engine Indicators and Diagrams*. The value of P in the formula, however, must be the total M. E. P. of the engine, reduced to its equivalent, acting on the area of the low-pressure piston.

CONDENSERS

24. It has been shown that the efficiency of the ideal engine, $\frac{T_1 - T_2}{T_1}$, may be increased by either raising the temperature T_1 or by lowering the temperature T_2. T_1 in the steam engine may be raised by increasing the boiler pressure; T_2 may be lowered by using a condenser. In non-condensing engines, the steam is exhausted into the atmosphere, and therefore the exhaust steam must have at least the pressure of the atmosphere; in practice, the back pressure of steam in a non-condensing engine is scarcely ever less than 16 pounds above vacuum, and is oftener 17 pounds or more. In good condensing engines, the back pressure is often as low as 2 pounds above vacuum.

25. Suppose that the boiler pressure of the steam is 80 pounds absolute, the temperature corresponding to this pressure is, from the Steam Table, 311.8° F., and the absolute temperature is, therefore, $460° + 311.8° = 771.8°$ F. The absolute temperature corresponding to a pressure of 17 pounds is $460° + 219.4° = 679.4°$ F., and corresponding to a pressure of 3 pounds is $460° + 141.6° = 601.6°$ F. The efficiency of the ideal engine, working between these temperatures, if non-condensing, is $\frac{T_1 - T_2}{T_1} = \frac{771.8 - 679.4}{771.8}$ $= 12$ per cent., nearly; if condensing at an exhaust pressure of 3 pounds absolute, the efficiency is $\frac{T_1 - T_2}{T_1} = \frac{771.8 - 601.6}{771.8}$ $= 22$ per cent.

26. The increase of economy by the use of the condenser may be shown in another manner. Let *A B C D E F*, Fig. 18, be an indicator diagram from a non-condensing engine. *M N* is the atmospheric line and *O X* the vacuum line. The back pressure, as shown by the diagram, is *O S*. The area

of the diagram represents to some scale the work done per stroke. Now let a condenser be attached to the engine. The back pressure will be lowered to OT, the line HK, instead of DE, now being the lower line of the diagram, and $ABCHKL$ will be the new diagram, its area, as before, representing the work done per stroke. Hence, by adding a condenser to the engine, the work per stroke has been increased by an amount represented by the area $FEDHKL$, the steam consumption remaining the same. Suppose the steam to be cut off at a point P, making the area of the diagram, $APGHKL$, equal to the area of the original

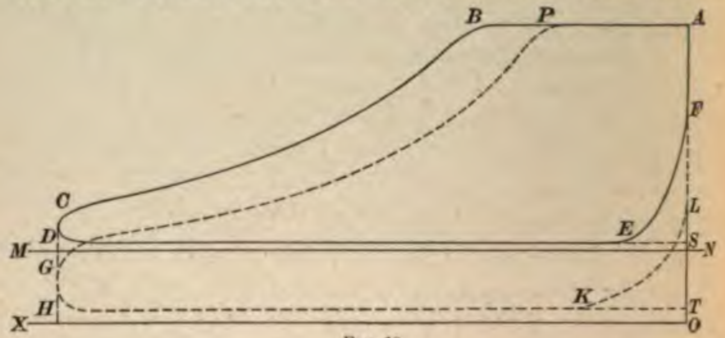


FIG. 18

diagram, $ABCDEF$. Then the work per stroke is the same in both engines, but the condensing engine uses an amount of steam per stroke represented by the length AP, while the non-condensing engine uses an amount represented by AB. Either case shows the economy of the condenser.

27. Types of Condensers.—There are two types of condensers in general use—the **surface condenser** and the **jet condenser.** In the former, the exhaust steam comes in contact with a large area of metallic surface which is kept cool by contact with cold water. In the latter, the exhaust steam, on entering the condenser, comes in contact with a jet of cold water. In either case, the entering steam is condensed to water, and in consequence a partial vacuum is formed. If a sufficient amount of cold water were used, the steam on entering would instantly condense and a practically

perfect vacuum would be obtained, were it not for the fact that the feedwater of the boiler always contains a small quantity of air, which passes with the exhaust steam into the condenser and therefore partially destroys the vacuum. To get rid of this air, the condenser is fitted with an air pump, which pumps out both the air and the water formed by condensation.

28. The Surface Condenser.—Fig. 19 is a perspective view of a Wheeler surface condenser; Fig. 20 is a sectional view of it. The cold condensing water is drawn from some water supply through *m*, and forced by the circulating

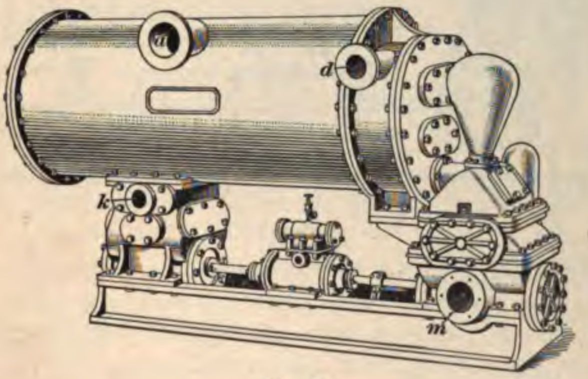


FIG. 19

pump *q* into the inlet *c* of the condenser. From *c*, the water is forced to the chamber *f*, and flows, as indicated by the arrows, through the inner tubes of the lower layer of double tubing to the left, and having passed through their entire length it returns through the space between the outside of the inner and the inside of the outer tubes into the chamber *g*. Fig. 21 shows more clearly the arrangement of this double tubing. From *g*, Fig. 20, it passes through *e* to *h*, and from *h* to *i* through the upper layer of double tubing, as already explained. From *i*, it is discharged through the nozzle *d*, Fig. 19, carrying with it all the heat it has received by coming in contact with the two layers of double tubing.

The nozzle at *a* is connected with the exhaust pipe of the steam cylinder of the engine. The movement of the air-pump piston *o* draws air through the orifice *b* from the condenser cylinder and discharges it through the valves and

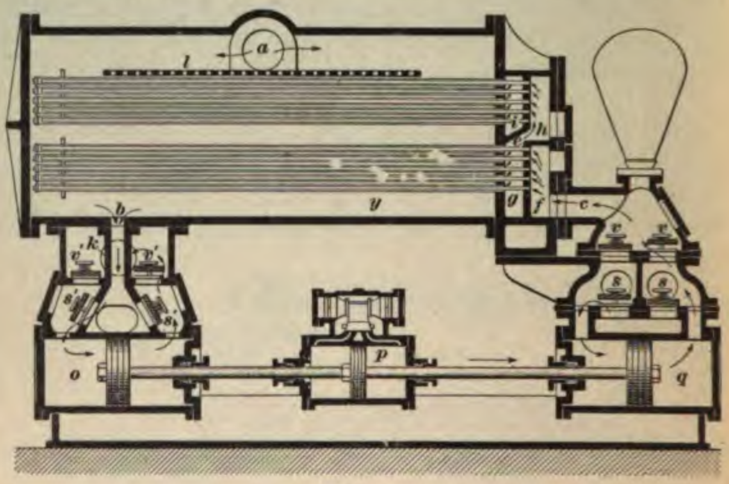


FIG. 20

nozzle *k* in the manner indicated by the arrows. The valves *s'* and *v'* are opened and closed automatically by the pressure of the air beneath them and by the pressure of the air and springs above them. A partial vacuum is created in

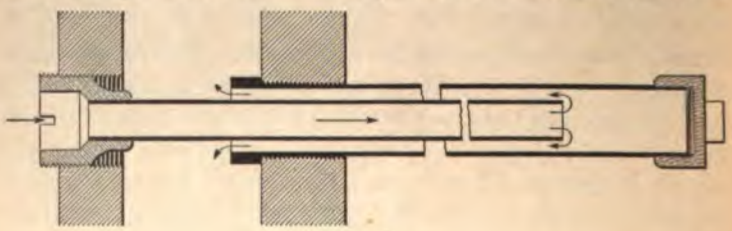


FIG. 21

the condenser cylinder *y* by the action of the air pump, thus sucking the condensed steam from the engine cylinder into the condenser cylinder.

As the exhaust steam enters the condenser cylinder through *a*, it first comes in contact with the perforated

scattering plate *l*, which protects the upper tubing from the damaging effect of direct contact with the exhaust steam. The steam comes in contact with the cold tubes, through which the cold water is being pumped, and condenses. As soon as this occurs, the condensed exhaust steam collects at the bottom of the condenser cylinder and runs through *b* into the air-pump cylinder, from which it is discharged, while still quite hot, and made use of as boiler feedwater. The temperature of this condensed exhaust steam will be less as the vacuum in the condenser cylinder is more complete. The advantage of making use of this condensed exhaust steam as boiler feedwater will be seen when it is remembered that it has a higher temperature than the ordinary feedwater, and will therefore require less fuel to reconvert it into steam.

In this condenser, the circulating and air pumps are run by an independent steam cylinder *p*. They are often connected directly to the main engine, and have their motion imparted to them by some of its moving parts, generally the crank-shaft.

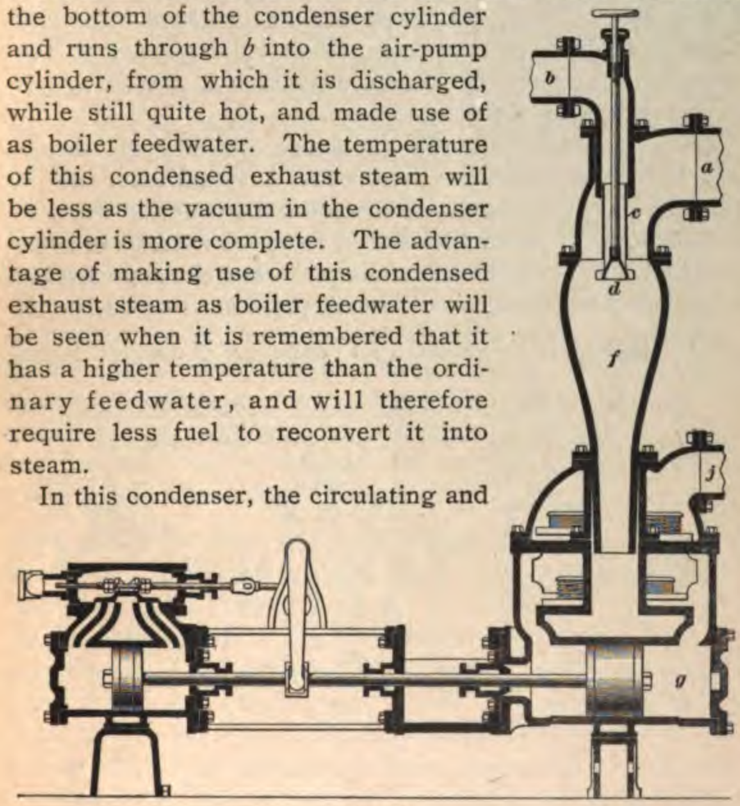


FIG. 22

29. The Jet Condenser.—In Fig. 22 is shown a section of a Worthington jet condenser. The cold water

enters the condenser at *b*, being drawn from the supply by the vacuum in the condenser, passes down the spray pipe *c*, and is broken into a fine spray by the cone *d*, by means of which the amount of injection water is also regulated. The exhaust steam comes in at *a* and, mingling with the spray of cold water, is rapidly condensed. The mixture of steam, water, and air is carried with a high velocity through the cone *f* into the pump cylinder *g*, whence it is forced by the pump through the discharge pipe *j*.

The jet condenser has the advantage of simplicity, cheapness, lightness, and small size; but the discharge water from it cannot be used for boiler feed, unless the injection water is itself fit to be so used. Where the injection water is fit to be used for boiler feed, the jet condenser is, therefore, preferable to the surface condenser; otherwise the latter may be better.

30. In Fig. 23 is shown a jet condenser connected to an

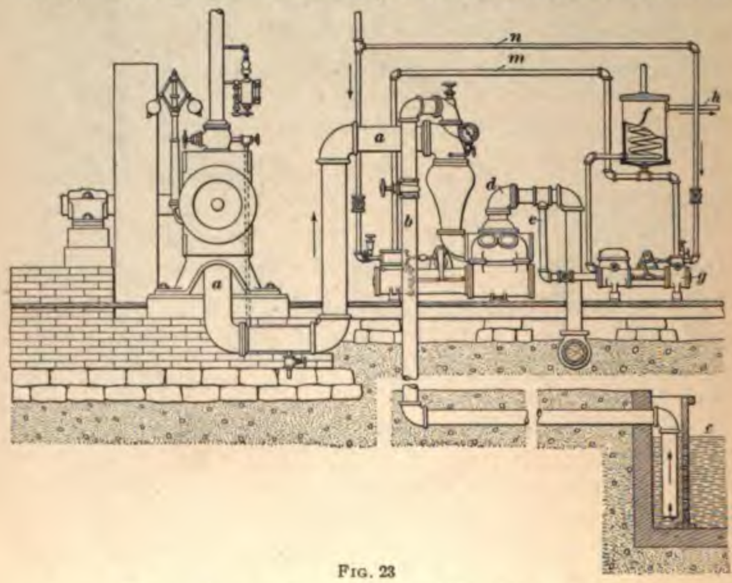


FIG. 23

engine. The exhaust pipe *a* leads directly to the condenser. The injection pipe *b* conveys water from the reservoir *c*. After the steam is condensed, the mixture of exhaust steam

and injection water is discharged through *d* into the sewer. A portion of this discharge, however, flows through *e* to the feed-pump *g*, which forces it through the coil in the heater *f* to the pipe *h* leading to the boiler. The exhaust from the two pumps is discharged into the feedwater heater through the pipe *m*. It will be noticed that water from the discharge pipe *d* enters the feed-pump under a slight head. This is because the water is heated by the exhaust steam, and hot water cannot be raised by a pump like cold water. A pipe *n* leading from the boiler supplies steam for both pumps.

31. The barometer column, or **siphon condenser,** is a type of jet condenser that differs from the common jet condenser in that no air pump is required. A circulating pump or head of water is needed to supply the injection water when the lift is more than 20 feet. The vacuum is produced by the condensation of the steam, and is maintained by a column of water flowing downwards through a vertical pipe of not less than 34 feet in length, having its lower end immersed in the water of the hotwell, into which the water is discharged.

32. An illustration and a description of an example of this type of condenser known as the Baragwanath condenser is here given.

Fig. 24 represents a sectional view, in which *a* is the exhaust pipe, *b* the injection pipe, *d* the long discharge pipe, or tail-pipe, and *e* the hotwell. The steam enters through exhaust pipe *a* and flows through the exhaust nozzle *f* into the condensing chamber *g*. Here it is met and condensed by the injection water that enters from the water-jacket *h* into the condenser in a thin conical sheet, flowing through the annular opening between the exhaust nozzle *f* and the prolongation of the shell of the condenser forming the inverted cone *i*. The injection water and water of condensation flow from the condensing chamber *g* through the throat *j* with such velocity as to carry with them the air that passes over with the steam and the injection water, and the uncondensed vapor. A vacuum is thus formed in the condensing

chamber g by the condensation of the steam and by the air and uncondensed vapor being carried out of the chamber by the stream of water.

The exhaust nozzle f is adjusted by means of the wheel and screw spindle k, and can be set so as to admit just the right quantity of injection water. An automatic atmospheric relief valve l is fitted for the purpose of discharging into the atmosphere any excessive amount of air, steam, or vapor that may accumulate in the exhaust pipe. A hotwell overflow or discharge pipe m is always connected to the hotwell.

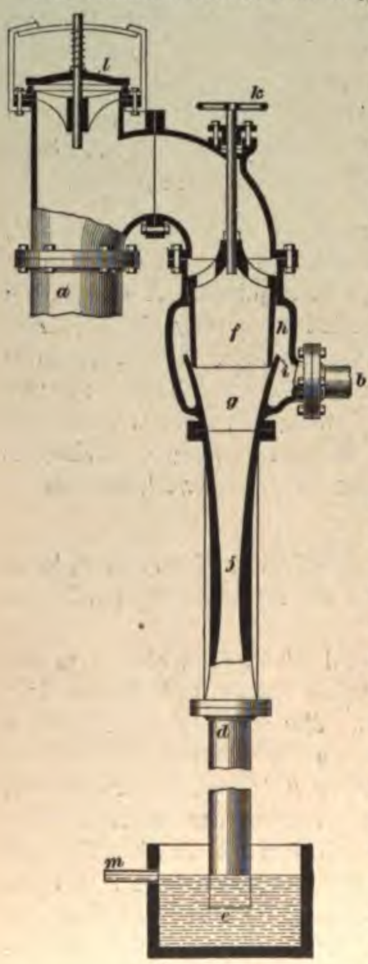


FIG. 24

33. Comparative Economies.—The important point to be considered as influencing the use of condensers is the greater economy of compound condensing engines as compared with ordinary non-condensing engines. The higher the cost of fuel and water, the greater will be the inducement to use condensing engines; but when the cost of fuel is low the interest on the cost of condenser equipment, pumping, maintenance, and attendance may equal the value of fuel and water saved; care should be taken, therefore, that this costly apparatus is installed only under conditions likely to render its use justifiable on the ground of economy. The

real gain by the use of condensers will vary according to the type of engines used, and can be estimated by comparing the pounds of steam per hour required to produce a horsepower with a condenser with the amount required to produce a horsepower without a condenser.

Table I shows the pounds of steam required by engines used in power stations under actual conditions. This table was originally prepared by the late Charles E. Emery, and has heen modified according to recent tests.

TABLE I

STEAM CONSUMPTION OF VARIOUS TYPES OF ENGINES

Type of Engine Name	Feedwater per Indicated Horsepower-Hour				Per Cent. Gained by Condensing
	Non-Condensing		Condensing		
	Probable Limits Pounds	Assumed for Comparison Pounds	Probable Limits Pounds	Assumed for Comparison Pounds	
Simple high-speed . . .	40 to 26	33	25 to 19	22	33
Simple low-speed . . .	32 to 24	29	24 to 18	20	31
Compound high-speed .	30 to 32	26	24 to 16	20	23
Compound low-speed .	25 to 18	25	20 to 12¾	18	28
Triple high-speed . . .	27 to 21	24	23 to 14	17	29
Triple low-speed . . .	24 to 17	20	18 to 12¾	15	25

CONDENSER CALCULATIONS

34. Pressure and Temperature.—The first point to be decided in the design of a condenser, is the degree of vacuum to be carried. If more than about 27 inches of vacuum is called for, the first cost of the condenser is very much increased. With the reciprocating engine, no practical gain in economy is to be expected from carrying the vacuum above 27 inches; for the condensed steam will then have a temperature less than 114° F., which is rather low for economical boiler feeding. Below this point, any heat extracted by the condenser will have to be replaced before the water is fed to the boiler. Hence, a lower vacuum than

27 inches is uneconomical and undesirable, unless, of course, the feedwater is taken from some other source than the condenser, or is warmed by heat that will otherwise go to waste. Even in such cases, it is doubtful whether the gain from the high vacuum is enough to justify the increased cost of the condenser; therefore, a condenser for a reciprocating engine would probably not be designed to carry more than 27 inches of vacuum.

On the other hand, the economy of a steam turbine is greatly increased by exhausting into a high vacuum, and a condenser for a steam turbine plant would therefore be designed for from 28 to 28.5 inches of vacuum. One point to be remembered in connection with the vacuum is that the temperature of the condensed steam discharged from the condenser is dependent on the pressure in the condenser. This temperature cannot be higher than that of saturated steam at the pressure in the condenser. The pressure in the vacuum chamber of a condenser is usually measured by a vacuum gauge, which resembles a steam gauge, but which records the difference of pressure, in inches of mercury, between the inside and the outside of the condenser. When the reading of the vacuum gauge is given in inches of mercury, the absolute pressure in the condenser may be found by the formula

$$p = \frac{14.7 \times (30 - r)}{30}$$

in which p = absolute pressure in condenser, in pounds per square inch;

r = reading of vacuum gauge, in inches of mercury.

When the absolute pressure of the steam in the condenser is known, the corresponding temperature can be found from the Steam Table.

EXAMPLE.—Find the highest possible temperature of condensed steam from a condenser carrying a vacuum of 26 inches.

SOLUTION.—Applying the above formula, $p = \frac{14.7 \times (30 - 26)}{30}$ = 1.96 lb. per sq. in. From the Steam Table, the temperature of saturated steam at 1.96 pounds per square inch absolute is 125.3° F.,

and this will be the highest temperature at which it will be possible to take the condensed steam from the condenser and carry a vacuum of 26 in. Ans.

JET CONDENSERS

35. Amount of Cooling Water Required.—The vacuum to be carried having been decided on, the next point to be determined, in the design of a jet condenser, is the quantity of cooling water required. This may be found by the formula

$$Q = \frac{H - (t - 32)}{t - t_1}$$

in which Q = number of pounds of cooling water required to condense 1 pound of steam;

H = total heat above 32° of 1 pound of steam at pressure at release;

t = temperature of mixture of cooling water and condensed steam on leaving the condenser;

t_1 = temperature of cooling water on entering the condenser.

EXAMPLE.—Steam is exhausted into a jet condenser from an engine cylinder at a pressure of 10 pounds absolute; the temperature of the cooling water on entering is 60° F., and on leaving 140° F. How much cooling water is required per pound of steam?

SOLUTION.—The total heat above 32° of 1 lb. of steam at 10 lb. absolute, from the Steam Table, is 1,140.9 B. T. U. Then, substituting the values of H, t, and t_1 in the above formula,

$$Q = \frac{1{,}140.9 - (140 - 32)}{140 - 60} = \frac{1{,}032.9}{80} = 12.91 \text{ lb. Ans.}$$

36. Size and Shape of Condenser.—The capacity of the condenser, that is, the volume of the chamber f, Fig. 22, should next be decided. If this is too small, there is danger that water will be thrown over to the engine cylinder, and it requires great care and watchfulness on the part of the attendant to prevent this; while, if the capacity is too large, it will take the air pump too long to create a vacuum. In practice, as a working compromise, the capacity is usually made from about one-fourth to one-half the volume of the cylinder that exhausts into the condenser. An average of

practice will probably be about one-third the volume of the cylinder. The smaller the cylinder and the higher the speed of the engine, the greater should be the ratio of condenser capacity to cylinder volume.

The necessary capacity can be put into any form that is convenient and desirable. Fig. 22 shows a common form; but jet condensers have been built in a great variety of forms to suit a great variety of circumstances. About the only points to consider in designing the form are that the inlet for exhaust steam is kept well above the point where the injection water discharges into the condenser, so as to minimize the danger of flooding the cylinder, and that the shape of the bottom will permit the water to drain away freely to the air-pump suction.

37. Velocity of Flow of Injection Water.—The velocity with which the water flows to the condenser should next be determined.

Let H = head at condenser, in feet;

h = height, in feet, to which condenser must lift injection water, which, in a condenser of the type shown in Fig. 22, is the vertical distance from the mouth of the internal pipe d to the level of the surface of the injection water supply;

p = absolute pressure in condenser, in pounds per square inch;

r = vacuum in condenser, in inches of mercury;

V = velocity of flow of injection water, in feet per second.

Then, as in Art. **34**, $$p = \frac{14.7 \times (30 - r)}{30} \qquad (1)$$

The difference of pressures inside and outside the condenser is therefore $14.7 - p$ pounds, which is equivalent to a head of water of $2.304\,(14.7 - p)$ feet, since each pound of pressure is represented by a column of water 2.304 feet in height. This would be the head of the condenser provided that the water supply was at the same level as the mouth of

the pipe d, Fig. 22. But if the water supply is lower, so that it must be lifted through a height h, in feet, the head will be decreased by that amount; so that

$$H = 2.304\,(14.7 - p) - h \qquad \textbf{(2)}$$

If the condenser is below the water level, the value of h becomes negative. According to *Kinetics of Fluids*, the velocity of flow of water under a head of H feet is

$$V = \sqrt{2gH} = 8.02\sqrt{H} \qquad \textbf{(3)}$$

EXAMPLE 1.—Find the theoretical velocity of flow into a condenser where h is 4 feet and the vacuum is 27 inches.

SOLUTION.—Applying formula **1**, $p = \frac{14.7 \times (30 - 27)}{30} = 1.47$ lb. per sq. in. Then, applying formula **2**, $H = 2.304\,(14.7 - 1.47) - 4 = 26.48$ ft. From formula **3**, then, $V = 8.02\sqrt{26.48} = 41.3$ ft. per sec. Ans.

EXAMPLE 2.—Find the theoretical velocity of flow into a condenser carrying 27 inches of vacuum, the point of discharge into the condenser being 11 feet below the water line.

SOLUTION.— $p = 1.47$ lb. per sq. in., as before. Applying formula **2**, $H = 2.304\,(14.7 - 1.47) - (-11) = 41.48$. Then, from formula **3**, $V = 8.02\sqrt{41.48} = 51.65$ ft. per sec. Ans.

38. The real velocity of flow of the water into the condenser will never be as great as just calculated, because of the resistance of the piping through which the water has to flow to reach the condenser. It is not necessary to know this diminution of velocity exactly, provided that a sufficiently great allowance is made for it. It is common practice to assume that the resistance of the piping will diminish the velocity to one-half its theoretical value, as calculated above, and this rule may be followed.

The velocity of flow into the condenser determines the necessary area of the injection orifice, because area of orifice × velocity of flow = volume of water passed through orifice in unit of time.

Let A = area of orifice, in square feet;
V = calculated or theoretical velocity of flow of water, in feet per second;
Q = number of pounds of injection water required per hour.

Then, $A V$ represents the number of cubic feet flowing per second, which is equivalent to 62.5 $A V$ pounds per second, or $3{,}600 \times 62.5\ A V$ pounds per hour. Hence, $Q = 3{,}600 \times 62.5\ A V$, or $A V = \frac{Q}{3{,}600 \times 62.5}$.

Assuming one-half the calculated velocity as the real velocity of the water, $\frac{A V}{2} = \frac{Q}{3{,}600 \times 62.5}$, whence

$$A = \frac{.0000089\ Q}{V} \qquad (1)$$

In the case of a circular orifice,

Let d = diameter of injection orifice, in inches.

Then $\frac{\pi d^2}{4} = 144\ A$, whence

$$d = 13.54\sqrt{A} \qquad (2)$$

The nearest size of standard pipe should be used.

In a case like that shown in Fig. 22, this size of pipe should be used for the internal injection pipe c, and the inside diameter of b should be made equal to the outside diameter of c.

The stem of the cone d should be given thread enough to open an area for the flow of water between the cone and the internal injection pipe equal to the cross-sectional area of the pipe b.

Let o = maximum opening of the cone d.

Then $\pi d o = \frac{\pi d^2}{4}$, whence

$$o = \frac{d}{4} \qquad (3)$$

The diameter of the exhaust nozzle a is that of the exhaust pipe of the engine.

SURFACE CONDENSERS

39. In the surface condenser, the exhaust steam and the injection water are kept separate throughout their course through the condenser; and the condensed steam leaves the condenser as fresh water, free from the impurities contained in the injection water. The water of condensation from a

surface condenser is therefore fit to be used as boiler feed, regardless of the quality of the water used to condense it. It is for this reason that the surface condenser, in spite of its greater complication, cost, size, and weight, as compared with the jet condenser, is used instead of the latter where the supply of injection water is unfit for use as boiler feed. Thus the surface condenser is used altogether in marine work, except for vessels navigating clean fresh water like that of the Great Lakes, in order to avoid the use of sea-water in the boilers.

40. The vacuum to be carried being known, the next point to decide, in the design of the surface condenser, is whether the steam shall be outside and the water inside the tubes, or the reverse. While each method presents some advantages, the balance is in favor of putting the steam outside and the water inside. This method is the common practice and may be followed in almost all cases. About the only exception is where every effort must be made to keep the engine room as cool as possible. In this case, the cold injection water against the outer shell tends to keep the room cool, and the water may well be put outside and the steam inside.

If the water is inside the tubes, it should enter at the bottom of the condenser and be discharged at the top. This brings the coldest water into contact with the partially condensed steam, and the warmest water into contact with the hot entering steam. When the water is outside the tubes, it is necessary to fit baffle plates on the water side to force the water into a definite and regular circulation, and to prevent it from going directly from inlet to outlet; also to prevent the water from arranging itself in layers according to temperature, with the coldest water on the bottom and the hottest water on top. The outlet should be well above the top row of tubes. A solid body of water above the top row of tubes is thus assured, and the accumulation of a stagnant body of hot water in the top of the condenser is prevented by its being continually drawn off by the circulating pump and replaced by cooler water from beneath.

Air tends to accumulate in the top of the water side of a surface condenser. This is particularly inconvenient where the water is inside the tubes, as the air fills the top rows of tubes and excludes the water, destroying their value as cooling surfaces. To prevent this, an air valve must be provided, as high up on the water side as possible, in all surface condensers, by which the air can be drawn off whenever it becomes troublesome. Drain valves and pipes should be provided at the bottom of both the air and the water side.

41. As the condensed steam from the surface condenser is generally pumped back into the boiler as feedwater, it is desirable to have it as hot as possible; but it must be remembered that it is impossible to get the feedwater from the condenser at a higher temperature than that of saturated steam at the absolute pressure existing in the condenser.

It will be considerably cooler than this if, after being condensed, it is allowed to lie in the bottom of the condenser and give up its heat to the circulating water. The heat thus given up is a total loss, and should be avoided by connecting the air-pump suction to the lowest point of the condenser and by shaping the bottom of the condenser so that the water will drain rapidly into the air-pump suction.

42. Amount of Cooling Water Required.—The amount of cooling or injection water required in the case of a surface condenser may be found by the formula

$$Q = \frac{H - (t - 32)}{t_2 - t_1}$$

in which Q = number of pounds of cooling water required to condense 1 pound of steam;

H = total heat above 32° of 1 pound of steam at pressure at release;

t = temperature of condensed steam on leaving condenser;

t_1 = temperature of cooling water on entering condenser;

t_2 = temperature of cooling water on leaving condenser.

EXAMPLE.—Steam exhausts into a surface condenser from an engine cylinder at a pressure of 6 pounds absolute; the temperature of the condensing water on entering is 55° F., and on leaving it is 100° F.; the temperature of the condensed steam on leaving the condenser is 125° F. How many pounds of cooling water are required per pound of steam?

SOLUTION.—The total heat of 1 lb. of steam at 6 lb. absolute, from the Steam Table, is 1,133.8 B. T. U. Then, substituting the values of H, t, t_1, and t_2 in the above formula

$$Q = \frac{1{,}133.8 - (125 - 32)}{100 - 55} = \frac{1{,}040.8}{45} = 23.13 \text{ lb. Ans.}$$

43. Cooling Surface.—It must now be decided whether to have the tubes horizontal, vertical, or inclined. The cooling surface is most efficient when the tubes are horizontal, and therefore they should always be so arranged, except when the best utilization of the space available compels a departure from this practice. The amount of cooling surface required can now be calculated. The following formula, known as **Whitham's formula,** is most commonly used for this purpose.

Let L = latent heat, in B. T. U., of saturated steam at the temperature T;

S = cooling surface, in square feet;

T = temperature, in degrees Fahrenheit, of saturated steam at absolute pressure existing in condenser;

t = mean temperature, in degrees Fahrenheit, of condensing water, which may be taken as the average of its temperatures on entering and leaving condenser;

W = total number of pounds of steam entering condenser per hour.

Then,
$$S = \frac{WL}{180\,(T - t)}$$

EXAMPLE.—Find the cooling surface required for the surface condenser of a triple-expansion engine of 6,000 I. H. P., total steam per hour, 96,000 pounds; vacuum, 26 inches of mercury; injection water raised from 70° F. to 100° F. in the condenser.

SOLUTION.—From the formula in Art. **34**, $p = \frac{14.7 \times (30 - 26)}{30}$ = 1.96 lb. By the Steam Table, T = 125.3° F. and L = 1,026.8 B. T. U.

$t = \frac{100 + 70}{2} = 85°$ F., and $W = 96{,}000$ lb. Therefore, substituting in the formula $S = \frac{W L}{180\,(T - t)}$,

$$S = \frac{96{,}000 \times 1{,}026.8}{180 \times (125.3 - 85)} = 13{,}589 \text{ sq. ft., nearly. Ans.}$$

44. Tubes.—The amount of cooling surface found necessary by the above formula should be provided in the tube surface exclusively, no value being attached to the surface of the tube-sheets. The tubes are always of brass. When the shell of the condenser is of cast iron, the tubes are sometimes tinned; but the benefit to be derived from tinning is not usually worth the expense. If the tubes are placed vertically, the amount of cooling surface should be at least one-third greater than that calculated by the formula in Art. 43. Where the tubes are screwed into the tube-sheets, or are otherwise tightly held in the tube-sheets, their unsupported length should not be more than 120 times the external diameter of the tubes. Where the ends are not tightly held, but are merely in stuffingboxes, the unsupported length of the tubes should not be more than 100 times their external diameter. If it is necessary to put the tube plates farther apart than this, the tubes must be supported at intermediate points, so as to keep the unsupported length down to the allowable maximum.

If the water circulates inside the tubes, the arrangement of tubes must be such that no injection water can escape from the condenser without passing through about 20 feet of tubing, while its velocity of flow through the tubes should not exceed about 400 feet per minute, as it will not become so heated that the minimum water supply will suffice if it goes through at a greater speed. The selection of an appropriate diameter, thickness, and length of tube must be made by trial.

45. Tube Fastenings.—The method of securing the tube ends in the tube-sheet must next be decided. Expanding the tubes, as in the case of boiler tubes, has been tried, but has proved a failure and has been abandoned, because both

the tubes and the tube plates are too soft to stand such treatment. The tubes are sometimes provided with a raised thread on at least one end, which end is screwed into the tube-sheet; but the usual method is to pack the tube ends in some way.

About the cheapest way of packing the tube ends is to drill the tube holes from $\frac{3}{16}$ to $\frac{1}{4}$ inch larger in diameter than the outside diameter of the tubes. In each hole is fitted, about the tube end, a ferrule or ring of very dry soft wood, of an easy driving fit. The ferrules need not project more than $\frac{1}{4}$ inch beyond the face of the tube plate on each side. When wet, the ferrules swell and pack the tube ends very tightly. If the condenser is allowed to stand dry for a considerable time, the ferrules are apt to shrink and come out of the holes. This is the principal objection to their use. Generally, a method that packs each tube separately is to be preferred to one that does not.

The method of tube packing employed determines the spacing of the tubes. With wooden ferrules, the pitch of the tubes, that is, the distance from the center of one tube to the center of the next, must be from $\frac{7}{16}$ to $\frac{1}{2}$ inch greater than the outside diameter of the tubes. With screwed glands, the tubes can be set closer, the pitch being from $\frac{3}{16}$ to $\frac{5}{16}$ inch greater than the outside diameter of the tubes. The usual arrangement of the tubes is in rows that make an angle of 60° with each other, as this is the most economical of space.

46. Condenser Shell.—The form of the condenser must next be decided. While the surface condenser does not lend itself so readily as the jet condenser to any desired form, many forms have come into use. The most common forms of the surface condenser are rectangular and cylindrical. The latter form is the lightest and also the cheapest, in consequence of the ease of manufacture of circular forms; but where it is necessary to save space, the rectangular form has the advantage. As soon as the diameter and spacing of the tubes and the form of the condenser have been determined, the necessary size of the condenser shell can be worked out on the drawing board by laying out the required number of

tubes at the necessary pitch and enclosing them by a perimeter of the desired form. The perimeter thus found shows the necessary cross-section of the condenser. Very little clearance need be allowed between the outer tubes and the condenser shell. If necessary, the outer tubes may have their centers no farther from the shell than the pitch of the tubes.

The shell may be either of cast iron or of brass. Cast iron is cheap and answers the purpose; but it is more subject to corrosion than brass, and so has to be thicker and heavier. Where economy in weight is important, brass shells are generally used. A brass shell may be either cast or made of brass sheets rolled to shape and riveted and brazed together. Wrought iron and steel should be altogether excluded from the condenser structure, because they corrode too rapidly.

47. Tube Plates.—The plates in which the ends of the tubes are held should be of brass, which may be either cast or rolled, the latter being the tougher and stronger. With wooden ferrules, or any other kind of packing that extends clear through the plate, the plate should be 1.625 times the outside diameter of the tube in thickness.

When the tube plate is of large area, it should be supported by brass or bronze stays running through the waterway to the adjacent head, to which the tube plate is tied. The stays do not pass through the air side of the condenser. The total cross-sectional area of the stays should be sufficient to carry the total load on the tube plate, which should be calculated as that due to the difference between the pressure of the head of water on the water side and the vacuum on the air side, with a factor of safety of about 6. The stays should be so distributed over the tube plate that each will support an area proportional to its own cross-section.

COOLING TOWERS

48. The **cooling tower** is a device used to lower the temperature of the injection water after it leaves the condenser, so as to allow the water to be used again and again for condensing purposes. It is so built as to divide the warm discharge from the condenser into a great number of thin sheets, or into sprays, thus exposing a very large amount of surface to the air, which does the cooling.

The cooling effect of the air is twofold. In the first place, it absorbs some of the heat from the water during their close contact with each other and the temperature of the water decreases while that of the air increases. In the second place, the air has an evaporative effect on the warm water, carrying a portion of it away in the form of vapor. The heat required to cause this evaporation comes from the warm water, and its temperature is thus lowered.

The mechanical subdivision and distribution of the water for cooling is obtained by different methods; as by passing it over suspended galvanized-wire mats; over many courses of thin vertical boards laid up like cribwork; or over many series of thin pipes. The amount of heat extracted from the water in passing through the cooling tower will vary according to the atmospheric temperature, the surface of water exposed, and the volume and velocity of the air passing through the tower; the reduction in temperature will be from 30° to 45° F., and sometimes even more. It will be necessary to add from time to time a sufficient amount of fresh water to replace the loss caused by evaporation. It will thus be seen that a plant can operate in almost entire independence of a city water supply. With any cooling tower, where the feedwater for the boilers is taken from that circulating through the tower, it will be necessary to utilize an oil extractor to remove the oil before the water is pumped into the boilers.

The following table shows the loss of water due to evaporation, the fall of temperature obtained, etc.

TABLE II

RESULTS OF TESTS ON COOLING TOWERS

	First Test	Second Test	Third Test	Fourth Test
Temperature of air entering tower, in degrees Fahrenheit	54.70	85.70	84.60	87.00
Atmospheric humidity, per cent. . .	37.50	51.00	63.00	42.60
Temperature of water delivered to tower in degrees Fahrenheit . . .	134.50	135.20	136.40	135.25
Temperature of water leaving tower, in degrees Fahrenheit	94.90	104.20	101.60	90.75
Number of degrees Fahrenheit water was cooled	39.60	31.00	34.80	44.50
Pounds of water supplied to tower .	12,531	12,508	10,713	6,304
Pounds of water lost by evaporation while passing through tower . . .	424	363	341	263

49. Air Circulation.—Three methods for obtaining air circulation through cooling towers are commonly used:

1. The air is forced in rapid circulation through the tower by means of a fan blower, which discharges fresh air into the lower part of the tower, whence it is deflected upwards through the film or spray of condensing water.

2. A chimney or vertical flue, rising from 50 to 100 feet above the cooling compartment of the tower, creates sufficient natural draft to draw a large volume of air through the cooling compartment, and allows the vapor from the cooling water to be carried off from the top of the chimney. This type of tower entirely avoids cost for daily operation of fans.

3. Where the ground space is sufficient, the cooling surface of the tower is distributed over a larger area, allowing free circulation of the air naturally through the tower; this also avoids the use of any mechanical means to circulate the air, and saves the cost of driving fans.

50. The Worthington Cooling Tower.—In the tower shown in Fig. 25, known as the Worthington cooling tower,

the water runs over the inside and outside surfaces of a large number of cylindrical tubular tiles c, c', c'' that rest on a grating d supported by a brick wall e extending around the tower inside the shell a. The warm discharge water from the condenser enters the tower through the pipe f, passes up the central pipe g, and is delivered on the upper layer of tiling and over the whole cross-section of the tower by the distributing device h, which consists of four pipes, radiating from the central pipe g, which are caused to revolve about the central pipe by the reaction of jets of water issuing from perforations on one side of each pipe. The water thus delivered spreads over the outside and inside surfaces of the walls of the tiling and presents a continuous sheet to the action of the air. The air is circulated by the fan b, driven by a small engine or electric motor. The air drawn in by the fan is deflected upwards by the plate l. The cooled water collects in the reservoir i, from which it is drawn off through the pipe j; m is a manhole to give access for inspection or repairs, and k is an overflow pipe from the reservoir.

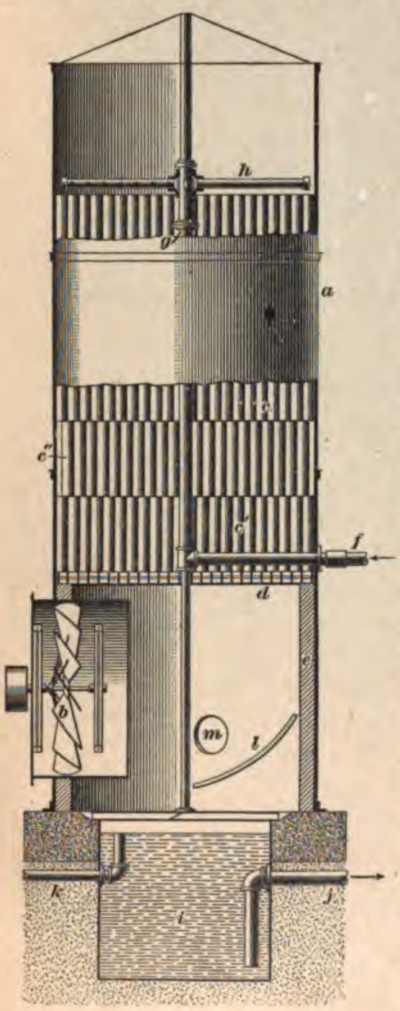


FIG. 25

STEAM TURBINES

(PART 1)

DESCRIPTION OF TURBINES

CLASSIFICATION OF TURBINES

1. General Principles.—The **turbine** is a machine by which the energy of a moving fluid, as steam or water, is transformed, producing rotary motion. The rotating part of the turbine is cylindrical in form, comprising a shaft carrying a wheel, to which are fastened blades, also called *vanes* or *buckets*, against which the moving fluid impinges. This wheel, known as a *turbine wheel*, is enclosed in a casing.

The energy of the steam, as it enters a steam turbine, may be in either of two forms, pressure energy or velocity energy. The **velocity energy**, that is, the kinetic energy, of steam is the energy that it possesses on account of its velocity and which the steam gives up to the turbine wheel, causing it to rotate. The **pressure energy** is the energy that is given up by the direct pressure of the steam against a resistance as the steam expands and does work. In the steam engine, the work is done on a piston; in the steam turbine, it is done by steam pressing against the vanes of the turbine wheel. These two kinds of energy furnish a basis for the classification of steam turbines.

2. Classification of Steam Turbines.—There are two principal classes of steam turbines—*velocity turbines* and *pressure turbines*. The distinction between these two classes may be shown by the diagrammatic sketches, Figs. 1 and 2.

In Fig. 1, the jet of steam from the nozzle *a* strikes the buckets of the turbine wheel *b*, causing the wheel to rotate on its axis. It will be noticed that the buckets may not be entirely filled with steam and that the steam may not be delivered to the entire circumference of the wheel at the same instant. Furthermore, it will be noted that as the steam issues from a nozzle into the chamber of the wheel in the form of a jet there will be no pressure change in the turbine buckets. The jet of steam necessarily leaves the nozzle at a high velocity and consequently the turbine wheel turns at a high velocity. Steam turbines of this class are therefore called **velocity turbines.**

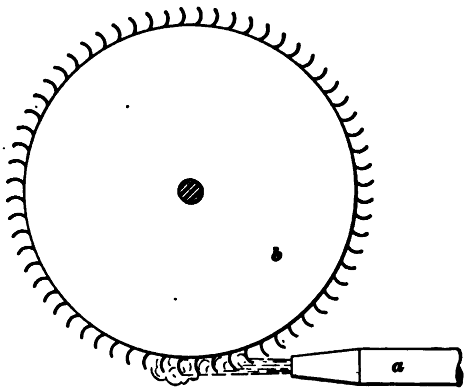


FIG. 1

In Fig. 2, the steam enters the space *a*, flows out through the guide vanes *b*, and then through the vanes of the moving wheel *c*. The wheel *c* is thus caused to rotate in the direction of the arrow *d*. In this type of wheel, it will be observed that the vanes run full of steam, and that there is a continual fall of pressure from the entrance of the steam until it leaves the turbine wheel. Steam turbines of this class are called **pressure turbines.**

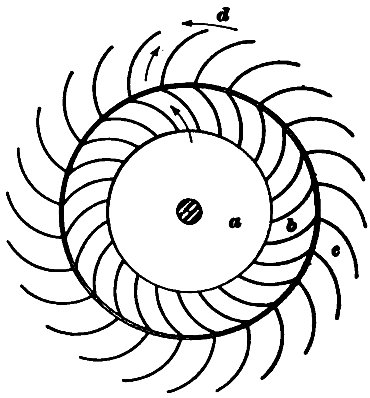


FIG. 2

Fig. 2, however, is introduced simply to illustrate the principle of the pressure turbine. In the actual turbine, the

guide vanes and the moving blades are arranged in alternate circles about the turbine axis, the direction of the flow of the steam being axial instead of radial, as will be explained later.

Velocity turbines may be subdivided as follows, according to the number of stages in which the steam is expanded: (1) *Single-stage expansion velocity turbines*, in which all the expansion of the steam takes place in a single stage in one set of nozzles, as in the De Laval steam turbine; (2) *few-stage expansion velocity turbines*, in which the steam is expanded in two, three, four, or five stages, as in the Curtis steam turbines; (3) *multiple-stage expansion velocity turbines*, in which the steam is expanded in many stages, as in the Rateau steam turbines.

The pressure type of steam turbine is always a multiple-stage expansion turbine, the number of stages reaching fifty to one hundred. The Parsons steam turbine is a representative of this class.

In referring to the various stages of expansion in a turbine, it is customary to omit the term "expansion" and speak of the single-stage velocity turbine (instead of the single-stage expansion velocity turbine); the few-stage velocity turbine; the multiple-stage velocity turbine.

3. Single-Stage Velocity Turbine.—The diagrammatic sketches, Figs. 3 to 6, will serve to explain the various steam turbines mentioned in the above classification. Fig. 3 represents the **single-stage velocity turbine**. The steam at a pressure p_1 is admitted to a nozzle shown at a, in which it expands to the final pressure p_2, and then passes into the chamber where the

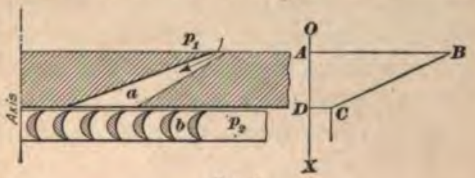


FIG. 3

turbine wheel b is located. The drop from the boiler pressure to the final condenser pressure is thus obtained in one or more nozzles. The pressure change is shown in the diagram at the right of Fig. 3. The line BC, laid off from OX

as a base line, shows the pressure change in the nozzle, AB representing the initial pressure p_1 and DC the final pressure p_2.

4. Few-Stage Velocity Turbine.—The **few-stage velocity turbine** is shown in Fig. 4. In this type, the steam expands in stages through the nozzles a_1, a_2, a_3, fixed in the partitions b_1, b_2, and b_3; there may be as many as four or five partitions. As there are only a few expansions, the drop in pressure in each is considerable, and the steam will enter the wheel chamber at a relatively high velocity. Instead of using the velocity energy in a single wheel, as in Fig. 3, two or more wheels are employed. Thus, the steam expands from p_1 to p_2 through a_1 and passes through the vanes w_1 on the first wheel at a very high velocity. This wheel runs at a slower speed than would be the case if only one wheel were used, so that the steam leaves the wheel with considerable velocity. The steam then passes through a set of fixed guide vanes g_1 and enters the vanes w_2 on another wheel, where a large part of the remaining velocity energy is given up. It then passes through the nozzle a_2 in the partition b_2, expanding to the pressure p_3 and increasing its velocity. The steam next passes through the wheel vanes w_3, the guide vanes g_2, the wheel vanes w_4, and expands through the nozzle a_3 to the pressure p_4 for the next stage.

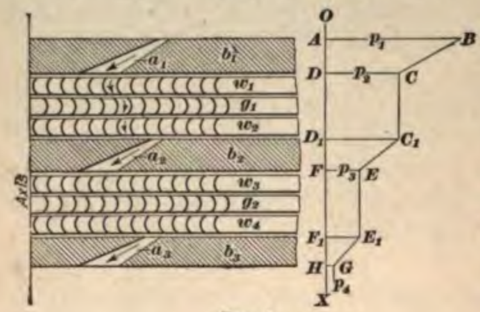


Fig. 4

The pressure remains constant in passing through the wheels, all the expansion taking place in the nozzles. The pressure changes are shown in the diagram at the right of Fig. 4, the pressures being laid off on lines at right angles to OX. AB represents the initial pressure, BC the expansion

from p_1 to p_2, $D C$ and $D_1 C_1$ the constant pressure p_2 in the first wheel chamber, $F E$ and $F_1 E_1$ the constant pressure p_3 in the second wheel chamber, and $H G$ the pressure p_4 in the third wheel chamber.

5. Multiple-Stage Velocity Turbine.—In the **multiple-stage velocity turbine** illustrated by the diagram in Fig. 5, the steam passes through nozzles a_1, a_2, a_3, etc. in the several partitions, which divide the interior of the turbine into wheel chambers, or cells. In each cell is a single wheel carrying radial buckets. In passing through the first nozzle, a_1, the

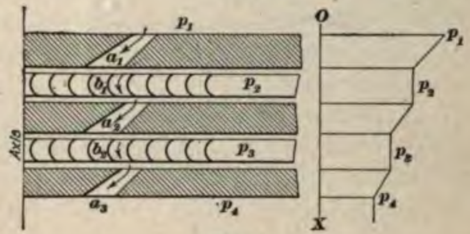


FIG. 5

steam expands from the pressure p_1 to p_2, acquiring a high velocity, which it imparts to the wheel b_1. The steam then expands from p_2 to p_3 through the nozzle a_2, increasing in velocity, and then strikes the vanes of the turbine wheel b_2; and so on through the other stages. There being a large number of stages in turbines of this type, much lower rotative speeds may be used than in single-stage velocity turbines.

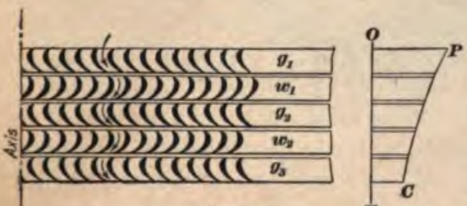


FIG. 6

The pressure changes are shown in the diagram at the right of Fig. 5, the pressures being laid off on lines at right angles to $O X$, as in the previous cases. The pressure drops from p_1 to p_2 in passing through the nozzle a_1, remains constant in the first chamber, drops from p_2 to p_3 in the nozzle a_2; and so on through the turbine.

6. Pressure Turbine.—The action of the pressure turbine is illustrated by Fig. 6. The steam passes in

succession through sets of guide vanes g_1, g_2, etc., and moving wheel vanes w_1, w_2, etc. The pressure drops continually through both stationary and moving vanes, as shown by the diagram to the right of Fig. 6, where the pressures from P to C are laid off at right angles to OX. The vanes run full of steam, whereas, in the velocity turbines, Figs. 3, 4, and 5, the blades need not be full of steam and the steam need not be admitted to the entire circumference of the turbine wheel.

It will be noted that in all the turbines here described the general direction of the flow of steam is parallel to the axis. The turbines in Figs. 4 and 5 have the buckets mounted on wheels that rotate in the cells, and are sometimes called *multicellular turbines*. In the turbine of Fig. 6, the movable vanes are inserted in the surface of a drum that is mounted on a shaft, while the stationary vanes are fixed in the casing that surrounds the drum. In the following articles, typical turbines of the different classes will be described.

THE DE LAVAL STEAM TURBINE

7. Action of the De Laval Steam Turbine.—The **De Laval steam turbine** is a single-stage turbine of the velocity type, having a single wheel carrying one row of radial buckets. The steam is delivered to the buckets in jets from nozzles placed at intervals around the circumference of the wheel, these being so shaped as to deliver the steam at the highest velocity attainable. Fig. 7 shows the general arrangement of the wheel, nozzles, and buckets. The turbine wheel is shown at *a* and the shaft, to which the wheel is rigidly attached, is shown at *b*. The buckets are shown at *c* with the outer ends forming a rim to prevent the steam from escaping in a radial direction. The outer ends of the buckets are represented broken away for a short distance in order to show to better advantage the form of the buckets and the way the steam passes between them. The turbine wheel is enclosed in a casing that is surrounded by a steam chamber. From this chamber, the steam enters tapered nozzles *d* and

expands as it passes through them and out of the large ends to the buckets. The nozzles are placed around the wheel so as to give a uniform distribution of the steam. They are beveled at the ends nearest the wheel, so as to fit closely to it and deliver the steam to the buckets without unnecessary waste.

During the expansion in the nozzle, the pressure of the steam falls until it is equal to the back pressure; that is, the total expansion of the steam occurs in the nozzle, and at the points where the jets impinge on the vanes the steam

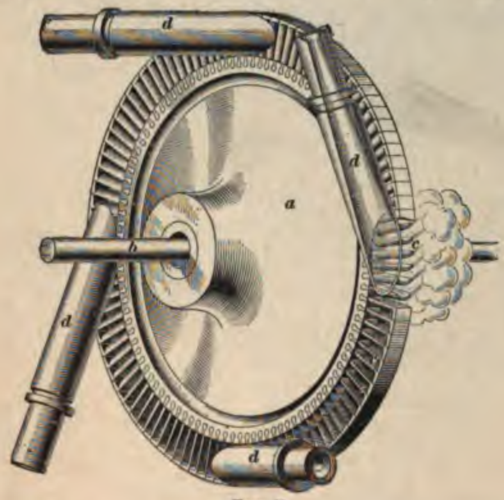
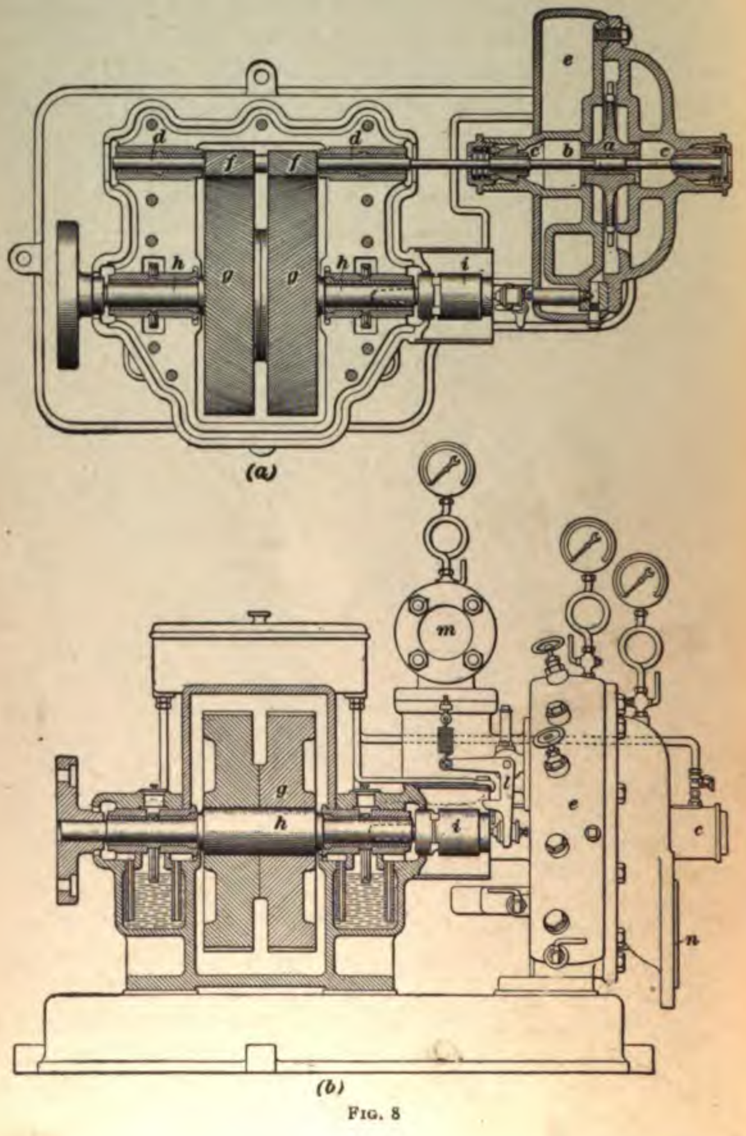


FIG. 7

pressure is equal to the pressure of the atmosphere, or of the condenser, as the case may be.

8. General Description.—The general arrangement of the parts of the De Laval steam turbine is shown in the sectional views in Fig. 8. The turbine wheel *a*, Fig. 8 (*a*), is mounted on the flexible shaft *b* running in the three bearings *c*, *d*, *d* and the so-called flexible bearing *c'*. The bearings *d*, *d* are rigid; the bearing *c*, however, is fitted in such a manner that it can accommodate itself to the bending of the shaft. The flexible bearing *c'* does not support the



(a)

(b)

FIG. 8

shaft, but merely acts as a stuffingbox to prevent the air from entering the wheel chamber when the turbine is running condensing, or the steam from escaping when running non-condensing. The casing *e* covers the turbine wheel.

The turbine wheel has such a high rotative speed that ordinary machinery cannot be connected directly to the shaft; hence, gearing is used to reduce the speed to a practical limit. The turbine shaft carries two pinions *f*, *f* that mesh with the gears *g*, *g*. These gears usually have ten times as many teeth as the pinions, thus making the speed of the gear-shaft one-tenth the speed of the turbine wheel. The gears have helical teeth, one set being right-hand and the other left-hand, as shown. By this difference in the inclination of the teeth, the end thrust of the shaft is practically eliminated. The gears are enclosed in a case, which also supports the bearings for the gear-shaft *h* and the receptacles for the oil for these bearings, as shown in Fig. 8 (*b*). The machine to be driven by the turbine is connected to the shaft *h*, either directly or by belt from a pulley on this shaft.

On one end, the shaft *h* carries a centrifugal governor *i*, which operates a double-seated throttle valve through a bell-crank lever shown at *l*. Steam is admitted to the turbine through the opening *m*, the exhaust passing out at *n* either to the atmosphere or to the condenser.

9. Steam Nozzles and Buckets.—Each nozzle of a De Laval turbine is set so that the steam enters the turbine

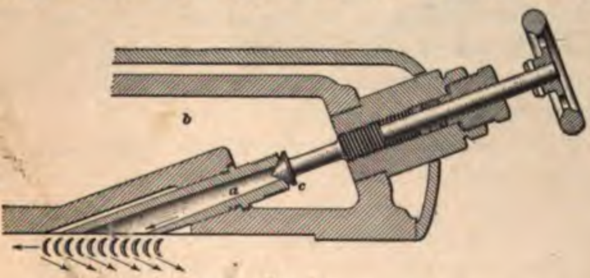


FIG. 9

at an angle of 20° to the plane of rotation of the turbine wheel. Fig. 9 shows a detail of the nozzle and its relation

to the buckets or blades; the nozzle, which is shown at *a*, is bored cylindrical for a short distance on the large end and with a uniform taper for the remainder of the length. The nozzles are stationary, uniformly distributed around the steam chamber *b*, and each is provided with a valve *c* for regulating the admission of the steam. Some or all of the nozzles may be used, according to the power required.

Fig. 10 shows the general appearance of the buckets as attached to the wheel. They are drop forgings, and the dovetail shank *b* is forced into a slot cut in the wheel rim. The center line of the shank is radial and the outer ends *a* of the buckets just touch each other. The inlet and outlet angles are made the same in the De Laval bucket, being 32° for the smaller sizes and 36° for the larger sizes. With these angles fixed, it is found that the best theoretical peripheral velocity of the wheel is about 950 feet per second for a steam velocity of 2,000 feet per second. Higher velocities bear about the same relation to each other.

FIG. 10

10. Velocities of the De Laval Turbine.—In the De Laval turbines that have been built, the actual peripheral velocity varies from about 1,400 feet per second in the larger sizes to about 500 feet per second in the smaller sizes. In comparison with existing machinery and other types of steam turbines, these velocities are exceedingly high.

The diameters of the turbine wheels differ according to the power they are to develop, and hence the number of revolutions must vary in order that the linear velocity at the circumference shall not be excessive. The speeds vary, in practice, from 10,000 to 30,000 revolutions per minute. These speeds are reduced approximately to one-tenth this amount by the helical gearing, giving a driving speed of from 1,000 to 3,000 revolutions per minute.

Fig. 8 shows a turbine of small horsepower using gears on one side of the pinions, as provided in the smaller types. In the larger types, it is necessary to use two sets of gears, one set on either side of the pinions. The pressure between the gears and pinions is thus divided, so that one-half the pressure is taken by each set of gears. Thus, the gear-pressure on one side of the pinion balances that on the other

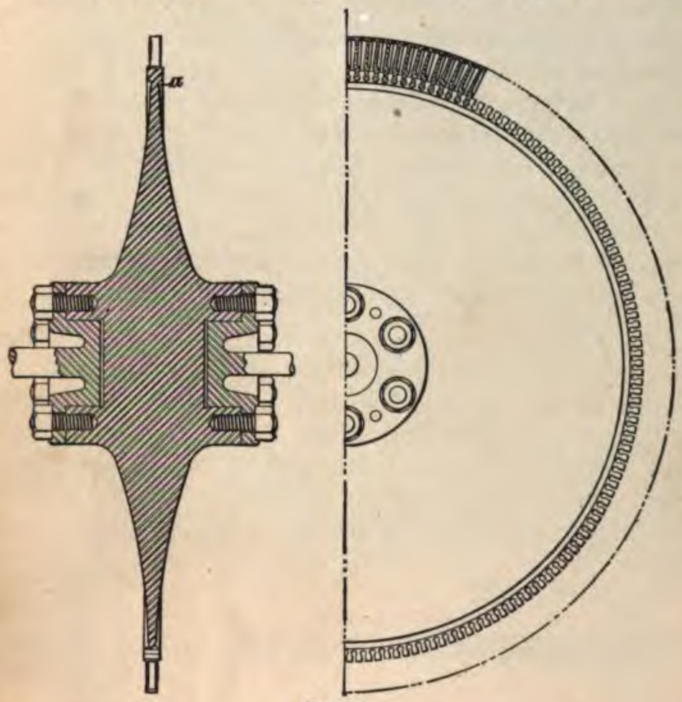


FIG. 11

side, and the pressure on the pinion bearings is greatly reduced.

11. De Laval Turbine Wheel.—In order to guard against possible damage from racing, the thickness of the wheel close to the periphery is greatly reduced, as shown at *a*, Fig. 11. The strength of the wheel is thus reduced, and it will rupture at this point first. If the wheel should

burst here, the rim holding the blades would be broken up into small pieces, which, on account of their size, would do little damage. At the moment the rim leaves the wheel, the stresses in the solid wheel body are considerably reduced. At the same time, the wheel becomes unbalanced and as the clearance around the hub is small, as shown in Fig. 8 (*a*), the hub comes in contact with the casing, which acts as a brake.

The high speeds have their influence also on the form of the hub of the wheel. In the smaller sizes of turbine, the wheel is forced on a tapered sleeve shrunk on the shaft, both shaft and sleeve passing entirely through the hub. In all sizes, the hub must be very heavy to resist the forces acting on it. In large turbines, this is especially important; and in order to have sufficient strength, the hub is not cut away for the shaft to pass through, the shaft being fastened to the ends of the hub by bolts, as shown in Fig. 11.

12. Flexible Shaft and Helical Pinions.—The flexible shaft and helical pinions are shown in Fig. 12. On account of the very high velocities, the shaft can be made very small. In the 5-horsepower turbine, for example, the shaft is only about ¼ inch in diameter at the smallest section, shown at *a*. This renders the shaft quite flexible and makes the turbine self-balancing, to a certain extent, at high

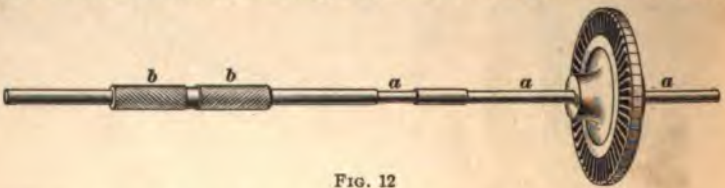


FIG. 12

velocities. The double helical pinions shown at *b*, *b* are intended to reduce the speed with the least possible vibration. The flexible shaft and helical gears are made of the best material. Great accuracy is also necessary in the form of the gear-teeth, but great strength is not required, as the force exerted by the teeth is not excessive, owing to the high velocities. The spindle of a 5-horsepower turbine,

for example, has a normal speed of 30,000 revolutions per minute, and the diameter of the pinion is only $\frac{3}{4}$ inch. The linear velocity of the teeth is therefore about 100 feet per second, and when delivering 5 horsepower the tangential force is only about $27\frac{1}{2}$ pounds.

13. Governor and Vacuum Valve.—The construction of the De Laval governor is shown in Fig. 13. It is a

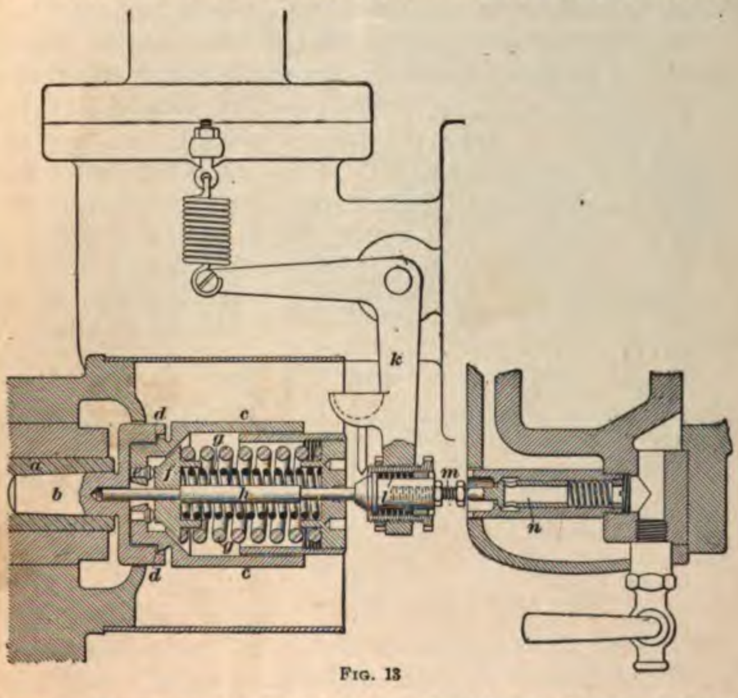


FIG. 13

spring governor attached to the end of the shaft *a* by a taper shank *b*, and carries the weights *c*, *c* hung on knife-edge bearings at *d*, *d*. These weights, which are semi-cylindrical, fly outwards under the action of centrifugal force, turning on the knife-edge pivots, and thus push the pins *e* to the right. These pins, bearing against the collar *f*, compress the springs *g*, *g* until their resistance is equal to the pressure caused by the centrifugal force. The stem *h* is at the same

time carried to the right, rotating the bell-crank *k* and causing the governor valve *v*, Fig. 14, to take a position corresponding to the speed of the governor. When the speed falls, the governor weights move inwards and the governor valve is opened farther by them, thus admitting more steam. When the speed rises, the centrifugal force increases and the governor weights move farther outwards; this partially closes the governor valve and thus reduces the amount of steam admitted.

The speed at which the turbine will run can be varied somewhat by changing the tension of the governor springs. The tension is increased to make the turbine run faster and decreased to make the turbine run slower.

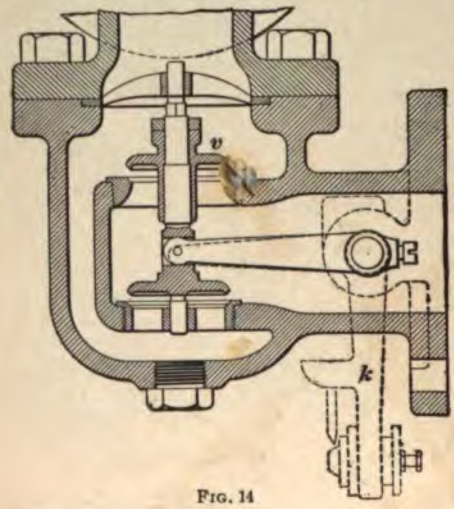


Fig. 14

When the turbine is running condensing, it has been found that the governor valve alone will not give a sufficiently close regulation during a sudden and excessive decrease of load. To assist the governor valve in controlling the speed under such conditions, the vacuum valve has been added. Its function is to admit air to the space in which the wheel revolves and thus impair or destroy the vacuum. The resistance of the air effectually checks the speed of the wheel. It is operated by the governor as follows: When the speed under a certain decrease of load becomes excessive, the governor first closes the governor valve entirely. If the governor weights continue to move outwards after this, they move the governor stem *h*, Fig. 13, farther to the right, compressing the spring *l*. The stud *m* then strikes the end of the vacuum valve *n* and

forces it inwards, thus admitting air. When the speed has been sufficiently reduced, the governor weights move inwards, and consequently the stud *m* moves away from the vacuum valve, allowing it to close. As the weights continue to move inwards, they open the governor valve and again admit steam to the turbine.

THE RATEAU STEAM TURBINE

14. The **Rateau turbine** is a leading example of the multiple-stage velocity type. The general arrangement of the turbine is shown in Fig. 15. It consists of a shell that is divided, by transverse partitions, into a number of cells,

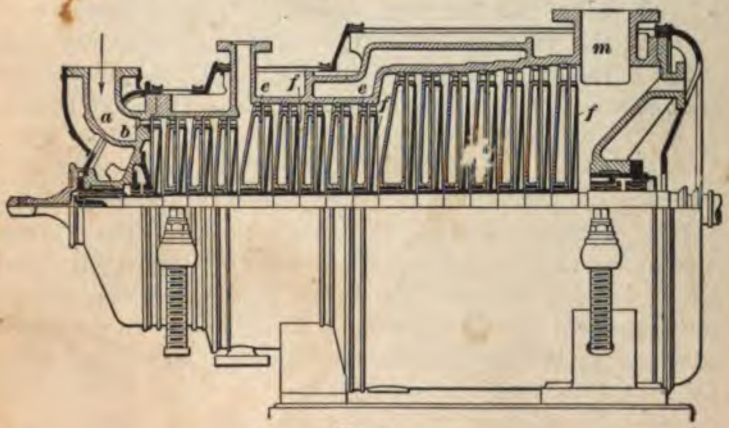


FIG. 15

each of which contains a wheel. All the wheels are keyed to a common shaft. In the figure, the partitions are denoted by *e*, *e*; the wheels by *f*, *f*. The partitions near the shell have passages that are oblique to the axis; and opposite these passages are the curved blades on the circumferences of the wheels. These blades are pressed out of sheet steel and riveted to the wheel, as shown in Fig. 16. The wheels themselves are made of thin steel plate pressed into conical form.

Steam enters at *a*, Fig. 15, and passes to the first wheel through the nozzle *b*. After passing through this wheel, it

is directed through the passages in the partition and passes through the second wheel. Then it passes through the next partition into the next cell, and so on. In any cell, the steam pressure is the same throughout, and the work done is due to the velocity of the jet of steam entering the cell through the passage in the partition. The drop of pressure from cell to cell occurs in this passage; there is no drop of pressure in the wheel. Hence, this is a velocity turbine. The total drop of pressure between the entering steam at a and the leaving steam at m is divided among fifteen or more cells. As the drop for each cell is relatively small, the velocity generated is small compared with the velocity in the single-stage De Laval turbine. Hence, this turbine may be run at moderate rotative speeds.

FIG. 16

CURTIS STEAM TURBINE

15. Action of the Curtis Turbine.—The **Curtis steam turbine** is a few-stage turbine of the velocity type. It is a vertical turbine; that is, the shaft is vertical and the turbine wheel revolves in a horizontal plane. The steam is admitted to the turbine through nozzles somewhat as in the De Laval turbine; it expands and acquires considerable velocity in passing through these nozzles, then passes through alternate sets of moving and fixed blades, and in so doing gives up part of its kinetic energy. It then enters another set of nozzles, expands and passes through a second series of fixed and moving vanes, in which more of its energy is given up.

The arrangement of blades and nozzles is shown in Fig. 17. The steam is admitted at the top, through the valves a, a which are controlled by the governor, and passes through the nozzles b, b, in which partial expansion takes place with considerable increase in velocity. It then strikes against the moving blades c, c and stationary blades d, d, which alternate with each other along the turbine wheel. The

moving blades are attached to wheels, fixed to the same shaft. After the steam leaves the last set of moving blades *c*, in turbines of only two stages, it exhausts into the atmosphere, when the turbine is running non-condensing.

When the turbine is running condensing, the steam passes through a second set of nozzles *e*, *e* into another series of movable and fixed blades, and then exhausts into a condenser. The part of the turbine in which the second expansion takes place is sometimes called the *low-pressure side*. The

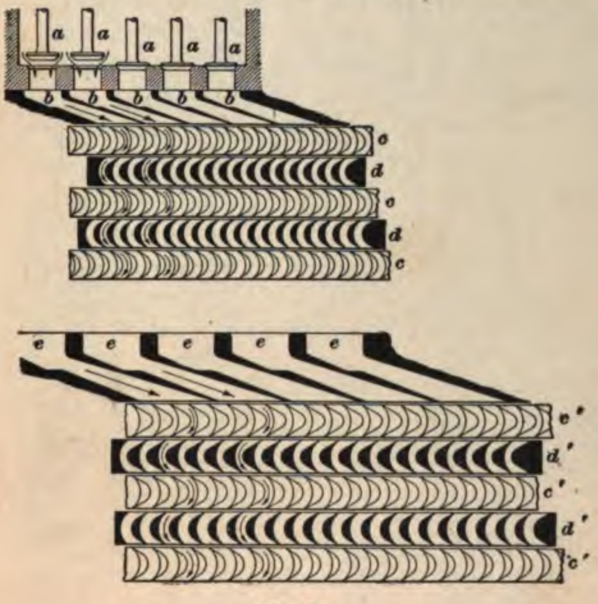


FIG. 17

moving blades c', c' on the low-pressure side are attached to wheels that are fastened to the main shaft, with the high-pressure wheels, so that they rotate together. The fixed blades d', d' alternate with the moving blades in the low-pressure side as in the high-pressure side.

16. Curtis Turbo-Generator.—Fig. 18 is a sectional view of one form of **Curtis turbo-generator**, showing the arrangement of the various parts. The shaft is vertical, the

dynamo A being mounted on top of the turbine B. The turbine is in two stages—an upper a and a lower b. Each stage contains three steel turbine wheels c with buckets, or

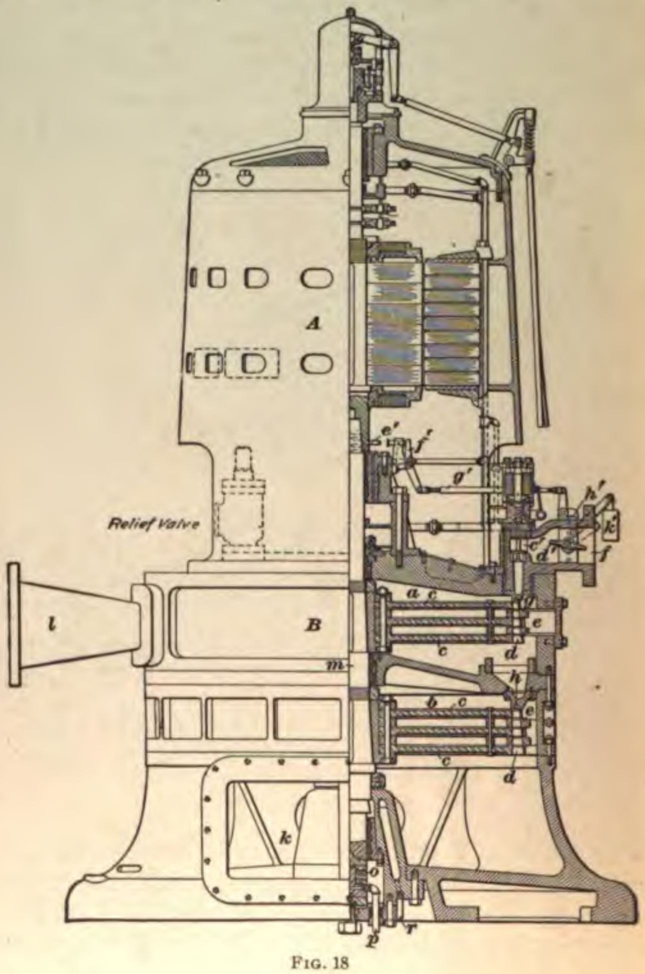


Fig. 18

vanes, d on the peripheries. The stationary vanes e between the wheels are supported by the turbine casing. Steam enters at f and passes through the nozzles at g, thus

impinging on the first wheel. The direction of flow is then reversed by the first set of stationary blades, the steam strikes the second wheel, and so on until it passes through the three wheels of the first stage. It then passes into the wheels of the second stage through openings at *h*, and finally passes through *k* into the condenser. In case a condenser is not used, the steam is exhausted through the exhaust connection *l*, the first stage only being used.

In order to obtain a high economy, it is necessary to operate turbines with a high vacuum; hence, they are not operated non-condensing if it is possible to avoid it. In some later types, a surface condenser is placed in the base of the turbine instead of being separate. By this arrangement, the liability of leakage between turbine and condenser is reduced.

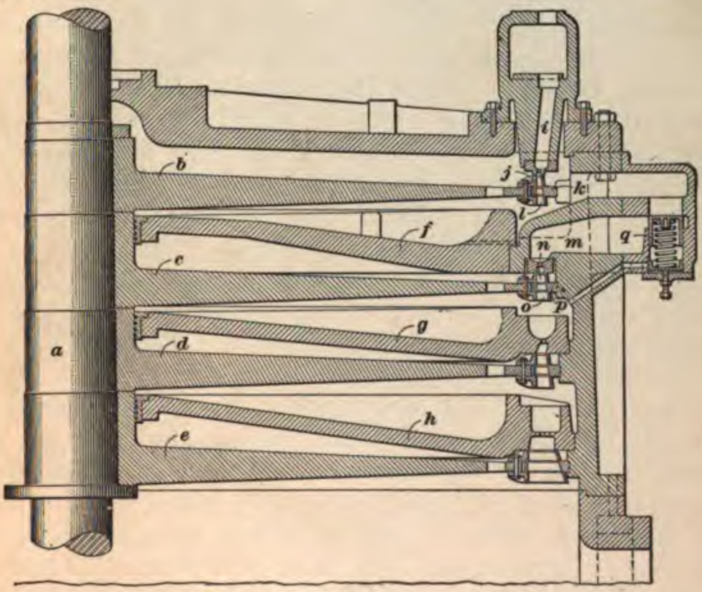


Fig. 19

17. Pressure and Velocity Stages.—In the Curtis turbine, the steam is expanded in several sets of nozzles, but the pressure in any one of the wheel chambers is constant.

Each wheel chamber has a different pressure; hence, each is spoken of as a pressure stage. In each wheel chamber, there are two or more sets of moving vanes, and each reduces by a certain amount the velocity and the kinetic energy of the steam; hence, each set of moving vanes is spoken of as a velocity stage.

The turbine shown in Fig. 18 has two pressure stages, with three velocity stages to each pressure stage. In more recent designs, four and five pressure stages have been employed, each with two velocity stages. Fig. 19 shows a section of a turbine with four pressure stages.

18. The Four-Stage Curtis Turbine.—The vertical shaft of the **four-stage Curtis turbine** is shown at *a*, Fig. 19, and the four wheels attached to the shaft are shown at *b*, *c*, *d*, and *e*. Between these wheels are partitions *f*, *g*, *h* that serve to separate the different pressure stages of the steam. The steam enters the turbine through the channel *i*, and then through an expanding nozzle that directs it against the first set of moving blades *j* at a high velocity. In passing through the blades *j*, the steam gives up energy and its direction of flow is changed; it then strikes the set of stationary blades *k* and its direction of motion is again changed to about the same direction it had when leaving the nozzle *i*. It then strikes a second set of moving blades *l*, giving up more energy. Both sets of moving blades *j* and *l* are attached to the wheel *b*, while the set of stationary blades *k* is fastened to the casing.

From the moving blades *l*, the steam passes through a valve *q* into the chamber *m* and expands through a set of short nozzles to the blades on the wheel *c*, which also carries two sets of buckets *n* and *o* on either side of a set of stationary buckets *p* fastened to the casing. The same process is repeated with the blades on the wheels *d* and *e*, the steam expanding through nozzles in the stationary partitions between the wheels.

19. Step Bearing of Curtis Turbine.—The turbine shaft *m*, Fig. 18, is supported on a thrust bearing *o* and oil

or water is forced in under the shaft through the pipe *p*. The oil or water flows up and out through the bearing and returns through the pipe *r*. The end of the shaft, therefore,

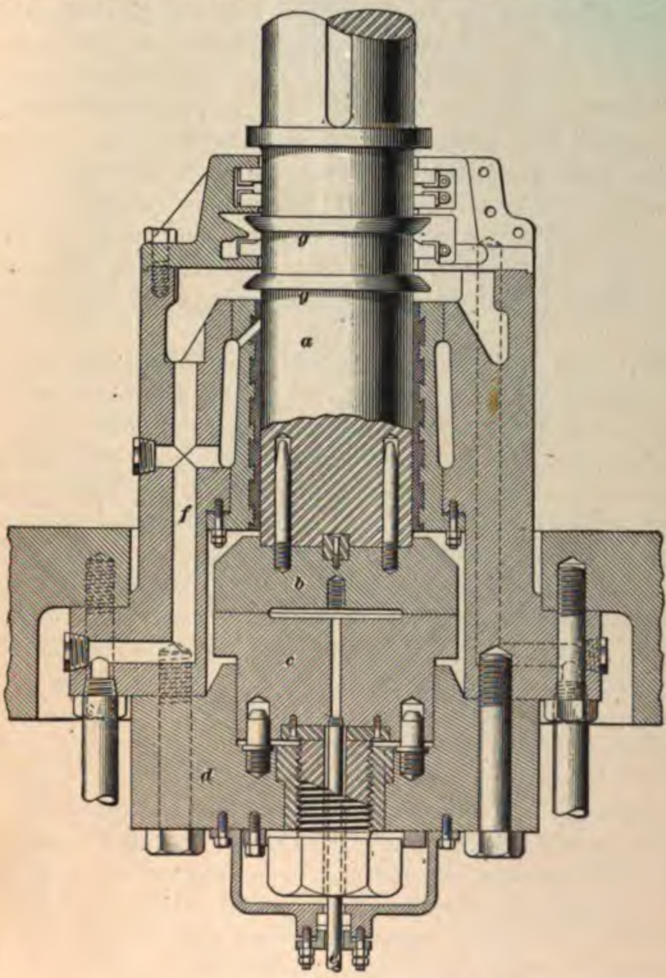


FIG. 20

rests on a thin film of oil or water, and the downward pressure on the bearing is taken up by the fluid pressure which is usually between 200 and 500 pounds per square inch.

The general arrangement of this bearing is shown more clearly in Fig. 20. The end of the shaft *a* is doweled to the cast-iron block *b*, which turns with it. The block *b* is supported by the block *c* on which it rotates. The block *c* is prevented from turning by being doweled to the base *d*. Both *b* and *c* are recessed about $\frac{1}{8}$ inch deep for about one-half their diameter. Oil or water is forced through the pipe *e* into this space with sufficient pressure to raise the entire weight on the shaft and cause a thin film of oil or water to flow all over the bearing surface between the blocks *b* and *c*. The oil works its way up around the shaft and then out through the passages *f*. The rings shown at *g*, *g* prevent the oil from working out above the step bearing.

20. The Governor.—Fig. 21 is a perspective view of the outside of the turbine showing the location of the governor and the way it is connected with the valves.

The governor *a* is located on the top of the machine and consists of centrifugal weights acting against a heavy spring. The movement of the weights operates the rod *b* connected to an electric controller *c*. The controller regulates an electric current through a series of electromagnets that operate small pilot valves *d*. These small pilot valves operate valves c', Fig. 18, which admit steam to the several nozzles. Any decrease in speed, caused by an increase in load, causes a corresponding movement of the governor. This moves the controller that operates the electromagnets, so as to bring more nozzles into action and thereby supply more power to carry the load. In addition to the regular throttle valve, the turbine is provided with a valve d', Fig. 18, that closes automatically when, for any reason, the speed becomes excessive. A centrifugal device arranged on the shaft at e' flies out when the speed rises above the proper amount, thus moving the lever f', pulling on the rod g', and releasing the catch h'. This allows the weight k' to drop and close the valve. Small adjustments in speed can be made by turning the small hand wheel *e*, Fig. 21, thereby slightly changing the action of the governor. In some of the turbines, particularly

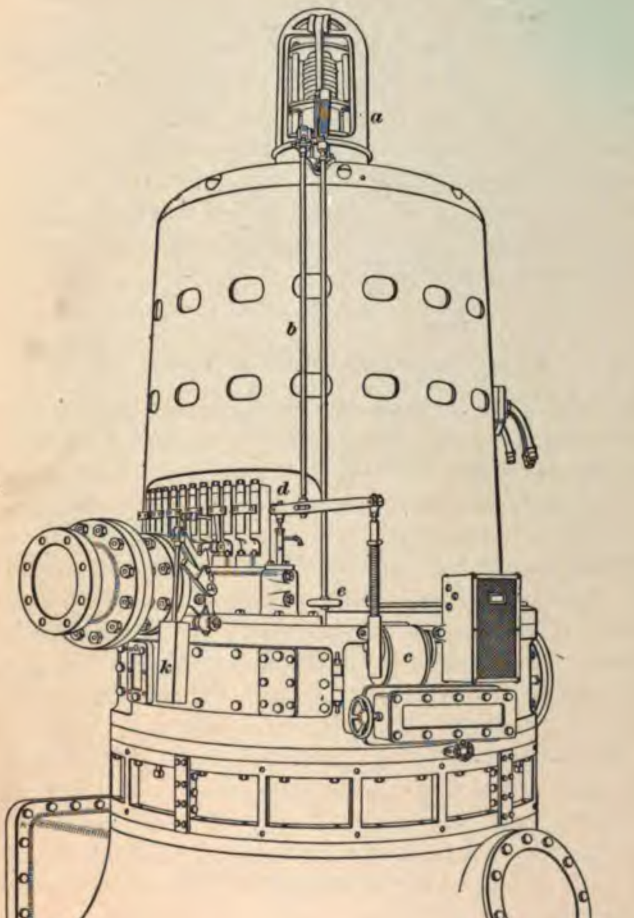


FIG. 21

in the larger sizes, this adjustment is made by means of a small electric motor controlled from the switchboard.

The governing of the four-stage turbine shown in Fig. 19 is essentially the same as that of the two-stage turbine. The governor acts on the first set of nozzles only; however, at *q*, there is a spring valve between the first and second sets of nozzles. When the pressure in the chamber of the first wheel reaches a certain point, this valve opens and admits steam to the second set of nozzles. There are no valves, however, between the second and third or between the third and fourth stages.

THE PARSONS STEAM TURBINE

21. General Arrangement of the Parsons Turbine. The leading example of the pressure type of turbine is the **Parsons turbine.** An outside view of one of this type, the Westinghouse-Parsons turbine, is shown in Fig. 22. Steam enters through the pipe *a*, passes up through a channel inside the casing *c* and the pipe *h* at the top of the turbine, and then through the governor valves *b*, *b'* into the turbine. The governor, which controls the valves *b*, *b'*, is inside the casing *d*; it is of the spring-loaded flyball type, with a dashpot to steady its action. The shaft *e* is rocked to and fro by an arm connecting it at one end to the governor, the other end being connected by an arm and link *f* to the shaft *i*, which is connected by arms to the pilot valves *g*, *g'* that operate the main steam admission valves. The action of the governor and pilot valves will be considered more fully later. At *j* is shown the dynamo connected to this turbine.

22. The section of this turbine, Fig. 23, shows its principal parts. The main shaft *a* carries all the rotating parts, including the three series of blades *b*, *c*, and *d*. The entire rotating part is generally called the **rotor.** The main bearings *e* and *f*, of the shaft, will be explained in detail later. The rotor is surrounded by a steam-tight casing *g*.

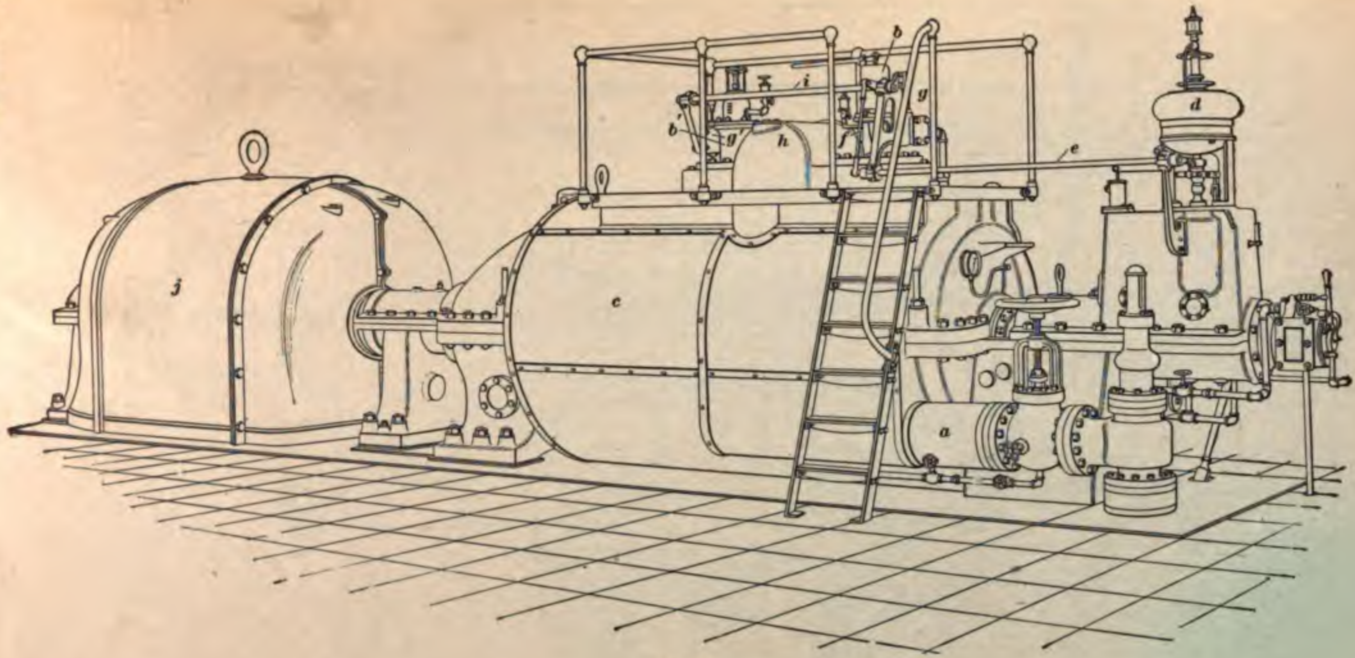


FIG. 22

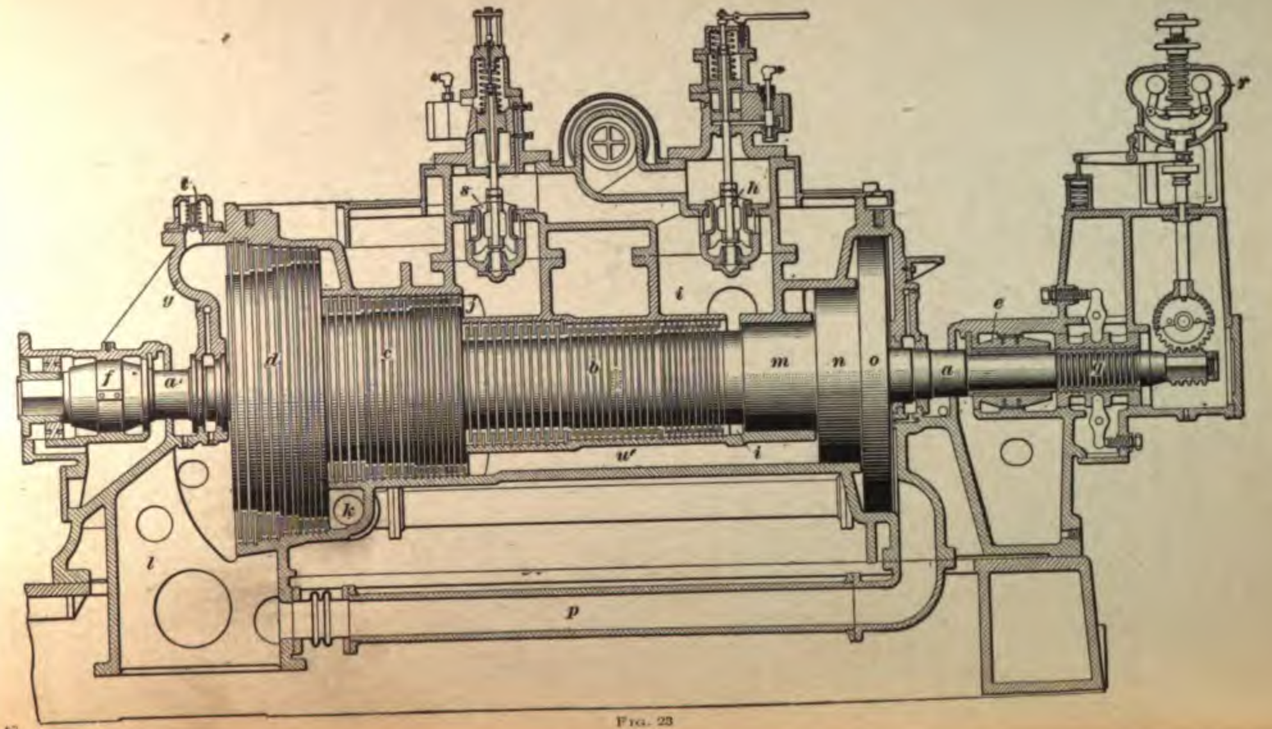


Fig. 23

The steam enters the turbine at boiler pressure through the valve *h* and fills the chamber *i*, *i*, which extends around the rotor. From the chamber *i*, the steam expands through a set of stationary blades, that so direct it that it strikes the moving blades and gives up energy to them. The stationary and moving blades alternate in sets, each set forming a ring around the rotor. The steam expands a small amount as it passes through each set of blades. Its velocity increases in passing through the stationary blades, and decreases, on account of the energy given up, in passing through the moving blades. The blades are radial, as in the De Laval turbine, but there are a large number of sets instead of only one. The length of the blades of one set is the same, and they are placed the same distance apart. In the first set of a series, they are comparatively short and close together, but their length and distance apart increase toward the next series. The purpose of this is to accommodate the expanding steam which requires an increased space as it expands.

After the steam has passed through the first series of blades *b* it enters the chamber *j* from which it enters the second series of blades *c*. After passing through the stationary and moving blades of this series and out into the chamber *k*, it passes through the third series of blades *d* and exhausts into the chamber *l*. From this chamber it goes either to the condenser or to the atmosphere.

23. In order to counteract the end thrust caused by the action of the steam on the rotor, the shaft is fitted with balance pistons *m*, *n*, and *o*. The diameter of *m* is about the same as that of the part *b* of the rotor, and the chamber *i* permits the same steam pressure to act on both, neutralizing the end thrust of the first section. In the same way, *n* is of the same diameter as *c* and is acted on by the same steam pressure, being connected by the passage *u*. Again, *o* is of the same diameter as *d*; one face is exposed to steam from the chamber *k* while the outer face is connected to the exhaust chamber by the pipe *p*, thus neutralizing the pressure on both sides of the part *b* of the rotor.

An adjustable bearing *q*, similar to a thrust block, confines the turbine shaft longitudinally, and at the same time resists any end thrust that may not be balanced by the balance pistons.

24. At *r* is shown the governor, which regulates the speed of the turbine by acting on the steam admission valve *h*. When the turbine must carry an overload and the supply of steam through the valve *h* is insufficient, the governor also causes the valve *s* to open and admit steam at boiler pressure to the second series of blades *c*. This increases the power of the turbine, but the steam is not used so efficiently, and the economy is therefore lowered somewhat.

A relief valve, shown at *t*, is provided to permit steam to escape to the atmosphere when the exhaust pressure becomes excessive.

25. Flexible Bearings.—The means adopted for preventing vibration is a careful balancing of the rotor combined with a construction permitting it to rotate about its axis of gravity. The shaft is made rigid, but the bearings are made flexible. It can readily be seen that if the axis of gravity of the rotor does not coincide with the axis of the shaft, the latter will move in a circular path when the rotor turns about its axis of gravity. In order to permit this rotation, and thus insure smooth running, the bearings are surrounded by loosely fitting concentric sleeves as shown at *e*, Fig. 23. The spaces between the sleeves are small and the sleeves are provided with holes through which oil can pass to the spaces. The oil, which is a yielding medium, forms a film around the shaft and sleeves and allows the bearing to move laterally, so as to accommodate itself to any lack of balance of the rotor. The bearing is prevented from rotating by suitable dowel-pins.

All bearings are lubricated by oil under a slight pressure, and the oil flows back by gravity to a reservoir. A small pump, driven from a worm-gear on the shaft, forces the oil from the lower to a higher reservoir, a few feet above the bearings. From this point the oil flows, under a pressure due to this

height, through pipes to the top of the bearings. Piping is also provided for the return of the oil from below the bearings to the lower reservoir.

26. The Governor.—The governor shown at *r*, Fig. 23, is of the spring-loaded flyball type. By adjusting the tension of the governor spring, the speed of the turbine can be varied within certain limits. The governor valve controlling the admission of steam to the turbine is not attached directly to the governor, but is operated by means of a pilot valve controlled by the governor. The work to be performed by the governor is therefore very light.

The governing mechanism is shown in detail in Fig. 24. The steam enters through the pipe *a* to the admission or governor valve *b*. This valve is attached to a rod *c* that carries at its upper end the piston *d* in the cylinder *e*. The spring *f* above the piston keeps the piston and valve in their lowest positions, in which the valve *b* is closed. Steam enters the cylinder *e*, below the piston, around the rod, which fits loosely in the opening *g*. When the turbine is not running, the pilot valve *h* stands in the position shown and is closed. When the throttle valve, connected to the pipe *a*, is opened, steam passes through the opening *g*, raising the piston *d* and the governor valve *b*, thus admitting steam to the turbine.

When the turbine is running, the pilot valve is operated, through a system of levers, by an eccentric on a shaft *k* that operates the governor, and that receives its power from the turbine shaft *j*. Owing to the high speed of the turbine shaft *j*, a worm and worm-gear are used to reduce the speed of the shaft *k* and therefore of the governor and pilot valve. The bevel gears *l* transmit the motion to the governor while the eccentric *m* gives motion to the pilot valve. Motion is transmitted from the eccentric rod through the arm *n* and a short link to the lever *o*, which has its fulcrum on the governor collar *r*. From the lever *o*, the motion is transmitted through another link and arm to the rocking-shaft *p*; from this rocking-shaft, the motion is given to the pilot valve by the arm and link shown at *q*.

While the turbine is running, the pilot valve is given an up-and-down motion by the eccentric *m*. Since the fulcrum of the lever *o* rises and falls as the flyballs *s*, *s* of the governor move outwards or inwards, the up-and-down motion of the

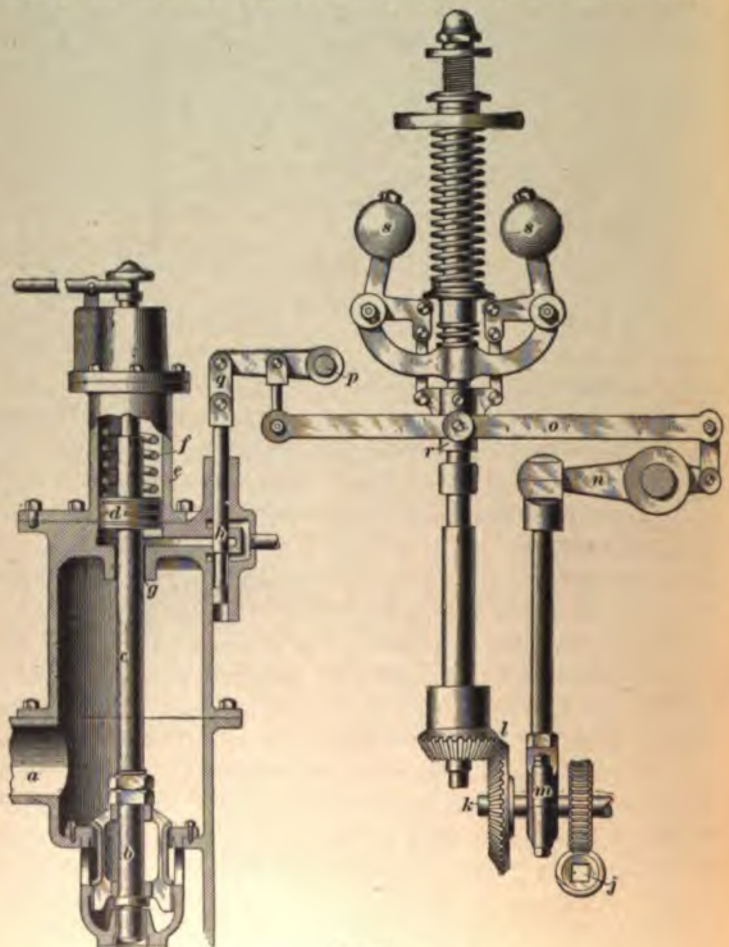


FIG. 24

pilot valve is made higher up when the flyballs of the governor are out, and lower down when the flyballs are in, near the spindle.

27. The action of the governor can best be studied by considering the action of the admission valve while the turbine is running at normal speed. With each revolution of the shaft *k*, the pilot valve is opened and closed. When closed, the steam pressure increases under the piston *d*, Fig. 24, which moves upwards and opens the steam admission valve *b*, thus opening the steam inlet. After a certain interval, the pilot valve opens and allows the steam to escape from beneath the piston; then the spring *f* immediately closes the admission valve, again shutting off the steam from the turbine. This alternate opening and closing of the admission valve occurs at intervals that depend on the speed of the turbine. It is thus seen that the steam is admitted in puffs.

The volume of steam admitted at each opening of the governor valve varies directly with the length of time the valve remains open. The object of the governor is to regulate this length of time, which is accomplished by causing the pilot valve to make its strokes in a higher or lower position, depending on the way in which the speed varies, thus changing the length of time the pilot valve, and therefore the time the admission valve remains open. Suppose that the speed of the turbine increases. Then the governor balls *s*, *s* fly outwards, and in doing so raise the governor collar *r*. As a result, the pilot valve *h* is also raised and makes its up-and-down strokes in a higher position. The exhaust port below the piston *d*, therefore, remains open for a greater length of time than before, with the result that the steam escapes and the spring holds the steam admission valve closed longer, or in other words, shortens the time during which steam is admitted to the turbine. The result of this decrease in the volume of steam admitted to the turbine is a reduction of its speed to the normal speed. When the speed drops below the normal, the pilot valve drops to a lower position, thus increasing the time during which steam collects below the piston *d*, and consequently increasing the time that the steam admission valve remains open. More steam is thus admitted at each opening of the

valve, which continues until the speed has increased to the normal.

The object of making the admission valve act intermittently, as described, is to admit the steam to the turbine at full boiler pressure, irrespective of the variation in load. With a full load, the puffs of steam admitted will be almost continuous.

OTHER STEAM TURBINES

28. The types of steam turbines that have been described do not embrace all the commercial forms. There are a large number of other turbines on the market, the action of which, however, is based largely on the same principles as those described, although the mechanism may differ widely in details of construction. The number of turbines is increasing rapidly, and it would not be practicable to illustrate and describe all the various forms in existence or in the course of development. The leading types described here embody the essential principles of turbine construction, and the application of these principles to the other forms will not be found difficult.

STEAM TURBINES

(PART 2)

STEAM-TURBINE CALCULATIONS

ENERGY CHANGES IN THE TURBINE

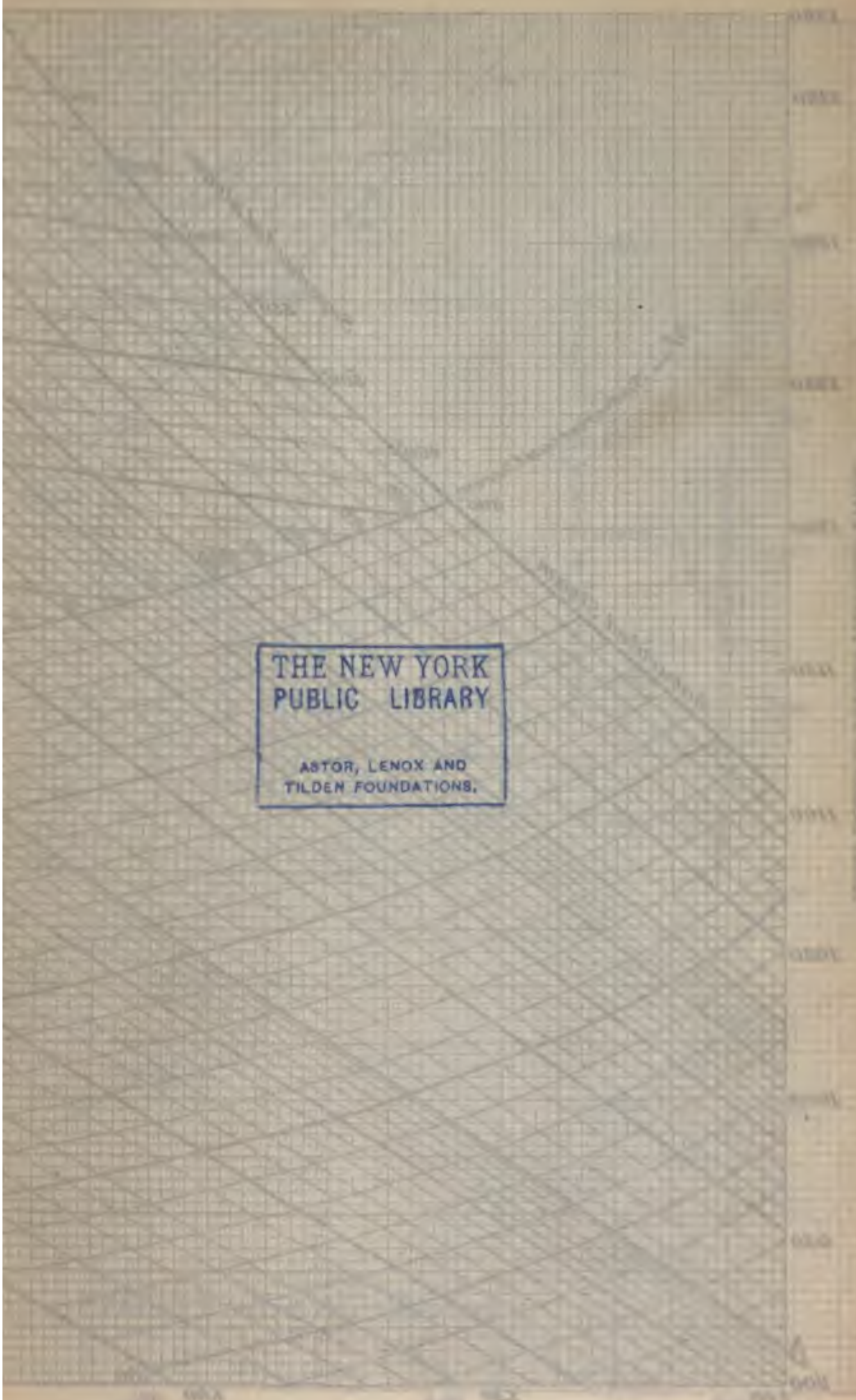
1. Heat Contents of the Steam.—As steam passes through a turbine it undergoes a considerable change of condition. It enters the turbine at a high pressure and may be superheated; as it leaves the turbine and enters the condenser it is at a much lower pressure and may contain some water. In any state, however, a unit weight, say a pound, contains a definite quantity of heat, called the **heat contents.** This quantity of heat may be defined as the number of British thermal units required to raise the temperature of 1 pound of water from 32° F. to the boiling point corresponding to the steam pressure, evaporate the pound of water at that temperature, and superheat it to the final temperature of the superheat. If the steam is just saturated, the contained heat is simply the total heat as given in the Steam Table. If the steam is wet and has the quality x, the contained heat is the heat required to raise the pound of water from 32° F. to the boiling point and to evaporate x per cent. of it.

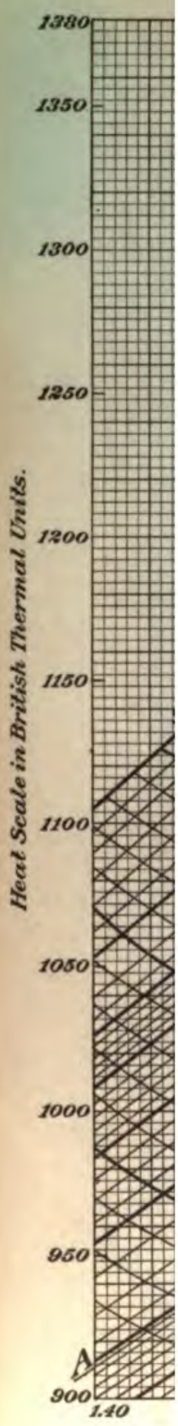
For any given state, the heat contents can readily be calculated. However, to obviate much tedious work, a Heat Chart has been prepared especially for this Section.

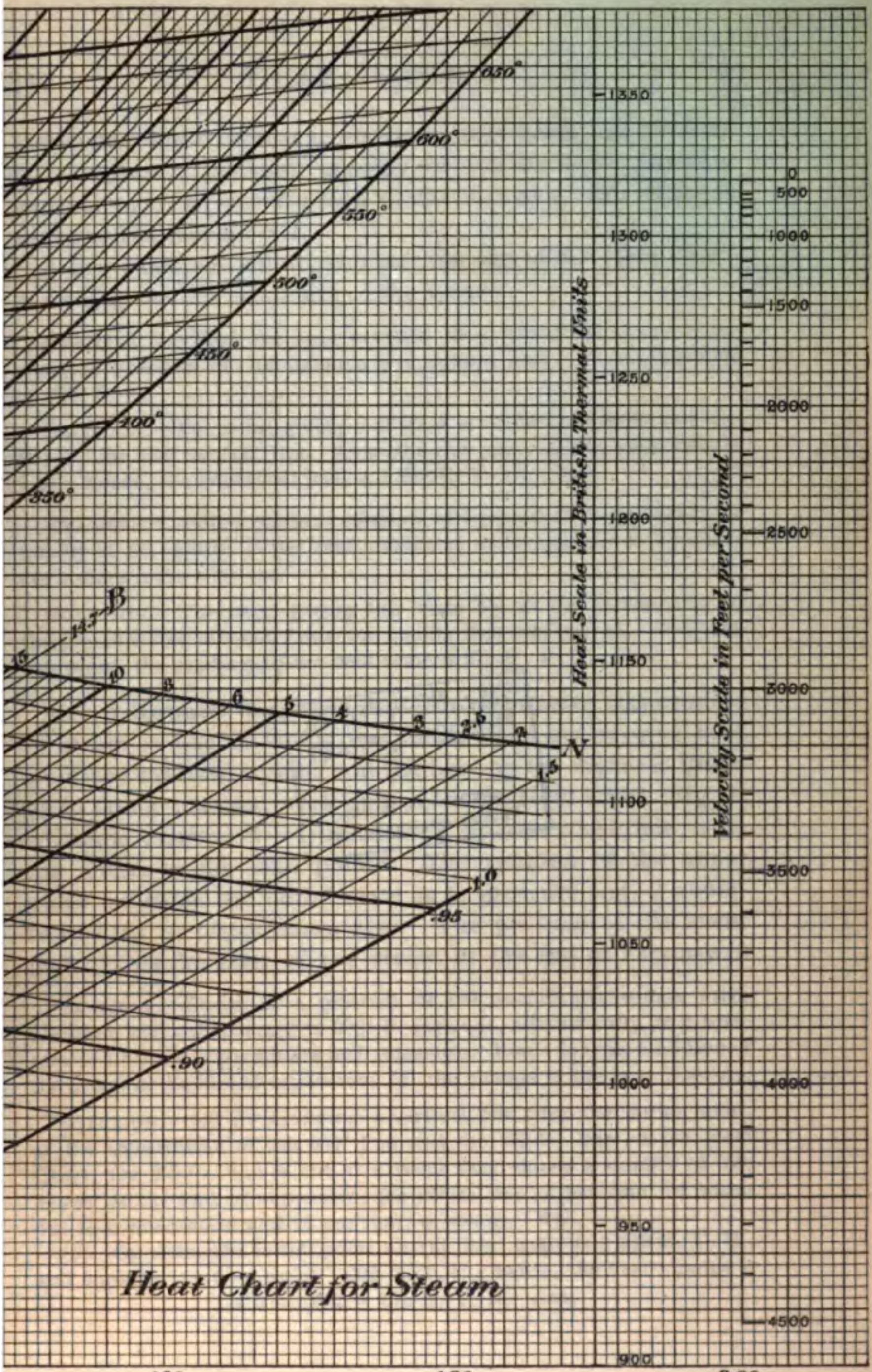
2. Heat Chart for Steam.—The **Heat Chart** shown in Fig. 1 has been constructed for the purpose of easily determining the quantity of heat, in British thermal units, in 1 pound of steam when its pressure and quality are known, and to determine the heat changes in the adiabatic expansion of steam. The form of the chart was originated by Mollier, but is plotted here so as to agree with the Steam Table given in *Entropy and Steam*. The heavy line MN represents the saturation points for steam at various pressures, and is called the saturation curve. Points located above this curve are said to be in the superheated region; those below it are said to be in the saturated region. All points on the line MN represent dry saturated steam. The lines that cross MN obliquely and nearly parallel to AB are pressure lines, with the pressures in pounds per square inch, absolute, given near the intersection of some of these lines with the saturation curve. The lines below and nearly parallel to MN give the quality of the steam. For example, points on the line marked *.90* represent steam with 10 per cent. of moisture and .90 per cent. of steam. The point in the saturated region where a quality line and a pressure line cross determines the state of wet steam; thus, the .95 line crosses the 75-pound line at a point that represents steam at an absolute pressure of 75 pounds per square inch and containing 95 per cent. dry steam and 5 per cent. moisture.

The lines above MN that are nearly horizontal represent superheated steam of the temperature in degrees Fahrenheit given at the ends of these lines. When steam is superheated, its state is represented by the point of intersection of the pressure line and the superheat line. If the steam has an absolute pressure of 150 pounds per square inch and is superheated to a temperature of 500° F., its state is represented on the Heat Chart by the point where the 150-pound line intersects the 500° line.

The ordinates of all points on the Heat Chart are heat units in British thermal units per pound of steam, as shown by the heat scale on the left margin; and the abscissas are entropy units, as shown by the scale on the lower margin.




Heat Scale in British Thermal Units.
1380
1350
1300
1250
1200
1150
1100
1050
1000
950
A
900
1.40



Heat Chart for Steam

Units

Horizontal lines represent constant heat lines, and vertical lines represent constant entropy or the lines of adiabatic expansion. The construction of the curves of this chart is explained in Appendix I.

A few examples will make clear the use of the Heat Chart in finding the heat contents of the steam.

EXAMPLE 1.—Find the heat contents of 1 pound of steam at an absolute pressure of 100 pounds per square inch and superheated to 500° F.

SOLUTION.—The point giving this state is at the intersection of the 100-lb. line and the 500° line. Opposite this point on the heat scale is 1,268; hence, the contained heat is 1,268 B. T. U. Ans.

EXAMPLE 2.—Find the heat contents of 1 pound of steam having a quality of .85, that is, 15 per cent. moisture, and a pressure of 3 pounds per square inch, absolute.

SOLUTION.—The point giving this state lies at the intersection of the pressure line marked *3* and the quality line marked *.85*. Opposite this point, the reading on the heat scale is 973 B. T. U. Ans.

3. Adiabatic Expansion.—An **adiabatic expansion** of steam is one in which the steam does not receive heat from any external source, and the heat of the steam is given up in doing external work only. In the Heat Chart, Fig. 1, adiabatic expansion is represented by a vertical line from the initial to the final points. The position of the upper point indicates the initial state as to pressure and, if superheated, as to temperature also. The position of the lower point gives the state of the steam at the end of expansion.

EXAMPLE.—Steam at a pressure of 150 pounds per square inch, absolute, and superheated to 600° F. expands, adiabatically, to a pressure of 2.5 pounds per square inch, absolute. Find: (*a*) the condition of the steam at the end of expansion; (*b*) the change in the heat contents.

SOLUTION.—(*a*) By dropping a vertical line from the initial point, at the intersection of the 600° line and the 150-lb. line on the Heat Chart, the 2.5-lb. line is cut at a point that shows a quality of .88; hence, at the end of expansion, the steam has 12 per cent. of water due to condensation and 88 per cent. of steam. Ans.

(*b*) Opposite the initial point on the heat scale is 1,312 B. T. U., and opposite the final point 1,002 B. T. U.; hence, the heat change is

1,312 − 1,002 = 310 B. T. U. Ans.

4. Volume of Steam.—If the point representing the condition of the steam is in the saturated region, the volume of 1 pound may be determined from the Steam Table. Thus, if V is the volume per pound, as given in the table, and x is the quality as determined from the Heat Chart, the volume v of 1 pound of the mixture of steam and water is given very nearly by the formula

$$v = x V \qquad (1)$$

For convenience in finding the volume of saturated steam at the ordinary condenser pressures found in steam-turbine practice, Table I is given.

TABLE I

VOLUMES OF SATURATED STEAM AT CONDENSER PRESSURES

Pressures Pounds per Square Inch, Absolute	Volumes Cubic Feet per Pound	Pressures Pounds per Square Inch, Absolute	Volumes Cubic Feet per Pound
1	334.6	$2\frac{1}{4}$	155.3
$1\frac{1}{4}$	270.9	$2\frac{1}{2}$	140.6
$1\frac{1}{2}$	227.9	$2\frac{3}{4}$	128.5
$1\frac{3}{4}$	197.0	3	118.4
2	173.6		

For points in the superheated region, the following formula will give the volume per pound:

Let p = pressure, in pounds per square inch, absolute;
T = absolute temperature = $t + 460°$ F.;
v_1 = volume of 1 pound of superheated steam, in cubic feet.

Then, $$v_1 = .591 \frac{T}{p} - .135 \qquad (2)$$

EXAMPLE.—(*a*) Find the initial volume per pound in the example of Art. 3. (*b*) Find the final volume.

SOLUTION.—(*a*) For the initial volume, substitute in formula **2**, $T = 460 + t = 460 + 600 = 1{,}060$, and $p = 150$. Hence,

$$v_1 = .591 \times \frac{T}{p} - .135 = .591 \times \frac{1{,}060}{150} - .135 = 4.0414 \text{ cu. ft.} \quad \text{Ans.}$$

(*b*) For the final volume at the pressure of 2.5 lb., apply formula **1**. The volume of 1 lb. of dry saturated steam at 2.5 lb. per sq. in., absolute, is about 140.6 cu. ft.; hence, substituting this and the quality of .88 in formula **1**,

$$v = .88 \times 140.6 = 123.7 \text{ cu. ft.} \quad \text{Ans.}$$

5. Theoretical Work of Steam.—Suppose a pound of steam to enter a heat motor—that is, either a steam turbine or a steam engine—in a given initial condition, expand adiabatically to some final condition, and then pass out of the motor. The steam brings into the motor the heat contents at the initial state, which may be denoted by H_1, and carries away with it the heat contents H_2 at the final state. If there were no losses of any kind, the difference $H_1 - H_2$ would represent precisely the heat transformed into work; hence, the theoretical work W, in foot-pounds per pound of steam, would be given by the formula

$$W = 778(H_1 - H_2) \qquad \textbf{(1)}$$

The theoretical steam consumption in pounds of steam per horsepower per hour is now easily obtained. One horsepower per hour is equal to $33{,}000 \times 60 = 1{,}980{,}000$ foot-pounds. Since 1 pound of steam produces $778(H_1 - H_2)$ foot-pounds, the theoretical weight, S_0, of steam required per horsepower per hour is given by the formula

$$S_0 = \frac{1{,}980{,}000}{778(H_1 - H_2)}$$

or

$$S_0 = \frac{2{,}545}{H_1 - H_2} \qquad \textbf{(2)}$$

EXAMPLE.—(*a*) Find the theoretical work of the adiabatic expansion of 1 pound of steam from a pressure of 150 pounds per square inch, absolute, and superheated to a temperature of 600° F., to a pressure of 2.5 pounds per square inch, absolute. (*b*) Find the weight of steam required per horsepower per hour.

SOLUTION.—(*a*) From the Heat Chart, the 150-lb. line crosses the 600° line at 1,312 B. T. U. The adiabatic expansion line—that is, the vertical line—from the initial to the final states crosses the 2.5-lb. line at 1,002 B. T. U. Hence, applying formula **1**,

$$W = 778(H_1 - H_2) = 778(1{,}312 - 1{,}002) = 241{,}180 \text{ ft.-lb.} \quad \text{Ans.}$$

(*b*) Apply formula **2**,

$$S_0 = \frac{2{,}545}{H_1 - H_2} = \frac{2{,}545}{1{,}312 - 1{,}002} = 8.21 \text{ lb., nearly.} \quad \text{Ans.}$$

6. Actual Work and Steam Consumption.—The effective work derived from 1 pound of steam is always less than the theoretical work. In a steam turbine, the losses are of two kinds: (1) those connected with the action of the steam, such as the friction in the nozzles and blades; (2) those not connected with the steam, such as journal friction. If the first losses are subtracted from the theoretical work, the remainder is the work that the steam actually does. This may be called the *indicated work* of the steam, because it corresponds to the work given by the indicator diagram in the steam engine. If the second set of losses is also subtracted, the result is the *effective*, or *brake*, *work*. Let W_1 and W_2 denote the indicated and brake work, respectively; then, the ratio of the indicated work to the theoretical work is the *indicated efficiency* E_1 of the motor, and

$$E_1 = \frac{W_1}{W} \qquad (1)$$

The *brake efficiency* E_2 is the ratio of brake work to the theoretical work, and is expressed by the formula,

$$E_2 = \frac{W_2}{W} \qquad (2)$$

These efficiencies refer to the theoretical work as a basis, and should not be confused with the theoretical efficiency of the ideal heat engine, $\frac{H_1 - H_2}{H_1}$, which is based on the heat H_1 carried into the turbine.

The steam consumption per indicated horsepower per hour is given by the formula

$$S_1 = \frac{1{,}980{,}000}{W_1}$$

or

$$S_1 = \frac{2{,}545}{E_1(H_1 - H_2)} \qquad (3)$$

The steam consumption per brake horsepower per hour is given by the formula

$$S_2 = \frac{1{,}980{,}000}{W_2}$$

or

$$S_2 = \frac{2{,}545}{E_2(H_1 - H_2)} \qquad (4)$$

EXAMPLE.—In the example of Art. **5**, the heat change is $H_1 - H_2 = 310$ British thermal units. (*a*) If the losses in the action of the steam are 30 per cent., what is the steam consumption per indicated horsepower per hour? (*b*) If the journal friction is 10 per cent. of the indicated work, what is the steam consumption per brake horsepower per hour?

SOLUTION.—(*a*) The indicated efficiency is 70 per cent., or $E_1 = .70$. Hence, applying formula **3**,

$$S_1 = \frac{2{,}545}{.70(H_1 - H_2)} = \frac{2{,}545}{.70 \times 310} = 11.728 \text{ lb. Ans.}$$

(*b*) The heat used in doing work, per pound of steam per indicated horsepower per hour, is $.70 \times 310 = 217$ B. T. U.; and, as 10 per cent. of this is lost in friction, only $217 \times .9 = 195.3$ B. T. U. is transformed into work at the brake. Hence, the brake efficiency, from formula **2**, is

$$E_2 = \frac{W_2}{W} = \frac{195.3 \times 778}{310 \times 778} = .63$$

The steam consumption per brake horsepower is found from formula **4**,

$$S_2 = \frac{2{,}545}{E_2(H_1 - H_2)} = \frac{2{,}545}{.63 \times 310} = 13.03 \text{ lb. Ans.}$$

7. Curve of Steam Consumption.—The curve in Fig. 2 shows the relation between the brake horsepower of the steam turbine at full load and the steam consumption.

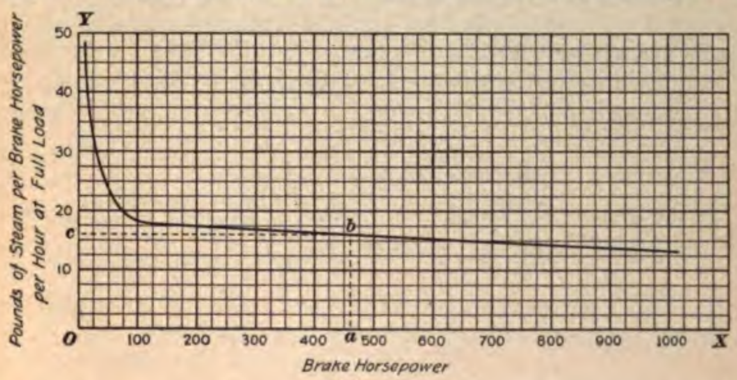


FIG. 2

The curve is plotted from the published results of tests of steam turbines of the types that have attained the greatest commercial success. The turbines above 100 brake horsepower that were considered were multiple-stage turbines. All the turbines examined used saturated steam of from 115 to

140 pounds per square inch gauge pressure, and all exhausted into a vacuum of from 26 to 28.5 inches of mercury. Better results than those shown by the curve can be obtained by the use of highly superheated steam.

Frequently, the power of the turbine is stated as so many kilowatts* (K. W.). In that case, dividing the number of kilowatts by .746 gives the horsepower. To allow for losses between the shaft and the dynamo terminals, the horsepower may be divided by .9 to give the brake horsepower of the turbine.

The use of the curve in Fig. 2 may be illustrated by the following example:

EXAMPLE.—Estimate the total steam consumption per hour of a condensing steam turbine of 310 kilowatts full-load capacity.

SOLUTION.— $310 \div .746 = 416$ H. P., nearly
$416 \div .9 = 462$ brake H. P., nearly

On the line OX, Fig. 2, locate the point a, for 462 as shown. Draw the line ab parallel to OY, intersecting the curve in the point b, and then from b draw bc parallel to OX, intersecting OY in c. Then, the steam required per brake horsepower per hour is represented by the distance Oc, which is about 16 lb., to the scale marked on the line OY. The total amount of steam, therefore, that is required by the turbine per hour is

$$16 \times 462 = 7{,}392 \text{ lb. Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the heat contents of 1 pound of steam at a pressure of 150 pounds per square inch and superheated to 550° F.
Ans. 1,286 B. T. U.

2. Find the heat contents of 1 pound of steam at a pressure of 2 pounds per square inch., absolute, and a quality of 83 per cent.
Ans. 947 B. T. U.

3. Find the volume of the pound of steam in example 1.
Ans. 3.844 cu. ft., nearly

4. Find the volume of the pound of steam in example 2.
Ans. 144 cu. ft., nearly

---

*The kilowatt is a unit of measurement for electrical power; 1 horsepower is equal to .746 kilowatt.

5. If the pound of steam in example 1 expands adiabatically to 3 pounds per square inch: (*a*) what is its quality in the final state? (*b*) what is its final heat contents? (*c*) what is its final volume?

Ans. $\begin{cases} (a) \text{ .873} \\ (b) \text{ 996 B. T. U.} \\ (c) \text{ 103.36 cu. ft.} \end{cases}$

6. From the change in heat contents of superheated steam from a pressure of 200 pounds per square inch and 550° F., to 3 pounds per square inch, absolute, find: (*a*) the theoretical work per pound of steam; (*b*) the theoretical steam consumption per horsepower per hour.

Ans. $\begin{cases} (a) \text{ 238,846 ft.-lb.} \\ (b) \text{ 8.29 lb., nearly} \end{cases}$

7. If the brake efficiency in example 6 is .59, find: (*a*) the effective work per pound of steam; (*b*) the steam consumption per brake horsepower per hour.

Ans. $\begin{cases} (a) \text{ 140,919 ft.-lb.} \\ (b) \text{ 14.05 lb., nearly} \end{cases}$

FLOW OF STEAM THROUGH NOZZLES

8. Velocity of Steam.—Suppose steam to flow through a nozzle or channel from a region of higher pressure p_1 to one of lower pressure p_2. If the expansion is adiabatic and there is no friction, the drop in heat contents $H_1 - H_2$ is used in increasing the kinetic energy of the jet of steam. It may be assumed that the velocity of the steam at the entrance to the nozzle is small and consequently may be neglected. If c_0 denotes the theoretical velocity with which the steam emerges from the nozzle, then, by the principles of mechanics, the kinetic energy per pound of steam is $\frac{c_0^2}{2g}$ foot-pounds. This is the precise equivalent of the theoretical work of the heat given up during expansion; hence, substituting $\frac{c_0^2}{2g}$ for W in formula **1**, Art. **5**,

$$\frac{c_0^2}{2g} = 778(H_1 - H_2)$$

or, as $g = 32.16$, the equation may be reduced to

$$c_0 = \sqrt{64.32 \times 778(H_1 - H_2)} = 223.7\sqrt{H_1 - H_2}$$

The heat drop $H_1 - H_2$ for a given adiabatic expansion is readily found from the Heat Chart.

EXAMPLE.—Find the theoretical velocity that will be generated by the adiabatic expansion of steam in a nozzle from a pressure of 150

pounds per square inch, absolute, and superheated to 600° F., to a pressure of 2.5 pounds per square inch, absolute.

SOLUTION.—Reference to the Heat Chart will show that $H_1 - H_2 = 310$ B. T. U.; hence, applying the formula,

$c_0 = 223.7\sqrt{H_1 - H_2} = 223.7\sqrt{310} = 3{,}939$ ft. per sec., nearly. Ans.

To reduce the work of calculation, a velocity scale, calculated by this formula, has been added at the right of the Heat Chart, by means of which the steam velocity may be laid off directly. The distance that measures the heat drop $H_1 - H_2$ along a line of adiabatic expansion is laid off from the top O of the velocity scale, down, and the velocity is read off directly from the scale in feet per second.

NOTE.—In this example, the heat drop is 310 British thermal units. Laying off, on the velocity scale, the distance that represents 310 British thermal units, the velocity is found to be about 3,940 feet per second, which differs but slightly from that found by calculation.

9. Friction in Nozzles.—Frictionless flow of steam through a nozzle is never attained in practice. There is always some friction between the steam and the nozzle walls, and this causes a reduction in the final velocity of the steam. The friction of the steam in the nozzle generates heat, most of which is returned to the steam, raising its quality. Hence, the friction work is not entirely lost. Reliable data on the friction of steam in nozzles are, however, lacking. From the few experiments that have been made, it is probably safe to assume a friction loss of 10 per cent. of the total heat drop as an average value for nozzles.

Friction losses in nozzles may now be readily taken into account in using the Heat Chart. The manner of doing this is shown in Fig. 3, which represents a portion of the Heat Chart. Suppose the point A, on the pressure line p_1, to represent the initial condition of the steam, when each pound contains H_1 British thermal units. Adiabatic expansion to the final pressure p_2 gives the lower point B where the vertical adiabatic line AB crosses the pressure line p_2. From B lay off $BC = \frac{1}{10}AB$ upwards to represent the friction loss in British thermal units; and from C draw the horizontal line CD intersecting the line p_2 at D. Then, the distance AC on AB represents the net drop in heat contents per pound of steam in expanding adiabatically through a nozzle from p_1 to p_2 with a friction loss CB. If AC is laid off on the velocity

scale, at the right of Fig. 3, it will give the velocity of the steam as it leaves the nozzle. The point D on the pressure line represents the final state of the steam, both as to heat

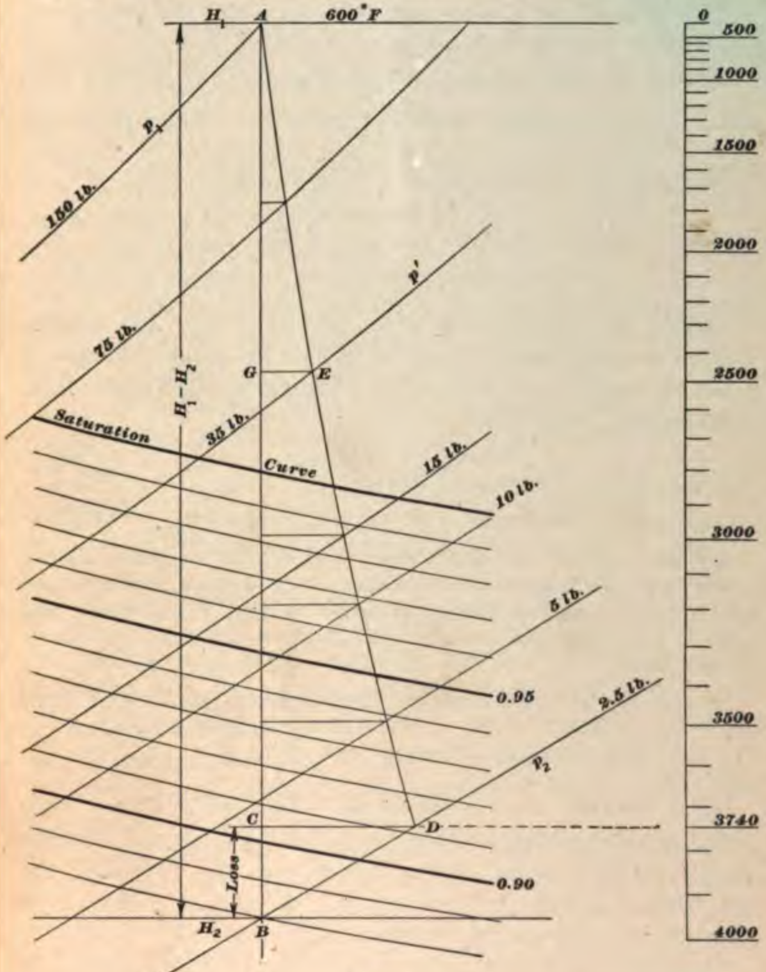


FIG. 3

contents and as to dryness. It will be observed, by referring to the quality lines through B and D, Fig. 3, that, while B and D are both on the pressure line p_2, D represents a higher quality

or a dryer state than B. This is because the work of friction has evaporated some of the moisture that was produced by the expansion.

The ratio between the actual velocity c_1 and the theoretical frictionless velocity c_0 may be obtained as follows: Let m represent the fraction of the heat drop $H_1 - H_2$ lost by friction; that is, m is the ratio $\frac{BC}{AB}$, Fig. 3. Then, from the kinetic-energy formula for 1 pound of steam,

$$\frac{c_1^2}{2g} = (1 - m)\frac{c_0^2}{2g}$$

or

$$c_1 = c_0\sqrt{1 - m} \qquad (1)$$

Taking the friction loss m as .10, $\sqrt{1 - m} = .95$, nearly. Hence, in most cases, it will be proper to assume a loss of 5 per cent. of the velocity in the passage through the nozzle, and formula **1** will become

$$c_1 = .95\,c_0 \qquad (2)$$

Then, by the formula of Art. **8**,

$$c_1 = .95 \times 223.7\sqrt{H_1 - H_2} = 212.5\sqrt{H_1 - H_2} \qquad (3)$$

EXAMPLE.—Find the actual velocity of exit of the steam from a nozzle when the initial state of the steam is a pressure of 150 pounds per square inch, absolute, superheated to 600° F., and the final state is 2.5 pounds per square inch, absolute, the heat loss due to friction being .1.

SOLUTION.—The heat drop is found from the Heat Chart to be $H_1 - H_2 = 310$ B. T. U. per lb. Hence, applying formula **3**,

$$c_1 = 212.5\sqrt{310} = 3{,}741 \text{ ft. per sec., nearly. Ans.}$$

10. Nozzle Cross-Sections.—The factors that enter into the determination of the proper cross-section of a steam nozzle, at any point of its length, are the pressure, the resulting jet velocity, the weight of steam flowing per second, and the specific volume of the steam.

Referring to Fig. 3, let a curve be drawn from A to D by locating other points in the same manner as that in which D was found. This curve will represent the successive conditions of the steam in its expansion in the nozzle. Thus, at some intermediate pressure p', the point E will represent

the condition of the steam and will show whether it is superheated or contains moisture. If E is projected horizontally to G, $A\,G$ gives the drop of heat contents; and, if laid off on the velocity scale, gives the velocity of the steam at the cross-section at which the pressure is p'. Now, let F denote the area of the cross-section of the nozzle, and V the volume of 1 pound at the pressure p' and condition denoted by the point E. The volume may be found by the formulas of Art. 4. The volume passing any cross-section is equal to the product of the area of the cross-section and the velocity. It is also equal to the number of pounds of steam flowing per second multiplied by the volume per pound.

Let G = number of pounds of steam flowing per second;
v = volume of 1 pound, in cubic feet;
c_1 = velocity, in feet per second;
F = area of cross-section, in square feet;
F_1 = area of cross-section, in square inches.

Then,
$$F c_1 = G v$$

Hence,
$$\left.\begin{aligned} F &= \frac{G v}{c_1} \text{ (when } F \text{ is in square feet)} \\ \text{or} \quad F_1 &= \frac{144\, G v}{c_1} \text{ (when } F_1 \text{ is in square inches)} \end{aligned}\right\} \qquad (1)$$

In these formulas, G must be known, v found from the condition of the steam, as indicated by the point E of Fig. 3, or on the Heat Chart, and c_1 found by laying off $A\,G$ on the velocity scale. In this way, the area of the cross-section of the nozzle can be found for any number of pressures between the initial and the final pressures.

It is not customary to apply this method for all the cross-sections; but if it is carried out, one peculiar thing will be noticed. The cross-section will be a minimum at the point where the pressure is .57 of the initial pressure. Thus, for an initial pressure of 100 pounds, absolute, the smallest section or *throat* of the nozzle will be at a pressure of 57 pounds. In the foregoing statement, it is assumed that the final pressure is less than 57 per cent. of the initial pressure. If this is not the case—that is, if the drop in pressure

through the nozzle is small, as in multiple-stage turbines —the walls of the passage will converge and the smallest section will be at the outlet end. Stated in brief, if p_2 is less than .57 p_1, the nozzle must have a throat and afterwards diverge as shown in Fig. 4; but if p_2 is greater than .57 p_1, the passage merely converges, as shown in Fig. 5.

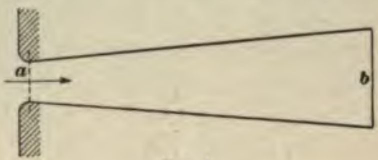


FIG. 4

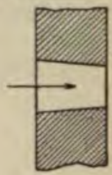


FIG. 5

Let p_1 = initial pressure, in pounds per square inch;
p' = throat pressure, in pounds per square inch.

Then, the statement of the throat pressure can be expressed as follows:

$$p' = .57\, p_1 \qquad (2)$$

11. Construction of the Nozzle.—In practice, the nozzle is bored out with a uniform taper, or conical, for the part in which the expansion takes place. This is shown in Fig. 4, where the taper is uniform from the throat a to the outlet b. The area of cross-section of the nozzle at the throat is determined from the pressure p' of formula **2**, Art. **10**, and at the large end it is found from the final pressure p_2, and the final condition represented by the point D, Fig. 3. The resulting conical form of nozzle gives good results if the axial length of the nozzle is not too short. On the other hand, too long a nozzle produces excessive loss by friction. The best length of nozzle is a matter for experimental determination. Such a length, however, as will give an angle of about 10° between the sides of the conical part of the nozzle is considered satisfactory and will be used here. A sharper divergence of the sides of the nozzle is liable to cause the stream of steam to separate from the sides of the nozzle and produce eddies.

Let o = diameter, in inches, at the large end, or the outlet diameter;

b = diameter, in inches, of the throat;

c = angle between opposite sides of the nozzle;

L = length of nozzle, in inches.

The length of the nozzle in the tapered portion is expressed by the formula

$$L = \frac{o - b}{2 \tan \frac{c}{2}} \qquad \textbf{(1)}$$

When the angle c is made 10°, the formula reduces to

$$L = 5.715(o - b) \qquad \textbf{(2)}$$

In either of these formulas, L may be in other units than inches when o and b are in the same units; that is, o, b, and L must always be in the same units.

EXAMPLE.—Let it be required to determine: (*a*) the outlet diameter, (*b*) the throat diameter, and (*c*) the length of a nozzle to deliver 982 pounds of steam per hour. The initial pressure of the steam is 150 pounds per square inch gauge and superheated to 480° F. The pressure at the outlet is 2 pounds per square inch, absolute. The energy loss due to friction is 10 per cent.

SOLUTION.—(*a*) The initial pressure, absolute, is

$$150 + 14.7 \text{ lb.} = 164.7 \text{ lb. per sq. in.}$$

From the Heat Chart, locate the initial point and drop a vertical line to the final pressure, as shown in Fig. 6 at $A\,B$. The point A is where the 164.7-lb. line crosses the 480° line; the observed contained heat at this point is, from the Heat Chart, H_1 = 1,251 B. T. U. The point B is on the 2-lb. line, and by its position shows that the final heat contents after adiabatic expansion is 948 B. T. U. Hence, the heat drop is $H_1 - H_2$ = 303 B. T. U. The friction loss is $303 \times .1 = 30.3$ B. T. U. This is laid off from B upwards to C; then, by projecting horizontally to the 2-lb. line, the point D is located, showing the final quality to be nearly .86 dry steam. The heat available for increasing the velocity is $303 - 30.3 = 272.7$ B. T. U. Laying this off on the velocity scale in Fig. 6 from *0* downwards, the exit velocity is found to be 3,690 ft. per sec. From Table I, the volume of 1 lb. of steam at 2 lb. per sq. in., absolute, is 173.6 cu. ft. Hence, the volume for a quality of .86 is from formula **1**, Art. **4**,

$$v = x\,V = .86 \times 173.6 = 149.296, \text{ say, } 149 \text{ cu. ft.}$$

Then, applying formula **1**, Art. **10**, in which $G = \frac{982}{60 \times 60}$; $v = 149$; and $c_1 = 3{,}690$,

$$F_1 = \frac{144\,G\,v}{c_1} = \frac{144 \times 982 \times 149}{60 \times 60 \times 3{,}690} = 1.5861 \text{ sq. in.}$$

If the nozzle is circular, the diameter that will give this cross-section is

$$o = \sqrt{\frac{F_1}{.7854}} = \sqrt{\frac{1.5861}{.7854}} = 1.4211 \text{ in. Ans.}$$

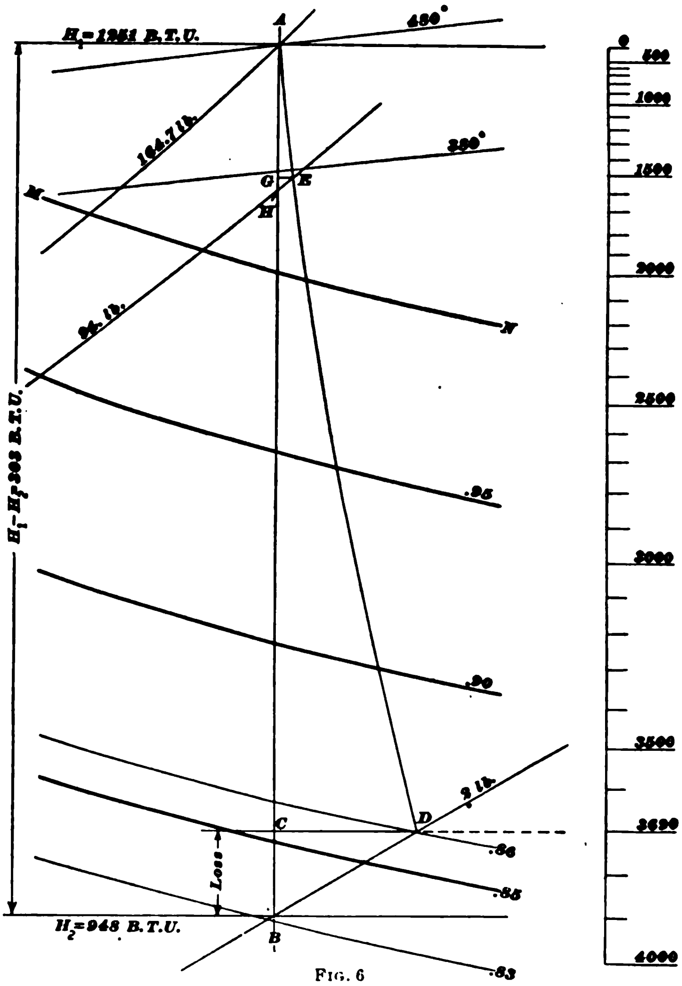


FIG. 6

(*b*) Applying formula **2**, Art. **10**, the throat pressure is

$$p' = .57 \times 164.7 = 93.879, \text{ say 94 lb. per sq. in.}$$

To get the throat cross-section, locate the point E on the 94-lb. line in Fig. 6, in the same manner as the point D was located on the

2-lb. line. This point is found to be in the superheated region at a temperature of about 380° F., or 840° absolute. Then, from formula **2**, Art. **4**, the volume v_1 per pound is

$$v_1 = .591 \times \frac{T}{p} - .135 = .591 \times \frac{840}{94} - .135 = 5.146 \text{ cu. ft.}$$

For this part of the expansion—that is, for $A\,G$—the heat drop is, by measurement, found to be 46 B. T. U.; while, from the velocity scale, the corresponding velocity is found to be 1,520 ft. per sec. Or, by taking the total heat drop $A\,H = 51$ B. T. U., and, using formula **3**, Art. **9**, $c_1 = 212.5\sqrt{51} = 1{,}518$ ft. per sec. Hence, by applying formula **1**, Art. **10**, for the throat,

$$F_1 = \frac{144\,G\,v}{c_1} = \frac{144 \times 982 \times 5.146}{60 \times 60 \times 1{,}520} = .133 \text{ sq. in., nearly}$$

Hence, for a circular nozzle, the diameter is

$$b = \sqrt{\frac{.133}{.7854}} = .4115, \text{ say } .41 \text{ in.} \quad \text{Ans.}$$

(*c*) In order to find the length, apply formula **2**,

$$L = 5.715\,(1.4211 - .4115) = 5.77, \text{ say } 5\tfrac{3}{4} \text{ in.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the theoretical velocity that will be developed by the adiabatic expansion of steam in a nozzle from a pressure of 200 pounds per square inch, absolute, and superheated to 700° F., to a pressure of 1 pound per square inch, absolute. Ans. 4,480 ft. per sec., nearly

2. Find the actual velocity, in example 1, when the heat loss due to friction is 10 per cent. of the heat drop. Ans. 4,256 ft. per sec., nearly

3. Find the outlet diameter of a nozzle to deliver 5 pounds of steam per minute from a pressure of 175 pounds per square inch, absolute, and superheated to 550° F. and expanding to a pressure of 2 pounds per square inch, absolute. Ans. $\frac{3}{4}$ in., nearly

4. Find the throat diameter for the nozzle in example 3. Ans. $\frac{1}{4}$ in., nearly

5. Find the length of the nozzle in examples 3 and 4. Ans. $2\frac{7}{8}$ in., nearly

---

ACTION OF THE JET ON THE BUCKETS

12. Velocities to be Considered.—In connection with the problems relating to the action of a jet of steam on the buckets of a turbine, there are three velocities that should be considered; namely, the velocity of the steam relative to the nozzle, that is, the absolute velocity of the

jet; the velocity of the bucket itself; the velocity of the jet relative to the moving bucket.

Let c_1 = absolute velocity of jet as it enters wheel chamber, in feet per second;

u = velocity of bucket, in feet per second;

w_1 = entrance velocity of jet relative to bucket, in feet per second;

w_2 = exit velocity of jet relative to bucket, in feet per second;

c_2 = absolute velocity of steam as it leaves turbine buckets, in feet per second;

a_1 = angle between c_1 and u;

e_1 = angle between w_1 and u;

a_2 = angle between c_2 and u;

e_2 = angle between w_2 and u.

The relation between these velocities is shown in Fig. 7. The velocity c_1 of the steam jet leaving the nozzle is laid off to some convenient scale from A to B, making the angle a_1 with the plane of the turbine wheel. Then, the velocity u of the turbine buckets is laid off from B to D to the same scale as c_1, making the angle $A\,B\,D$ equal to a_1 and having the direction $D\,B$, as indicated by the arrowhead. The closing side $A\,D$ of the triangle gives the velocity w_1 of the jet relative to the buckets. The angle $A\,D\,E = e_1$ gives the angle between w_1 and u. This is based on the principles of the resolution of velocities.

When the velocities c_1 and u and the angle a_1 are known, the velocity w_1 and the angle e_1 may be found by the following formulas derived from trigonometry:

$$w_1 = \sqrt{c_1^2 + u^2 - 2\,c_1\,u\cos a_1} \qquad (1)$$

and

$$c_1 \sin a_1 = w_1 \sin e_1$$

or

$$\sin e_1 = \frac{c_1 \sin a_1}{w_1} \qquad (2)$$

As the steam jet passes through the wheel, its direction is changed by the curved surfaces of the buckets, and the final velocity of the steam relative to the buckets is given by w_2, Fig. 7, laid off from A to F, making the angle e_2 with the plane of the turbine wheel. The angle e_2 depends on

the angle of the exit side of the bucket. In this case, it is assumed that the angles of the bucket on the entrance and exit sides are the same and e_2 is made equal to e_1. Without friction, $w_2 = w_1$. The final absolute velocity c_2 is found by the graphic method, by laying off $FG = u$ from the end of w_2, and completing the triangle, when $AG = c_2$.

The final velocity c_2 may be calculated by a formula similar to formula **1**,

$$c_2 = \sqrt{w_2^2 + u^2 - 2\,w_2\,u\cos e_2} \qquad \textbf{(3)}$$

The angle a_2 may be found from a formula similar to formula **2**,

$$w_2 \sin e_2 = c_2 \sin a_2$$

or

$$\sin a_2 = \frac{w_2 \sin e_2}{c_2} \qquad \textbf{(4)}$$

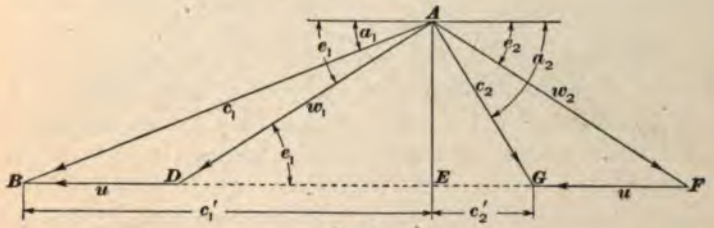


FIG. 7

EXAMPLE.—Steam at a velocity of 3,000 feet per second enters a turbine wheel at an angle of 20°; the angles e_1 and e_2 of the turbine buckets are equal. Find: (*a*) the final absolute velocity of the steam if u = 1,200 feet per second. (*b*) the exit angle a_2 of the steam.

SOLUTION.—(*a*) Applying formula **1**, c_1 = 3,000; u = 1,200; and $\cos a_1 = \cos 20° = .93969$. Hence,

$$w_1 = \sqrt{c_1^2 + u^2 - 2\,c_1\,u\cos a_1}$$
$$= \sqrt{(3{,}000)^2 + (1{,}200)^2 - 2 \times 3{,}000 \times 1{,}200 \times .93969}$$
$$= 1{,}917 \text{ ft. per sec., nearly}$$

From formula **2**,

$$\sin e_1 = \frac{c_1 \sin a_1}{w_1} = \frac{3{,}000 \times .34202}{1{,}917} = .53524$$

Hence, $e_1 = 32° \, 21' \, 36''$, and $\cos e_1 = .84470$.

Next apply formula **3**, with $e_2 = e_1$; $\cos e_2 = .84470$; $w_2 = w_1$ = 1,917; and u = 1,200. Then,

$$c_2 = \sqrt{w_2^2 + u^2 - 2\,w_2\,u\cos e_2}$$
$$= \sqrt{(1{,}917)^2 + (1{,}200)^2 - 2 \times 1{,}917 \times 1{,}200 \times .84470}$$
$$= 1{,}108 \text{ ft. per sec. Ans.}$$

(*b*) Applying formula **4**, with $w_2 = 1{,}917$; $c_2 = 1{,}108$; and $\sin e_2 = .53521$.

$$\sin a_2 = \frac{w_2 \sin e_2}{c_2} = \frac{1{,}917 \times .53524}{1{,}108} = .92604$$

Hence, $a_2 = 67° \, 49' \, 33''$. Ans.

13. Velocities With Friction Considered.—In the actual flow of the steam through turbine buckets, there is considerable decrease of the theoretical velocities on account of the friction between the steam and the blades. The amount of this friction is uncertain; but if friction factors are assumed it is possible to allow for friction in the velocity diagrams, as shown in Fig. 8. The theoretical frictionless velocity is laid off

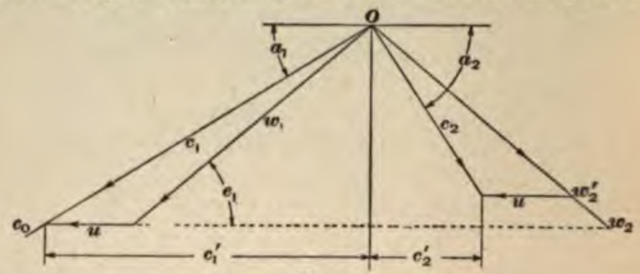


FIG. 8

from O equal to c_0. The actual velocity on leaving the nozzle is, from Art. **9**,

$$c_1 = c_0 \sqrt{1 - m} \qquad \textbf{(1)}$$

As before, w_1 is readily found by laying off u and completing the triangle. Without friction, $w_2 = w_1$; but with friction w_2 is decreased to the smaller velocity w_2'. In general,

$$w_2' = k \, w_1 \qquad \textbf{(2)}$$

in which k = friction factor varying from .8 to possibly .95.

The final velocity c_2 is found by laying off u from w_2' and completing the triangle. In the examples, k will be taken as .9. When the velocities c_1 and u, the angle a_1, and the coefficient k are known, it is possible to calculate the final velocities as in Art. **12**. The formulas for w_1 and $\sin e_1$ apply without change; that is,

$$w_1 = \sqrt{c_1^2 + u^2 - 2 c_1 u \cos a_1} \qquad \textbf{(3)}$$

and

$$\sin e_1 = \frac{c_1 \sin a_1}{w_1} \qquad \textbf{(4)}$$

The formulas for c_2 and sin a_2 become

$$c_2 = \sqrt{(.9\,w_1)^2 + u^2 - 1.8\,w_1\,u\cos e_2} \qquad (5)$$

and

$$\sin a_2 = \frac{.9\,w_1 \sin e_1}{c_2} \qquad (6)$$

EXAMPLE.—Solve the example of Art. **12** when an allowance is made for friction.

SOLUTION.— $c_1 = 3{,}000$ ft. per sec.; $a_1 = 20°$; $u = 1{,}200$ ft. per sec. As the formula for w_1 does not change, $w_1 = 1{,}917$ ft. per sec.; $\sin e_1 = .53524$; and $\cos e_1 = .84470$. Hence, to find c_2, apply formula **5**,

$$c_2 = \sqrt{(.9\,w_1)^2 + u^2 - 1.8\,w_1\,u\cos e_1}$$
$$= \sqrt{(.9 \times 1{,}917)^2 + (1{,}200)^2 - 1.8 \times 1{,}917 \times 1{,}200 \times .84470}$$
$$= 958.8 \text{ ft. per sec.}$$

Applying formula **6**,

$$\sin a_2 = \frac{.9\,w_1 \sin e_1}{c_2} = \frac{.9 \times 1{,}917 \times .53524}{958.8} = .96313$$

Hence, $a_2 = 74° 23' 38''$. Ans.

14. Work of the Jet.—By giving up its energy, the jet of steam that enters the turbine does work in turning the turbine wheel. As the condition of the steam at entrance and at exit is known, it is desirable to derive some basis for calculating the work done. Work is expressed in foot-pounds and signifies force, in pounds, acting through a distance, in feet. The force that the steam exerts on the turbine buckets is that due to the change in its velocity in the direction of motion of the turbine wheel. It is known, from the principles of mechanics, that force equals mass multiplied by acceleration. Referring to Fig. 7, it will be seen that c_1' is the component of c_1 in the direction of motion of the turbine wheel, and that c_2' is the component of c_2 in the same direction. As c_1 and c_2 are velocities in feet per second, and c_1' and c_2' are opposite in direction, $c_1' + c_2'$ is the change, in feet per second, of the component of the velocities c_1 and c_2 in the direction of motion of the turbine wheel. Acceleration is change in velocity per second; hence, $c_1' + c_2'$ is the acceleration of the steam, in feet per second in the direction of the moving buckets. The mass of 1 pound is $\frac{1}{g}$; hence, the force exerted by 1 pound of steam

having its velocity changed from c_1 to c_2, as shown in Figs. 7 and 8, is $\frac{c_1' + c_2'}{g}$.

Let P = force, in pounds, exerted by 1 pound of steam;
g = acceleration of gravity = 32.16;
$c_1' + c_2'$ = acceleration;
W_1 = work of 1 pound of steam in 1 second;
u = velocity of turbine buckets, in feet per second.

c_1' and c_2' may be found from c_1 and c_2 when cos a_1 and cos a_2 are known (see Figs. 7 and 8). Thus, $c_1' = c_1 \cos a_1$ and $c_2' = c_2 \cos a_2$; hence,

$$c_1' + c_2' = c_1 \cos a_1 + c_2 \cos a_2 \qquad (1)$$

The force P exerted by 1 pound of steam with the acceleration $c_1' + c_2'$ is

$$P = \frac{c_1' + c_2'}{g} = \frac{c_1 \cos a_1 + c_2 \cos a_2}{g} \qquad (2)$$

This pressure of the steam acts on the turbine buckets, moving them u feet in 1 second; and, as the force, in pounds, multiplied by the distance, in feet, through which the force acts gives the work, in foot-pounds,

$$W_1 = Pu = \frac{u(c_1' + c_2')}{g} = \frac{u(c_1 \cos a_1 + c_2 \cos a_2)}{g} \qquad (3)$$

For accurate results, friction should be taken into account in finding c_1' and c_2'. This formula applies equally well to velocity and to pressure turbines.

EXAMPLE.—What is the work of 1 pound of steam expanding through a nozzle from a pressure of 150 pounds per square inch, absolute, and a temperature of 600° F., to an absolute pressure of 2.5 pounds per square inch flowing into a turbine at an angle of 20°, while the turbine runs at 1,400 feet per second? The angles of the turbine blades are the same on both the entrance and the exit sides.

SOLUTION.—The heat drop from the Heat Chart is $H_1 - H_2 = 310$ B. T. U. In the example of Art. **9**, the velocity, when considering friction in the nozzle, was found to be c_1 = 3,741 ft. per sec. Angle a_1 = 20°. Applying formula **3**, Art. **13**,

$$\begin{aligned} w_1 &= \sqrt{c_1^2 + u^2 - 2c_1 u \cos a_1} \\ &= \sqrt{(3{,}741)^2 + (1{,}400)^2 - 2 \times 3{,}741 \times 1{,}400 \times .93969} \\ &= 2{,}472 \text{ ft. per sec.} \end{aligned}$$

Applying formula **4**, Art. **13**,

$$\sin e_1 = \frac{c_1 \sin a_1}{w_1} = \frac{3{,}741 \times .34202}{2{,}472} = .51760$$

Then, $\cos e_2 = \cos e_1 = .85563$.

Taking friction into account, and applying formula **5**, Art. **13**,

$$c_2 = \sqrt{(.9 w_1)^2 + u^2 - 1.8\, w_1\, u \cos e_2}$$
$$= \sqrt{(.9 \times 2{,}472)^2 + (1{,}400)^2 - 1.8 \times 2{,}472 \times 1{,}400 \times .85563}$$
$$= 1{,}257 \text{ ft. per sec.}$$

Applying formula **6**, Art. **13**,

$$\sin a_2 = \frac{.9\, w_1 \sin e_1}{c_2} = \frac{.9 \times 2{,}472 \times .51760}{1{,}257} = .91611$$

Hence, $\cos a_2 = .40092$.

In order to find the pressure P, apply formula **2**,

$$P = \frac{c_1 \cos a_1 + c_2 \cos a_2}{g} = \frac{3{,}741 \times .93969 + 1{,}257 \times .40092}{32.16} = 125 \text{ lb.}$$

The work is then found by applying formula **3**,

$$W_1 = Pu = 125 \times 1{,}400 = 175{,}000 \text{ ft.-lb. Ans.}$$

15. Efficiency.—If W represents the theoretical work of the jet, then, from the formulas in Arts. **5** and **8**, the theoretical work per pound of steam is

$$W = \frac{c_0^2}{2g} \tag{1}$$

or

$$W = 778(H_1 - H_2) \tag{2}$$

The effective work at the brake is W_2, which is obtained from formula **3**, Art. **14**, by subtracting the friction of the machine. The indicated efficiency E_1 is, by formula **3** of Art. **14** and formula **1**,

$$E_1 = \frac{W_1}{W} = \frac{2u(c_1' + c_2')}{c_0^2} \tag{3}$$

$$\text{or } E_1 = \frac{u(c_1 \cos a_1 + c_2 \cos a_2)}{778g(H_1 - H_2)} = \frac{u(c_1 \cos a_1 + c_2 \cos a_2)}{25{,}020(H_1 - H_2)} \tag{4}$$

The brake efficiency E_2 is

$$E_2 = \frac{W_2}{W} \tag{5}$$

EXAMPLE.—What was the indicated efficiency in the example of Art. **14**?

SOLUTION.—Applying formula **2** and remembering that $W_1 = 175{,}000$ and that $H_1 - H_2 = 310$,

$$E_1 = \frac{W_1}{W} = \frac{175{,}000}{778 \times 310} = .726, \text{ or } 72.6 \text{ per cent., nearly. Ans.}$$

EXAMPLES FOR PRACTICE

1. Steam issues from a nozzle with a velocity of 3,600 feet per second, and enters a turbine wheel at an angle of 20°. The turbine buckets are the same on both the inlet and the outlet sides. Find the final absolute velocity of the steam as it leaves the turbine wheel, if the turbine wheel runs at 1,400 feet per second and the friction is neglected. Ans. 1,362 ft. per sec.

2. What is the final velocity in example 1 when the friction loss in the buckets is 10 per cent.? Ans. 1,173 ft. per sec.

3. What is the angle between the steam as it leaves the turbine blades and the plane of the turbine wheel, in example 2? Ans. 70° 51′ 30″

4. Find the work done by 1 pound of the steam on the turbine buckets in example 2. Ans. 164,000 ft.-lb.

5. What is the indicated efficiency in examples 2 to 4, with the theoretical velocity of the steam 3,800 feet per second? Ans. 73 per cent.

SINGLE-EXPANSION TURBINE

16. The first item needed in the design of a steam turbine is the weight of steam to be used per hour. The probable steam consumption can be estimated from known results obtained from similar turbines. The curve shown in Fig. 2 may be used as a rough guide. From the steam consumption and the horsepower, the weight of steam per hour can be found. The initial pressure, the degree of superheat, if any, and the condenser pressure must be known. Suitable friction coefficients based on experiments with existing turbines should be assumed. Knowing the steam consumption and the horsepower, a convenient number of nozzles can be chosen and the weight of steam flowing through each nozzle found. The nozzle calculations can then be carried out as in Arts. **10** and **11**.

17. Velocity of Steam Relative to Buckets.—It is now necessary to determine the form of the buckets. This can be done advantageously by laying out the steam and bucket velocities on a drawing board and drawing a bucket larger than full size. The line bc, Fig. 9, represents the

direction of motion of the buckets past the mouth of the nozzle, and its length represents the linear velocity u of the mid-height of the buckets to a given scale.

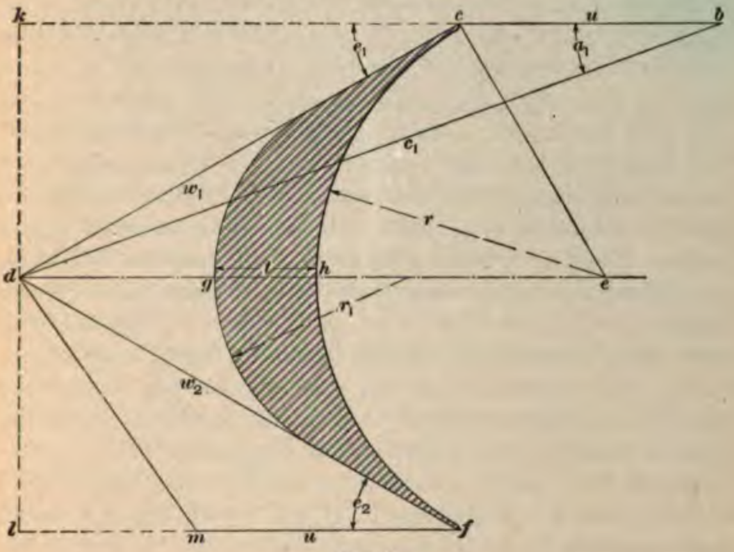


FIG. 9

The bucket velocities used in the De Laval turbine are shown in Table II.

TABLE II

BUCKET VELOCITIES IN DE LAVAL TURBINES

Horsepower of Turbine	Revolutions per Minute	Diameter of Wheel From Center to Center of Blades Inches	Bucket Velocity Feet per Second
5	30,000	3.94	515.75
30	20,000	8.86	773.18
100	13,000	19.68	1,116.32
300	10,600	29.92	1,383.84

Referring again to Fig. 9, the line $b\,d$ represents the direction and linear velocity c_1 of the steam on emerging from the

nozzle. The direction is that of the axis of the nozzle, and the velocity is that calculated for the steam at the outlet of the nozzle and is laid off to the same scale as u. The nozzles in the De Laval turbine are so placed that their axes, in all sizes of turbines, stand at an angle of 20° to the plane of the wheel; that is, the angle a_1 between u and c_1 is 20°. As in the case of a triangle of forces, a triangle of relative velocities may be drawn. Complete the triangle by drawing cd; this side represents the velocity w_1 of the steam relative to the moving buckets to the same scale that bc represents the velocity of the turbine and bd the absolute velocity of the steam. The backs of the inlet sides of the buckets should be parallel to w_1. It is more important for the backs of the turbine buckets to be parallel to the flow of the entering steam than for the faces against which the steam must strike. If the steam strikes the backs and is deflected, it tends to retard the turbine, causing loss; but if it strikes the faces, it gives energy to the turbine without causing loss.

The relative velocity w_1 and the angle e_1 that this relative velocity makes with the plane of the wheel, may be scaled approximately from the drawing or calculated by trigonometry, as in formula **1**, Art. **12**. In the De Laval turbine, the angle e_1 is usually about 30°.

When the angle a_1 is taken as 20°, formula **1**, Art. **12**, becomes

$$w_1 = \sqrt{c_1^2 + u^2 - 1.87938\ c_1 u} \qquad \textbf{(1)}$$

The angle e_1 is found from its sine by formula **2**, Art. **12**; and when $a = 20°$ the formula becomes

$$\sin e_1 = \frac{.34202\ c_1}{w_1} \qquad \textbf{(2)}$$

Experimental results show that u, the velocity of the turbine wheel, should not exceed 1,400 feet per second in practice.

Example.—If the steam, on leaving the nozzle of a turbine, has a velocity of 3,621 feet per second, and the velocity of the turbine buckets is 1,260 feet per second, with an angle of 20° between the axis of the nozzle and the plane of the turbine wheel, what is the velocity of the steam relative to the buckets, and what is the angle that the backs of the buckets should make with the plane of the wheel?

SOLUTION.—Applying formula **1**,

$$w_1 = \sqrt{c_1^2 + u^2 - 1.87938\ c_1\ u}$$
$$= \sqrt{13{,}111{,}641 + 1{,}587{,}600 - 1.87938 \times 3{,}621 \times 1{,}260}$$
$$= 2{,}475 \text{ ft. per sec., nearly. Ans.}$$

Then, from formula **2**,

$$\sin e_1 = \frac{.34202\ c_1}{w_1} = \frac{.34202 \times 3{,}621}{2{,}475} = .50039$$

Hence, from the Table of Natural Sines, the angle $e_1 = 30°$, nearly. Ans.

18. Form of the Buckets.—In the design of buckets, it is customary to make the inlet and outlet sides of the buckets the same for single-stage velocity turbines. This is done for convenience in manufacturing and to reduce the end thrust.

Referring to Fig. 9, the line de is drawn from d parallel to u, and w_2 is laid off below de on df equal to w_1, the angle edf being made equal to the angle cde. From c, draw ce at right angles to cd; then, with e as a center, draw the arc h with a radius r slightly less than ce. The amount that r is less than ce is for the purpose of giving a little thickness to the edge of the bucket; this amount depends on the judgment of the designer. In the absence of better data, r may be taken from .95 to .98 of ce. The radius r may be calculated by trigonometry. Thus,

$$r = k_1 \times \frac{w}{2 \cos e_1}$$

in which k_1 = a constant, varying from .95 to .98;
w = width of bucket, in inches;
r = radius, in inches;
e_1 = angle between w_1 and u.

The curve h will then be the face of the bucket. While this may not be the exact theoretical curve, it should give the desired change in direction to the flow of the steam without shock or excessive friction. The thickness t is next determined; this depends on the necessities of mechanical construction and on the width and height of the buckets; it may be made about .2 the width of the bucket, provided that the bucket is then stiff enough. If the buckets are thick

enough, the stresses in them will be small, as the force acting on each bucket is small. In Fig. 9, $g\,h$ is $.2\,c\,f$, and the radius r_1 is selected so that the arc g is drawn tangent to w_1 and w_2; $c\,g\,f$ then forms the outline of the back of the bucket.

The bucket section shown in Fig. 9 is larger than that used in practice, but this was done for the purpose of showing how to get the outline of the bucket, and it may be reduced to a more desirable size without affecting the principles of its construction. The width of bucket varies considerably in practice, and up to the present time no expression for this dimension has been formulated, the designer depending on his judgment. The practical limits for width vary from .2 to 1 inch.

EXAMPLE.—Let it be required to draw a section of a bucket .5 inch wide and .1 inch thick, when the steam enters the wheel at an angle of 20°, and with a velocity of 3,621 feet per second.

SOLUTION.—In the solution to the example of Art. **17**, the angle e_1 was calculated as 30°. Two parallel lines $b\,c$ and $d\,e$, Fig. 10, are first drawn, at a distance of $w = .5$ inch apart, and the direction of w_1 at the angle e_1 is laid off from f. At a point directly opposite f, the direction of w_2 is laid off at the angle $e_2 = e_1$ to $d\,e$. The position of the center line is then determined by the intersection of w_1 and w_2, and r is calculated from the formula

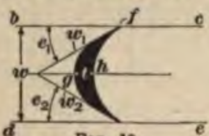


FIG. 10

$$r = k_1 \frac{w}{2 \cos e_1} = .98 \frac{.5}{2 \times .866} = .283 \text{ in., nearly, say } \tfrac{9}{32} \text{ in.}$$

The thickness t is then laid off as $\frac{1}{10}$ in. and the back of the bucket completed by drawing the circular arc g tangent to w_1 and w_2.

19. Passage Between Buckets and Height of Buckets.—In Fig. 11 are shown two buckets on an enlarged scale. The thickness t being known, the distance d between the buckets should be assumed, the dimension being based on data determined by experiment, so as to secure the best results. The distance d being determined, a pair of adjacent buckets should be drawn to scale and the perpendicular distances d_1 and d_2 between the buckets at the points where the steam enters and leaves the wheel, respectively, should be measured.

Let o = diameter, in inches, of nozzle at outlet end;
d = length, in inches, of space between buckets at mid-height (see Fig. 11);
t = thickness, in inches, of buckets at mid-height;
h = height, in inches, of buckets;
c_1 = velocity, in feet per second, of steam on leaving nozzle;
w_1 = velocity, in feet per second, of steam relative to buckets;
a_1 = angle between center line of nozzle and plane of turbine wheel, usually taken as 20°;
D = diameter, in inches, of turbine wheel at mid-height of buckets;
n = number of buckets in complete circumference.

The position of the nozzle in relation to the turbine buckets is shown in Fig. 12. The nozzle is shown at m, the buckets at b, the angle that the nozzle makes with the buckets at a_1, and the outlet diameter at o. The section shown is

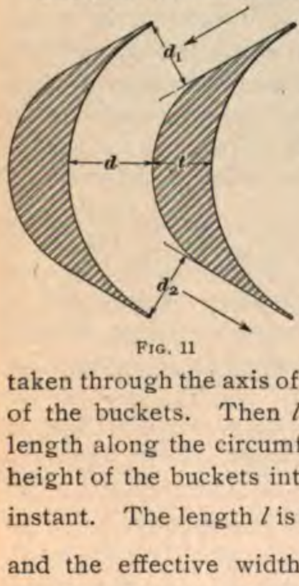


FIG. 11

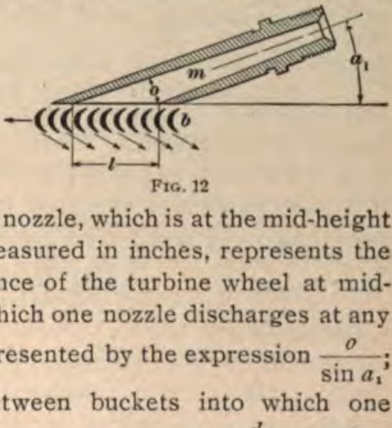


FIG. 12

taken through the axis of the nozzle, which is at the mid-height of the buckets. Then l, measured in inches, represents the length along the circumference of the turbine wheel at mid-height of the buckets into which one nozzle discharges at any instant. The length l is represented by the expression $\frac{o}{\sin a_1}$; and the effective width between buckets into which one nozzle discharges steam at any one instant is $\frac{d}{d+t} \times \frac{o}{\sin a_1}$. This expression multiplied by the height h of the buckets gives $\frac{d\,o\,h}{(d+t)\sin a_1}$, the area of cross-section, in square

inches, between the buckets into which one nozzle discharges steam. Dividing this by 144 gives the area, in square feet; and multiplying the quotient by the relative velocity w_1, in feet per second, gives $\frac{d\,o\,h\,w_1}{144\,(d+t)\sin a_1}$, which represents the volume of steam, in cubic feet, passing through the turbine per second.

The section of the end of the nozzle is an ellipse, with l, Fig. 12, as the longer axis; and since the ellipse is not as wide at the ends as at the middle, it is apparent that more steam passes into the blades at the middle of the nozzle than at the ends.

The height of the blades must be great enough to receive the greatest quantity that may enter. If the nozzles were square, with the sides equal to the diameter, the distribution of the steam would be the same as it actually is at the middle of the circular nozzle. In determining the height of the blade, therefore, it may be supposed that the nozzle is square and has an area at the largest portion equal to the square of the diameter. As c_1 is the velocity of the steam, in feet per second, as it leaves the nozzle, and o is the diameter of the nozzle at the outlet end, in inches, $\frac{c_1 o^2}{144}$ gives the volume of steam, in cubic feet, that, according to this assumption, leaves the nozzle per second. As the steam does not expand in passing through the buckets, the volume that leaves the nozzles must be equal to that passing through the buckets. Placing them equal,

$$\frac{d\,o\,h\,w_1}{144\,(d+t)\sin a_1} = \frac{c_1 o^2}{144}$$

Solving for h,

$$h = \frac{c_1 o\,(d+t)\sin a_1}{d\,w_1} \qquad (1)$$

When $a_1 = 20°$, $\sin a_1 = .34202$, and formula **1** becomes

$$h = .34202 \times \frac{c_1 o(d+t)}{w_1 d} \qquad (2)$$

The diameter D of the turbine wheel from center to center of buckets depends on practical convenience and is to be

decided by experience. The number of buckets is then given by the formula

$$n = \frac{3.1416\,D}{d + t} \qquad (3)$$

If the values found by this method are practically unsuitable, the designer must use his judgment in making changes that will give the most practical proportions.

EXAMPLE.—(*a*) If the outlet diameter of a steam nozzle for a single-stage turbine is 1.38 inches, the velocity of steam at exit is 3,621 feet per second, the angle of the nozzle to the turbine wheel is 20°, and the relative velocity of the steam to the buckets is 2,475 feet per second, what is the height of buckets if the thickness is .1 inch and the space between buckets .179 inch? (*b*) How many buckets will be used if the wheel is 32 inches in diameter?

SOLUTION.—(*a*) Applying formula **2**, the height of the buckets is

$$h = .34202 \times \frac{c_1\,o(d + t)}{w_1\,d} = .34202 \times \frac{3{,}621 \times 1.38(.179 \div .1)}{2{,}475 \times .179} = 1.076 \text{ in. Ans.}$$

In practice, the height would probably be made at least 1.375 in., which is approximately the outlet diameter of the steam nozzle.

(*b*) The number of buckets is determined by applying formula **3**,

$$n = \frac{3.1416\,D}{d + t} = \frac{3.1416 \times 32}{.179 + .1} = 360. \text{ Ans.}$$

MULTIPLE-EXPANSION TURBINES

20. General Method of Procedure.—If a steam turbine is to have several pressure stages, the total drop of heat contents $(H_1 - H_2)$ will be divided among them. With a few-stage turbine, like the Curtis, it is generally possible and advisable to give each stage about the same drop and therefore derive from each about the same amount of work. With a many-stage turbine, like the Rateau velocity turbine or the Parsons pressure turbine, this is not practicable, and more work is done in the stages near the condenser.

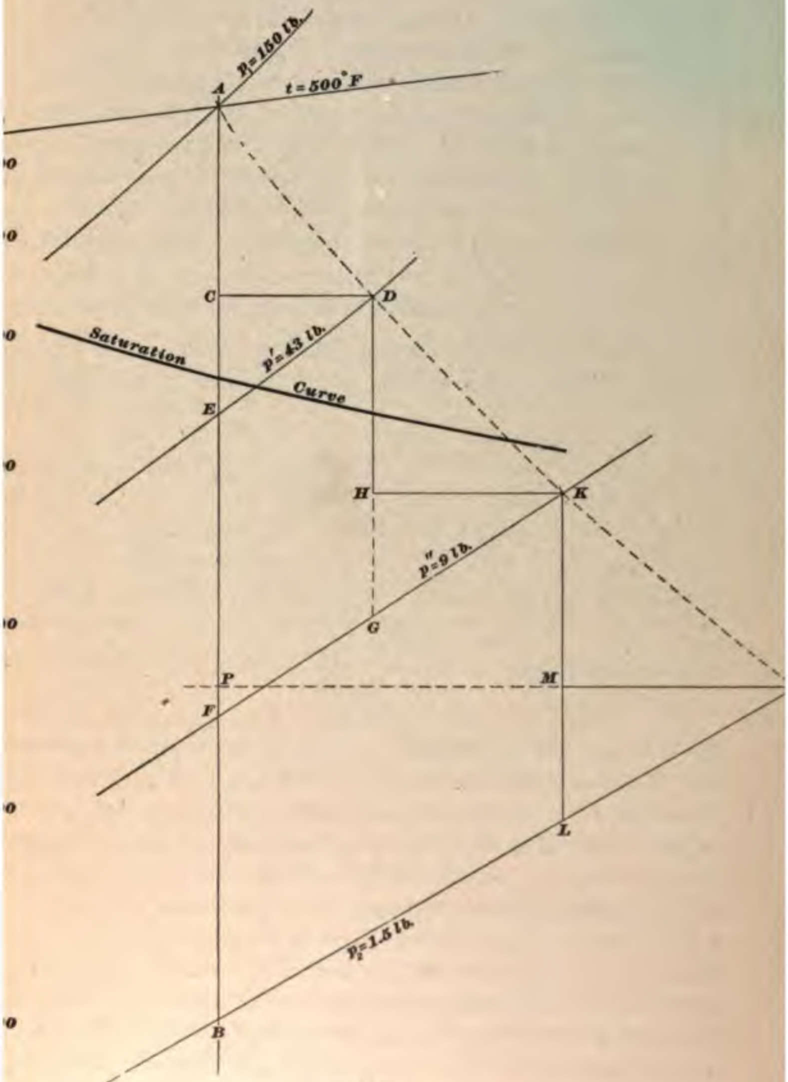
In beginning the design of a turbine, it is necessary to know the initial steam pressure, the degree of superheat, if any, and the condenser pressure. The intermediate pressures in the several cells can then be determined from the way in which the heat drop is divided. Referring to Fig. 3,

the heat drop after subtracting the friction is represented by AC. For a two-stage turbine, the heat drop will be divided into two very nearly equal parts. Let AC be divided into two very nearly equal parts AG and GC; project G to E on the curve AD; and thus determine the intermediate pressure p'. Now, with the pressures p_1 and p', a single-stage turbine may be designed for the first cell; and, with p' and p_2 as extreme pressures, a second single-stage turbine may be designed for the second cell. Evidently, the same course of procedure may be followed for three, four, five, or more stages.

21. Calculations for a Curtis Turbine.—In order to make the method outlined in Art. **20** clear, and also to show the method of solving the problems of multiple velocity stages, an example will be worked out. Such problems may be solved either graphically or by calculation. The graphic method is the simpler, and, as the errors it introduces are within the limits of the accuracy of our knowledge of what takes place in the turbine, it is to be preferred. Enough of both methods, however, will be given here to enable either method to be used.

Let the turbine work with steam at a pressure of 150 pounds per square inch, absolute, and superheated to 500° F., and with a condenser pressure of 1.5 pounds per square inch, absolute. Let the turbine have three pressure stages—that is, three wheels running in three cells—and two velocity stages to each pressure stage. Let the peripheral velocity of the wheels be 400 feet per second. Throughout the turbine, it will be assumed that 10 per cent. of the velocity of the steam is lost in friction in each set of buckets.

In Fig. 13, which is reproduced directly from the Heat Chart, Fig. 1, the point A is located from initial conditions and adiabatic expansion to condenser pressure gives the point B. The total drop AB is found to be 316 British thermal units. If this is divided into three equal parts, each stage will have, say, 105 British thermal units. Of course, this division may be varied slightly if found desirable. The



$p_1 = 150$ lb.
$t = 500°F$
A
C
D
Saturation
Curve
$p' = 43$ lb.
E
H
K
$p'' = 9$ lb.
G
P
M
F
L
$p_2 = 1.5$ lb.
B

FIG. 13

points E and F are thus located on the line AB so that $AE = EF = FB = \frac{1}{3}AB$. The point E will lie on the 43-pound line and the point F on the 9-pound line, very nearly.

Starting with the first cell, the drop in heat contents AE is 105 British thermal units. Taking 10 per cent. friction in the nozzle, the effective heat drop used in giving velocity to the steam jet is $105 \times .9 = 94.5$ British thermal units. Laying this off on the velocity scale at the left, as shown at R, gives a velocity of about 2,175 feet per second.

The velocity may also be calculated by using formula **3**, Art. **9**:

$$c_1 = 212.5\sqrt{H_1 - H_2} = 212.5\sqrt{105} = 2{,}177 \text{ feet per second}$$

22. Next, construct the velocity diagram shown in Fig. 14. Let the angle between the jet and the plane of the wheel be

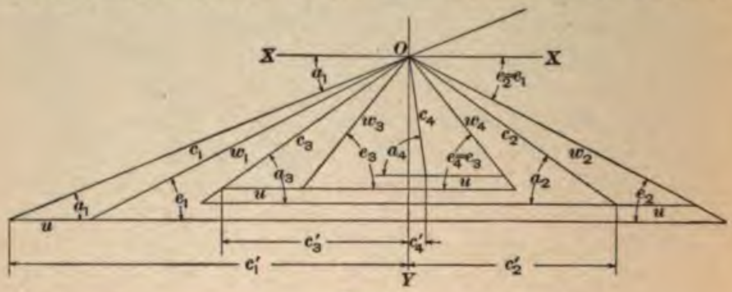


FIG. 14

$22\frac{1}{2}°$. Then lay off c_1 to some convenient scale to represent 2,177 feet per second, and making an angle $a_1 = 22\frac{1}{2}°$ with XX. Then lay off $u = 400$ feet per second and complete the triangle, finding the velocity $w_1 = 1{,}815$ feet per second, nearly, as the velocity of the jet relative to the buckets. Next, lay off a line equal in length to w_1, making the angle e_2 with XX equal to the angle e_1. Then, .9 of its length represents w_2 and measures about 1,633 feet per second. Combining w_2 and u gives c_2, the absolute velocity of the steam as it leaves the first set of moving blades and enters the guide vanes. Measuring c_2 it is found to equal 1,290 feet per second.

In passing through the guide vanes, the direction is changed from c_2 to c_3, making the angle a_3 equal to the

angle a_2; but, as there is 10 per cent. lost in friction, $c_3 = .9\,c_2$ or 1,161 feet per second. The velocity relative to the fixed vanes is the absolute velocity with which the steam enters the second set of moving blades. Combining this velocity c_3 with u, the velocity of the blades, gives w_3 as the velocity of the steam relative to the second set of moving buckets at entrance. Laying off a line equal to w_3 on the other side of $O\,Y$, and making the angle e_4 equal to the angle e_3, and deducting 10 per cent. for friction, gives w_4 as the relative velocity of the steam as it leaves the second set of moving blades. Combining the velocity w_4 with u gives c_4 as the absolute velocity of the steam as it leaves the second set of moving blades. Scaling w_3, it is found to equal 865 feet per second, w_4 equals about 780 feet per second, and c_4 equals about 615 feet per second. It is desired to find the work per pound of steam in this stage. To do this, find the components of c_1, c_2, c_3, and c_4 in the direction of $X\,X$; they are c_1', c_2', c_3', and c_4'. Scaling these values, it is found that c_1' equals 2,010 feet per second, c_2' equals 1,050 feet per second, c_3' equals 940 feet per second, and c_4' equals 90 feet per second, nearly. Substituting these values in formula **3**, Art. **14**, the work of the first wheel is

$$W_1 = \frac{u\,(c_1' + c_2')}{g} = \frac{400\,(2{,}010 + 1{,}050)}{32.16} = 38{,}060 \text{ foot-pounds}$$

The work for the second wheel is, also,

$$W_1 = \frac{u\,(c_3' + c_4')}{g} = \frac{400\,(940 + 90)}{32.16}$$
$$= 12{,}811 \text{ foot-pounds, nearly}$$

Hence, the total work of the first stage is, by the graphic method,

$$38{,}060 + 12{,}811 = 50{,}871, \text{ say } 51{,}000, \text{ foot-pounds, nearly}$$

23. The entire solution may be calculated by applying the preceding formulas as follows: knowing c_1, a_1, and u, it is possible to find w_1 from formula **1**, Art. **12**,

$$w_1 = \sqrt{c_1^2 + u^2 - 2\,c_1\,u\cos a_1}$$
$$= \sqrt{(2{,}177)^2 + (400)^2 - 2 \times 2{,}177 \times 400 \times .92388}$$
$$= 1{,}814 \text{ feet per second}$$

Then, $w_2 = .9\,w_1 = .9 \times 1{,}814 = 1{,}632.6$ feet per second.

In order to find c_2, it is necessary to find $\cos e_2$ or $\cos e_1$, as they are equal. Applying formula **2**, Art. **12**,

$$\sin e_1 = \frac{c_1 \sin a_1}{w_1} = \frac{2{,}177 \times .38268}{1{,}814} = .45926$$

Hence, from the Table of Natural Sines and Cosines, $\cos e_1 = \cos e_2 = .8883$.

To find the velocity c_2, apply formula **5**, Art. **13**,

$$c_2 = \sqrt{(.9\,w_1)^2 + u^2 - 1.8\,w_1 u \cos e_2}$$

$$= \sqrt{(.9 \times 1{,}814)^2 + (400)^2 - (1.8 \times 1{,}814 \times 400 \times .8883)}$$

$$= 1{,}290 \text{ feet per second}$$

Then, $c_3 = .9\,c_2 = .9 \times 1{,}290 = 1{,}161$ feet per second. In order to find w_3, it is necessary to find $\cos a_2 = \cos a_3$. Hence, applying formula **6**, Art. **13**,

$$\sin a_2 = \frac{.9\,w_1 \sin e_1}{c_2} = \frac{.9 \times 1{,}814 \times .45926}{1{,}290} = .58123$$

From the Table of Natural Sines and Cosines, $\cos a_2 = \cos a_3 = .81374$.

To find the velocity w_3, apply formula **3**, Art. **12**,

$$w_3 = \sqrt{c_3^2 + u^2 - 2\,c_3 u \cos a_3}$$

$$= \sqrt{(1{,}161)^2 + (400)^2 - 2 \times 1{,}161 \times 400 \times .81374}$$

$$= 867 \text{ feet per second}$$

Then, $w_4 = .9\,w_3 = .9 \times 867 = 780.3$ feet per second.

In order to find c_4, it is necessary to find $\cos e_4 = \cos e_3$ by applying formula **4**, Art. **12**,

$$\sin e_3 = \frac{c_3 \sin a_3}{w_3} = \frac{1{,}161 \times .58123}{867} = .77833$$

Hence, by the Table of Natural Sines and Cosines, $\cos e_3 = .62786$.

In order to find c_4, apply formula **3**, Art. **12**,

$$c_4 = \sqrt{w_4^2 + u^2 - 2\,w_4 u \cos e_4}$$

$$= \sqrt{(780.3)^2 + (400)^2 - 2 \times 780.3 \times 400 \times .62786}$$

$$= 614 \text{ feet per second}$$

The $\cos a_4$ will also be needed to find c_4'; hence, applying formula **6**, Art. **13**,

$$\sin a_4 = \frac{.9\,w_3 \sin e_3}{c_4} = \frac{.9 \times 867 \times .77833}{614} = .98914$$

From the Table of Natural Sines and Cosines, $\cos a_4 = .14695$.

The values of c_1', c_2', c_3', and c_4' may now be calculated as in Art. **14**,

$$c_1' = c_1 \cos a_1 = 2{,}177 \times .92388 = 2{,}011 \text{ feet per second}$$
$$c_2' = c_2 \cos a_2 = 1{,}290 \times .81374 = 1{,}050 \text{ feet per second}$$
$$c_3' = c_3 \cos a_3 = 1{,}161 \times .81374 = 945 \text{ feet per second}$$
$$c_4' = c_4 \cos a_4 = 614 \times .14695 = 90 \text{ feet per second}$$

By comparing these values with those obtained by the graphic method, it will be seen that they agree fairly well, and may be considered as checking one another.

24. If the total work for the first cell is recalculated for these values, it will be found to be 51,033 foot-pounds. Using the 51,000 previously found by the graphic method, the heat equivalent is found to be 51,000 ÷ 778 = 65.6 British thermal units, nearly. Hence, of the total heat drop, 105 British thermal units, of the first stage, 65.6 British thermal units has been changed into useful work. The remainder, 39.4 British thermal units, has been expended in friction.

Now, from *A*, Fig. 13, lay off *A C* equal to 65.6 British thermal units, and draw *C D* horizontally to cut the pressure line p' in *D*. Then *D* represents the state of the steam as it enters the second set of nozzles. As the saturation curve, laid off as in the Heat Chart, Fig. 1, passes between *D* and *E*, it shows that *E* is in the saturated region and *D* is in the superheated region. Hence, the friction has evaporated all the moisture and has superheated the steam.

The steam starts into the second pressure stage in the state represented by *D*; hence, to find the heat drop, the vertical line *D G* is drawn to the 9-pound line. Measuring the length of *D G* to the heat scale on the Heat Chart, Fig. 1, shows the heat drop in the second stage to be 112 British thermal units. Deducting 10 per cent. for nozzle friction leaves 100.8 British thermal units available for producing velocity; and, from the velocity scale, either on the Heat Chart or on Fig. 13, by laying off 100.8 British thermal units or .9 *D G*, the exit velocity from the nozzles will be 2,250 feet

per second, nearly. This may be checked by applying formula **3**, Art. **9**,

$$c_1 = 212.5\sqrt{H_1 - H_2} = 212.5\sqrt{112} = 2{,}249 \text{ feet per second}$$

25. The velocity diagram for the second stage is now drawn as shown in Fig. 15. For convenience, all velocities have been drawn on one side of the axis. The velocity c_1 with which the steam enters the first wheel is the same as the exit velocity from the nozzles, equal to 2,250 feet per second. The velocity u of the turbine buckets is the same as before, equal to 400 feet per second, and the angle a_1 equals $22\frac{1}{2}°$. Draw w_1 to represent the velocity of the steam relative to the buckets at entrance; measuring w_1, it is found to equal

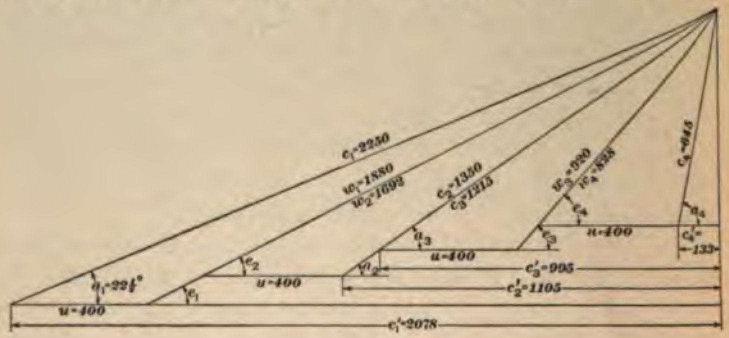


FIG. 15

1,880 feet per second, nearly, to the same scale that c_1 equals 2,250 feet per second. The velocity of the steam relative to the buckets as it leaves the first wheel, allowing 10 per cent. for friction, is w_2, equal to .9 w_1, or 1,692 feet per second. Combining w_2 with u, the absolute velocity c_2 of exit from the first wheel is found to be 1,350 feet per second. The absolute velocity c_3 of entrance to the second wheel, allowing 10 per cent. for the friction in the guide vanes, is .9 c_2, equal to 1,215 feet per second. Combining c_3 and u, the velocity w_3 of the steam relative to the second wheel at entrance is found, by measurement, to be 920 feet per second, and the relative velocity w_4 at exit, allowing 10 per cent. for friction, is 828 feet per second. Combining w_4 with u gives the final absolute velocity c_4

of exit from the second wheel as 645 feet per second. Projecting c_1, c_2, c_3, and c_4 to lines parallel to u, that is, parallel to the direction of motion of the wheel, gives c_1' equal to 2,078 feet per second; c_2' equal to 1,105 feet per second; c_3' equal to 995 feet per second; and c_4' equal to 133 feet per second.

Owing to slight inaccuracies in drawing and measuring lines, the calculated values will not be likely to agree exactly with the plotted values, but either set of values will be close enough for practical purposes.

26. The velocities of the second stage may be calculated by applying the formulas in Arts. **12** to **14**. First, from formula **3**, Art. **13**, the relative velocity w_1 is found to be

$$\begin{aligned} w_1 &= \sqrt{c_1^2 + u^2 - 2\,c_1\,u\cos a_1} \\ &= \sqrt{(2{,}250)^2 + (400)^2 - 2 \times 2{,}250 \times 400 \times .92388} \\ &= 1{,}887 \text{ feet per second} \end{aligned}$$

Then, from formula **4**, Art. **13**,

$$\sin e_1 = \frac{c_1 \sin a_1}{w_1} = \frac{2{,}250 \times .38268}{1{,}887} = .4563$$

From the Table of Natural Sines and Cosines, $\cos e_1 = .88982$.

The relative velocity $w_2 = .9\,w_1$, or $.9 \times 1{,}887 = 1{,}698$ feet per second. The absolute velocity c_2 is found from formula **3**, Art. **12**, to be

$$c_2 = \sqrt{w_2^2 + u^2 - 2\,w_2\,u\cos e_2} = 1{,}354 \text{ feet per second}$$

The absolute velocity c_3 equals $.9\,c_2$, or 1,219 feet per second, nearly. Using formula **4**, Art. **12**,

$$\sin a_2 = \frac{w_2 \sin e_2}{c_2} = .57223$$

From the Table of Natural Sines and Cosines, $\cos a_2 = .82009$.

Next solve for w_3 by formula **3**, Art. **13**, remembering that $a_2 = a_3$,

$$w_3 = \sqrt{c_3^2 + u^2 - 2\,c_3\,u\cos a_3} = 920 \text{ feet per second, nearly}$$

Then, $w_4 = .9\,w_3 = .9 \times 920 = 828$ feet per second.

Applying formula **4**, Art. **13**, gives $\sin e_3 = \dfrac{c_3 \sin a_3}{w_3} = .75820$. From the Table of Natural Sines and Cosines,

$\cos e_3 = .65202$. Applying formula **3**, Art. **12**, for c_4, remembering that $e_3 = e_4$,

$c_4 = \sqrt{w_4^2 + u^2 - 2\,w_4\,u\cos e_4} = 643$ feet per second, nearly

Applying formula **4**, Art. **12**, for $\sin a_4$,

$$\sin a_4 = \frac{w_4 \sin e_4}{c_4} = .97634$$

From the table, $\cos a_4 = .21625$. Then, solving for the components of the velocities c_1, c_2, c_3, and c_4 in the direction of motion of the wheel, as in Art. **14**,

$$c_1' = c_1 \cos a_1 = 2{,}250 \times .92388 = 2{,}079 \text{ feet per second}$$
$$c_2' = c_2 \cos a_2 = 1{,}354 \times .82009 = 1{,}110 \text{ feet per second}$$
$$c_3' = c_3 \cos a_3 = 1{,}219 \times .82009 = 1{,}000 \text{ feet per second}$$
$$c_4' = c_4 \cos a_4 = 643 \times .21625 = 139 \text{ feet per second}$$
$$\text{Total, } 4{,}328 \text{ feet per second}$$

By the graphic solution, the total would be 4,311 feet per second, which checks very closely for such work. For convenience, use 4,300 feet per second.

The total work for the second stage is found as indicated in formula **3**, Art. **14**,

$$W_1 = \frac{u\,(c_1' + c_2' + c_3' + c_4')}{g} = \frac{400 \times 4{,}300}{32.16} = 53{,}483,$$

say 53,500, feet per second.

27. The heat equivalent of this work is $53{,}500 \div 778 = 68.77$ British thermal units, nearly. Now, from D, Fig. 13, lay off DH on DG, to represent 68.77 British thermal units. Draw HK horizontally to the 9-pound pressure line, giving K as the point representing the state of the steam entering the third set of nozzles. Drop the vertical from K to L on the 1.5-pound line, representing the adiabatic expansion from p'' to p_2, and measure its length by the heat scale on the Heat Chart, Fig. 1. The heat drop KL is found to be practically 112 British thermal units, the same as in the second stage. Hence, the velocity diagram for the third stage will be the same as in the second stage and may be represented by Fig. 15.

The work of the third stage will be the same as that of the second stage.

28. To determine the final condition of the steam, lay off KM, Fig. 13, equal to 68.77 British thermal units, and draw MN horizontally to cut the line p_3 at N. Then, N represents the final state of the steam entering the condenser. Reference to the Heat Chart, Fig. 1, shows a quality of about .94. Producing MN to P on AB gives AP as the total useful heat drop, which is equal to 203 British thermal units, nearly.

The total work of all three wheels is

$$51{,}000 + 53{,}500 + 53{,}500 = 158{,}000 \text{ foot-pounds}$$

The heat equivalent of this is $158{,}000 \div 778 = 203$ British thermal units, nearly. Hence, of the 316 British thermal units available from the adiabatic expansion, 203 British thermal units has been utilized. The indicated efficiency is, therefore,

$$E_1 = 203 \div 316 = .6424, \text{ or } 64.24 \text{ per cent.}$$

If, of the 203 British thermal units changed to work, 20 per cent., say, is used in overcoming journal friction and other losses not connected with the steam, the friction loss will be $203 \times .20 = 40.6$ British thermal units. This leaves $203 - 40.6 = 162.4$ British thermal units for work at the brake; the brake efficiency is, therefore,

$$E_2 = 162.4 \div 316 = .514 \text{ nearly, or } 51.4 \text{ per cent.}$$

The steam consumption per brake horsepower is, from formula **4**, Art. **6**,

$$S = 2{,}545 \div 162.4 = 15.67 \text{ pounds}$$

29. Nozzle Calculations.—From the probable steam consumption and the known required capacity of the turbine, the weight of steam required per second can be determined. Dividing this by the number of nozzles employed gives the weight of steam delivered to each nozzle. Then, having for each stage the initial and final pressures and the condition as given in Fig. 13, the nozzle can be proportioned precisely as was explained in Arts. **10** and **11**.

Suppose that the turbine, in the example worked out in Arts. **21** to **28**, is to deliver 1,500 kilowatts at full-load capacity. The corresponding brake horsepower will be,

approximately, $\frac{1{,}500}{.746 \times .9} = 2{,}234$, and the steam required per second will be $\frac{2{,}234 \times 15.67}{60 \times 60} = 9.72$ pounds. If this is passed through twenty-four nozzles, each must take a flow of $9.72 \div 24 = .405$ pound per second.

30. Blade Calculations.—The velocity diagram, Fig. 14, gives the angles for the blades of the first stage. Thus, in Fig. 16, the outer edges of the first blade are

FIG. 16

parallel to w_1 and w_2, Fig. 14; those of the fixed intermediate blade are parallel to c_2 and c_3; and those of the last blade are parallel to w_3 and w_4. The blades may now be laid off as in Figs. 9 and 11.

The required height of blade for any stage may be calculated from the known weight and condition of the steam flowing. Thus, in the example considered in Arts. 21 to 28, the steam flows into the condenser at a pressure of 1.5 pounds per square inch, absolute, and a quality of .94. At this pressure, the volume per pound of the steam is 227.9 cubic feet, nearly (see Table I). Hence, the volume flowing through the nozzles per second is

$$9.72 \times 227.9 \times .94 = 2{,}082 \text{ cubic feet, nearly}$$

If the turbine makes 1,200 revolutions per minute, the mean circumference of the turbine wheel for a peripheral velocity of 400 feet per second must be

$$400 \times 60 \div 1{,}200 = 20 \text{ feet} = 240 \text{ inches}$$

Suppose that the buckets are spaced .875 inch apart, and that the clear space for the exit of steam (dimension d_s, Fig. 11) is .5 inch. There are $240 \div .875 = 274$ buckets on the circumference; hence, the clear space for the exit of the steam is $274 \times .5 = 137$ inches. As determined in Art. **26**, the final exit velocity w_4 relative to the last buckets was 828 feet per second. At the exit of the steam from the second row of moving buckets the space between the buckets is probably filled with steam, and the total area between the buckets may be found by dividing the total volume per second by the velocity at exit. Hence, for a flow of 2,082 cubic feet per second, the total sectional area through the blades must be $2{,}082 \div 828 = 2.51$ square feet $= 361.4$ square inches, nearly

The height of the blades may now be found by dividing the area through the buckets by the clear space at exit. Thus, the height of the last row of buckets in the third stage $= 361.4 \div 137 = 2.638$, say $2\frac{5}{8}$ inches.

With smaller circumferential velocities at the same number of revolutions per minute, the diameter of the wheel will be smaller, and in consequence the blades must be larger. Evidently, then, with more pressure stages (which calls for smaller circumferential speed), the blade lengths throughout must be greater.

31. Examples of Blade Construction.—Fig. 17 shows the outline of the backs of the blades used on a Curtis steam turbine of 1,500 kilowatts rated capacity, making 900 revolutions per minute at maximum efficiency. The initial steam pressure is 150 pounds per square inch gauge; the steam pressure at the end of expansion from the first set of nozzles is about equal to that of the atmosphere, and at the end of the second row of nozzles the steam pressure is 1 pound, absolute, the expansion being divided into two stages. There are four rows of blades in each stage, that is,

four velocity stages to each pressure stage. The section of each blade is composed of arcs and tangents, as shown in Fig. 17. With the line AB as the axis of the turbine, the angle b is 20° in the first row and 60° in the last row of blades. The angle c is 30° in the first row and 60° in the last row. The rows of blades are numbered consecutively in the direction of the steam flow. The arrows a show the direction of flow through the blades.

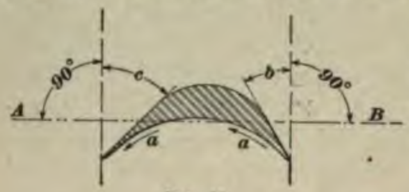


Fig. 17

Fig. 18 shows other details of the same blades. The width is shown at a, which is 1.3125 inches in the high-pressure, or first, stage of the expansion and 1.625 inches in the low-pressure, or second, stage. The dimension b shows the distance between centers of blades, which is .75 inch between the high-pressure and 1.0625 inches between the low-pressure blades. The blades are drawn down to a rather sharp edge, where steam enters and leaves. A shroud that joins the outer ends of the blades is shown at c, and the main wheel is shown at d. The width of the stationary blades is equal to the width of the moving blades in both stages. The space between the rows of moving blades exceeds the width of stationary blades by about $\frac{1}{8}$ inch in both stages. There is the same clearance for both moving blades and fixed blades.

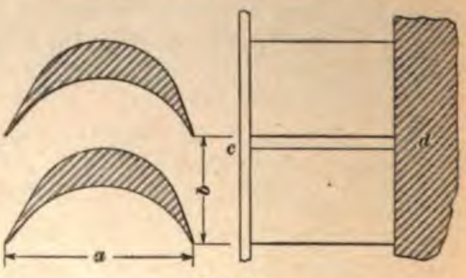


Fig. 18

Since the velocity of flow of the steam is decreased in stages, that is, by every row of moving blades through which it passes, there must be a greater area of cross-section of channel for it to flow through in the later rows of blades than in the earlier ones. This can be provided by continually increasing the height of the blades or by

increasing both the height and the space between the blades. The following are approximate dimensions for the heights of successive rows of blades, as indicated by the letters in

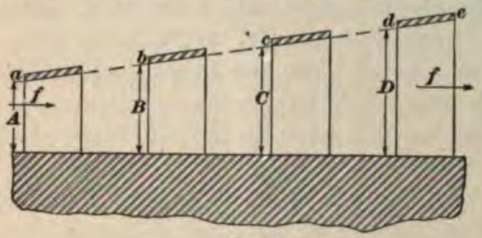


FIG. 19

Fig. 19, for the turbine just described. The arrows *f*, *f* show the direction of flow of the steam.

A = 1 inch in the high-pressure, or first, stage
A = $1\frac{5}{16}$ inches in the low-pressure, or second, stage
B = $1\frac{5}{16}$ inches in the high-pressure, or first, stage
B = $1\frac{7}{8}$ inches in the low-pressure, or second, stage
C = $1\frac{7}{16}$ inches in the high-pressure, or first, stage
C = $2\frac{7}{16}$ inches in the low-pressure, or second, stage
D = $1\frac{13}{16}$ inches in the high-pressure, or first, stage
D = 3 inches in the low-pressure, or second, stage

The heights of the blades, where the steam leaves them, are such that *a b*, *b c*, *c d*, and *d e* form an approximately straight line, *d e* being approximately *c d* produced.

32. Footstep Bearing of Curtis Turbine.—For a description of the footstep bearing of the Curtis turbine, see *Steam Turbines*, Part 1.

It is intended to have the shaft always run on an oil or water film between the bearing surfaces, as no metals would long stand actual contact under such severe conditions. The oil or water is pumped into the bearing under considerable pressure. This pressure, in the 1,500-kilowatt turbine just noted, is 380 pounds per square inch of bearing surface, and reaches a value of 1,000 pounds per square inch in the 5,000-kilowatt Curtis turbine. Under this pressure, the fluid is forced between the rubbing surfaces of the bearing, and keeps them from coming into metallic contact with

each other. The thickness of the interposed film of oil is about .003 inch.

The surfaces of the bearing are of cast iron, since this material wears quite uniformly even under actual metallic contact. Hardened steel and brass have been tried, but have been abandoned in favor of cast iron, because they cut badly when metallic contact occurs.

It is essential that the interior of the bearing surfaces should be bored out, leaving a bearing only in a circumferential rim. This is done to secure an approximately equal rubbing velocity on all parts of the bearing with consequent approximation to equality of wear, and to provide a surface against which the oil or water may act in order to relieve the bearing surface of the pressure of the revolving parts.

CALCULATION OF A PRESSURE TURBINE

33. Data Required.—In the calculation of a pressure turbine, it is necessary to have the following data: (1) Initial pressure and condition and final pressure of steam; (2) absolute velocity of steam through the blades at successive stages; (3) exit angles of both fixed and moving blades; (4) peripheral velocity of blades on the different steps of the rotor; (5) friction losses through the turbine.

Regarding the assumptions to be made, the following points may be observed: The peripheral velocity must not be chosen at too high a figure. Since, in a pressure turbine, there must be admission for the entire circumference, the first set of blades must necessarily be short. For a given number of revolutions per minute, a large peripheral velocity means a correspondingly large drum diameter, and this means short blades. With blades that are too short, the clearance between the blade ends and the casing becomes too large a percentage of the blade length; that is, the leakage loss becomes too great.

The exit angles may be from 20° to 30° for both guide and rotating vanes. Small exit angles permit the use of fewer stages; but, because of the longer channels between the blades, the friction per stage is increased.

The steam velocity c must be kept within definite limits. With c small, too many stages are required; while, with c large, too great friction results.

As practical limits, the peripheral velocity u at the first stage may vary from 100 to 135 feet per second. In the later stages, the peripheral velocity may be considerably increased, because the steam volume is larger. The steam velocity c may vary from $2u$ to $3.5u$.

34. Heat Drop in Any Stage.—If at any stage the steam velocity c, with which the steam leaves the guide vanes, and the peripheral velocity u are known, the heat transformed into work in that stage may be determined by the simple graphic construction shown in Fig. 20. From O lay off $OA = c$, making the exit angle a with the plane of the wheel: then lay off $AB = u$, the peripheral velocity of the wheel, draw $OB = w$, and prolong AB to E on the vertical through O. With E as a center and EA as a radius, strike an arc from A downwards and drop the perpendicular BD to intersect the arc in D. Measure the length $BD = f$ as a velocity to the same scale as the velocities c and u. Then, the work W_1, per pound of steam in this stage is

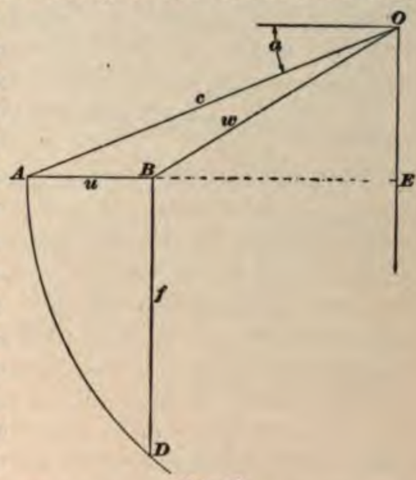


FIG. 20

$$W_1 = \frac{f^2}{g} \qquad (1)$$

The derivation of formula 1 is explained in Appendix II.

The heat drop h required to produce this work is

$$h = \frac{f^2}{778g} = \left(\frac{f}{158}\right)^2 \qquad (2)$$

This construction holds good when the fixed and moving vanes have the same form, so that the heat drop is about the same in each.

The following formulas may be used in place of the graphic construction:

Let c = absolute velocity, in feet per second, of steam entering moving blade;

w = absolute velocity of steam, in feet per second, on leaving blades;

u = velocity of blades, in feet per second;

a = angle between c and u;

e = angle between w and u.

Then,
$$c \sin a = w \sin e \qquad (3)$$
$$w = \sqrt{c^2 + u^2 - 2\,c\,u \cos a} \qquad (4)$$

The formula for the work becomes
$$W_1 = \frac{u(c \cos a + w \cos e)}{g} \qquad (5)$$

Formula **5** is similar to previous formulas for the work of flowing steam, as shown in Art. **14**. The absolute velocity of the steam on leaving the vanes is assumed to be approximately equal to the relative velocity on entering, because the forms of the blades are the same for both fixed and moving wheels and equal expansions take place in both.

If the heat drops in all the stages were equal, the number of stages would be determined by dividing the total heat drop $H_1 - H_2$, after deducting friction, by the drop h per stage. Actually, the work in the stages near the condenser is several times that in the earlier stages. Hence, a curve should be derived to show the values of h at the various stages, and the mean ordinate of this curve will give the mean heat drop per stage. The method of procedure may best be shown by the working out of an actual case.

35. Let a pressure turbine be designed to run with steam at a pressure of 180 pounds per square inch, absolute, and superheated to 550° F., and a condenser pressure of 1.5 pounds per square inch, absolute. The friction through the turbine is assumed to be .25 of the total work of the

steam, and the exit angles of the blades are to be $22\frac{1}{2}°$. The rotor will be built up of three drums of different diameters. The three mean peripheral speeds of the drums will be taken as 125, 175, and 300 feet per second, respectively. The steam velocity of the first stage of the turbine will be taken as 250 feet per second, and the exit velocity of the steam from the last stage as 900 feet per second.

In Fig. 21, AB is laid off to represent the length of the rotor, and it is assumed, for the purpose of calculation merely, that the stages are equally spaced along the axis.

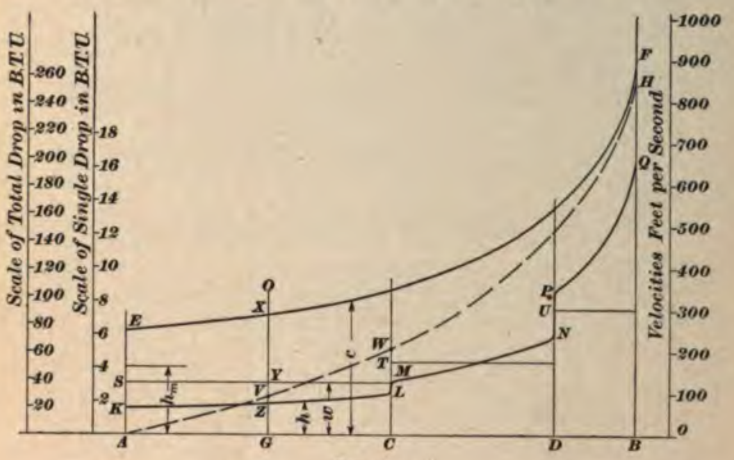


FIG. 21

In the actual construction, the stages may be placed as desired. AE is laid off perpendicular to AB, to represent the steam velocity c in the first stage, that is, 250 feet per second. BF is laid off in a similar manner to represent the the final velocity, 900 feet per second, of the last stage. Between E and F, a curve may be drawn, to represent the velocities of the steam at the different points along the rotor. Such a curve is shown at EXF, and the ordinates of this curve represent the velocities of the steam in the stages situated at the corresponding points along the rotor. The velocities of the steam, and consequently the ordinates of the curve, should increase rather slowly at first, but more

rapidly at the large end where the volume of steam per pound is very greatly increased. The exact determination of the curve EXF is practically impossible, but the steam velocities should be so chosen as to give the smallest friction losses; hence, the ordinates of the curve should be kept as short as possible.

36. The axis AB of the rotor is divided at C and D to represent the drums, AC being the first, CD the second, and DB the third. These divisions should be so made that the ratios of the steam velocities to the peripheral velocities shall be between 2 and 3.5. The peripheral velocities are also laid off as follows:

The peripheral velocity of the first drum is $AS = 125$ feet per second.

The peripheral velocity of the second drum is $CT = 175$ feet per second.

The peripheral velocity of the last drum is $DU = 300$ feet per second.

Now, at any section, as OG, the absolute velocity c of the steam is GX, and the peripheral velocity u is GY. The scale of both velocities is shown at the right of Fig. 21. With these known velocities and the angle a equal to $22\frac{1}{2}°$, lay out a geometric construction like Fig. 20, and determine the segment f. Find the value of f by means of the velocity scale at the right of Fig. 21; then find the heat drop h from formula **2**, Art. **34**, or use formulas **3**, **4**, and **5**, Art. **34**, to find the work in foot-pounds, and divide this by 778 to get the heat drop in British thermal units for the stage at OG. This heat drop in British thermal units is laid off on OG as GZ according to some convenient scale, such as that marked *Scale of Single Drop in B. T. U.* at the left of Fig. 21. This construction is repeated for several sections along the axis AB, especially at the sections C and D; and as a result, the curve of single drop $KLMNPQ$ is obtained. The term single drop is applied to this curve because the ordinates represent the heat drop in a single stage in British thermal units at the point along the axis where the ordinate is taken,

or, what amounts to the same thing, the work done in that stage. It should be noted that in the first stage the drop is less than 2 British thermal units, while in the last stage it is more than 16 British thermal units. Now measure the area between the curve $KLMNPQ$ and AB, either by the use of a planimeter or by ordinates, and divide the area by the length AB to determine the mean ordinate. The mean ordinate will give the average heat drop per stage h_m, which in this case is found to be 4.2 British thermal units.

37. The number of stages required can now be determined. From the Heat Chart, Fig. 1, the total adiabatic drop from initial to final condition of the steam—that is, 180 pounds and 550° F. to 1.5 pounds—measures 336 British thermal units, nearly. But of this 25 per cent. is lost in friction through the vanes; hence, the available heat drop is $336(1 - .25) = 252$ British thermal units. Now, since the mean drop per stage is 4.2 British thermal units, the number of stages required is $252 \div 4.2 = 60$, nearly.

Before it is possible to determine any blade dimensions, the capacity and probable steam consumption of the turbine must be known. Suppose that the turbine is required to develop 1,000 horsepower at the brake, and that the losses due to leakage, journal friction, etc. are 25 per cent. of the energy given up by the steam. Since, for each pound of steam, 252 British thermal units is changed into work, the net heat that gets into work at the brake is $252(1 - .25) = 189$ British thermal units.

The steam consumption, from formula **4**, Art. **6**, is $S = 2{,}545 \div 189 = 13.47$ pounds, nearly. The weight of steam required per second is, therefore,

$$\frac{1{,}000 \times 13.47}{3{,}600} = 3.74 \text{ pounds, nearly}$$

38. From the weight flowing per second, the volume flowing per second through any stage can be found, provided that the condition of the steam as regards pressure and superheat or moisture at that stage is known. The condition may be obtained, approximately, as follows: As in

Fig. 13, locate, on the heat diagram, the initial point A and the final point B. Lay off $A P$ = 252 British thermal units, the available heat drop, and project P to N on the condenser pressure line. Then N represents the final condition of the steam. Through A and N draw a condition curve as $A D K N$.

Returning, now, to Fig. 21, it is necessary to find the curve of the total heat drop $A H$. As was noted previously, the area $A K L M N P Q B A$ represents the total drop through the turbine, $H_1 - H_2$ = 252 British thermal units. The area $A K Z G$ will, to the same scale, represent the heat drop from the first stage to the stage at the section $O G$; likewise, the area $A K L C$ represents the total heat drop through the first drum, and so on. Let these areas be measured, and let the areas be laid off as ordinates; thus, $G V$ = the area $A K Z G$, $C W$ = the area $A K L C$, etc.

In this way, the curve of total heat drop $A V W H$ is obtained. The last ordinate $B H$, of course, represents 252 British thermal units. The scale for the total heat drop in British thermal units is shown at the left of the figure.

39. Now return to the condition curve on the Heat Chart, as shown in Fig. 22. From the initial point A, on the pressure line p_1, lay off the length $A B$ equal to the total drop $G V$. Fig. 22 is made double the scale of Fig. 21, for convenience. Project horizontally from B to the assumed condition curve, and thus locate the point C and the pressure line p_c. Likewise, lay off $A D = C W$ of Fig. 21, and on Fig. 22 project D to E on the condition curve, determining the pressure line p_e. The points C, E, etc. thus found give the pressure and condition of the steam at the various stages, as $O G$, C, etc. The curve

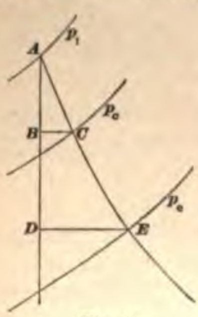


FIG. 22

of pressures is shown in Fig. 23; the ordinates of this curve represent the pressures per square inch of the steam at different points along the axis $A B$ that represents the length

of the rotor. It will be noted that the curve of pressures is nearly a straight line, showing that the drop in pressure is nearly uniform. The pressure at the end of the first drum is 75 pounds per square inch, absolute, and at the end of the second drum about 18 pounds per square inch, absolute, according to the scale of pressures on the left of Fig. 23.

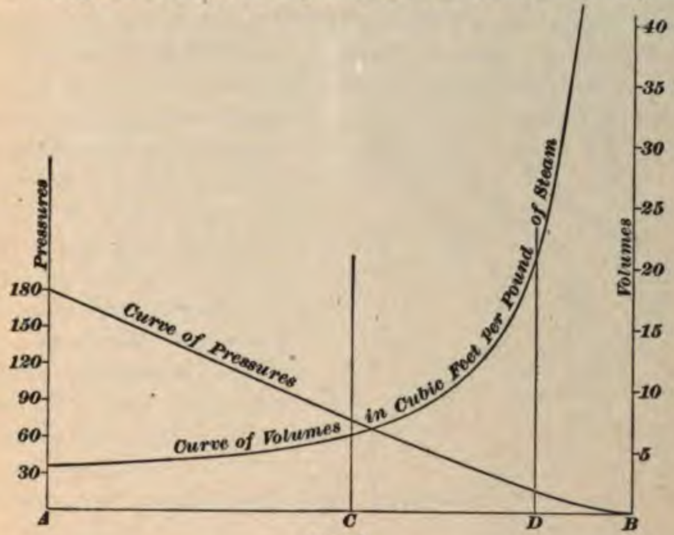


FIG. 23

Since the pressures and conditions of the steam are known for various points along the turbine, the volumes, in cubic feet per pound, of steam can be found by the formulas of Art. **4.** The curve of these volumes is also laid off in Fig. 23, the ordinates of the curve representing the volumes in cubic feet for those points along the axis at which the ordinates are taken.

40. Referring to Fig. 24, which shows two rows of blades, one for a moving wheel and one for a guide wheel, the velocity c of the steam as it leaves the guide vanes makes an angle of $22\frac{1}{2}°$ with the plane of the wheel. The component of this velocity in the direction of the axis is c', and is given by the formula

$$c' = c \sin 22\tfrac{1}{2}° = .38268\, c \qquad \textbf{(1)}$$

In Fig. 24, let e = the pitch of blades;
n = number of blades per wheel;
l = length of blades.

Then, the area A of the ring of blades is given by the formula

$$A = nel \qquad (2)$$

Not all the area A of this ring is available for the passage of the steam. The thickness t of the blades must be subtracted from the pitch e to give the space b and then part of the remaining area is not effective because of the interference of the edges of the next wheel. It is probably safe, however, to take two-thirds of the area A as the effective area for the passage of the steam. If, now, this effective area is multiplied by the velocity component c' perpendicular to it, the product V_1 gives the volume, in cubic feet, of steam flowing through the wheel per second.

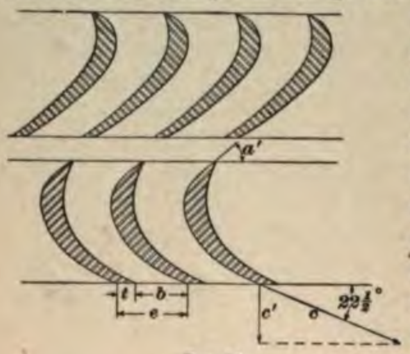


FIG. 24

$$V_1 = \tfrac{2}{3}\, c'\, nel \qquad (3)$$

The volume is also expressed by Gv, where G is the weight flowing per second, and v is the volume, in cubic feet, of a pound of the steam as shown by the curve, Fig. 23. Hence,

$$\tfrac{2}{3}\, c'\, nel = Gv$$

or

$$l = \tfrac{3}{2}\,\frac{Gv}{c'ne} \qquad (4)$$

But ne is the mean circumference of the circle of blades on the wheel and may be denoted by S. Hence,

$$l = \tfrac{3}{2}\,\frac{Gv}{c'S} \text{ (in feet)} \qquad (5)$$

or

$$l_1 = 18\,\frac{Gv}{c'S} \text{ (in inches)} \qquad (6)$$

In formulas **5** and **6**, G is known, v can be found for any stage from the curve of Fig. 23, and c' can be measured from

the curve EF, Fig. 21. The circumference S can be found as soon as the revolutions per minute have been decided. In this case, let the revolutions per minute be 1,500. Then, for the small drum,

$$S_1 = 125 \times 60 \div 1{,}500 = 5 \text{ feet}$$

For the intermediate drum,

$$S_2 = 175 \times 60 \div 1{,}500 = 7 \text{ feet}$$

For the large drum,

$$S_3 = 300 \times 60 \div 1{,}500 = 12 \text{ feet}$$

41. The blade lengths can now be determined by applying formula **5**, Art. **40**. As an example, take the section

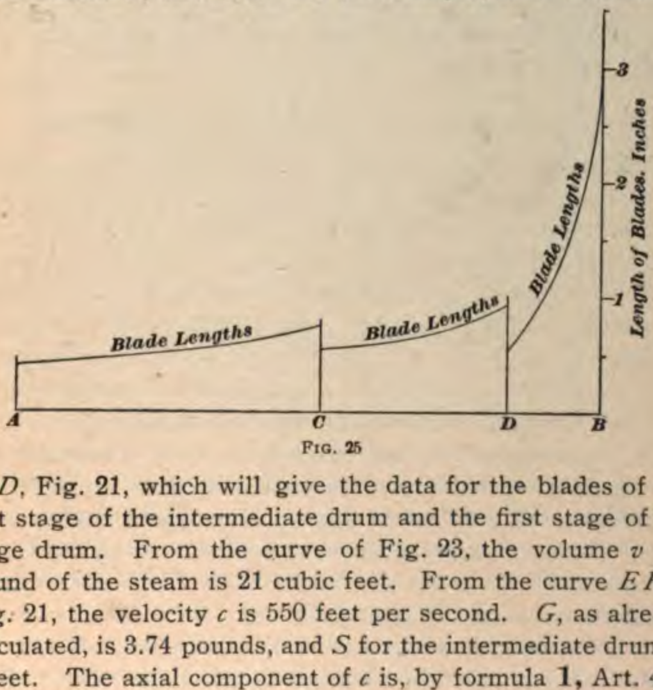


FIG. 25

at D, Fig. 21, which will give the data for the blades of the last stage of the intermediate drum and the first stage of the large drum. From the curve of Fig. 23, the volume v per pound of the steam is 21 cubic feet. From the curve EF of Fig. 21, the velocity c is 550 feet per second. G, as already calculated, is 3.74 pounds, and S for the intermediate drum is 7 feet. The axial component of c is, by formula **1**, Art. **40**,

$$c' = .38268 \times 550 = 210.474, \text{ or } 210 \text{ feet per second, nearly}$$

Hence, the length, from formula **5**, Art. **40**, is for the last stage of the intermediate drum,

$$l = \frac{18\,G\,v}{c'\,S} = \frac{18 \times 3.74 \times 21}{210 \times 7} = .962 \text{ inch}$$

For the first stage of the large drum, the length is found by using the same formula with $S = 12$, so that

$$l = \frac{18 \times 3.74 \times 21}{210 \times 12} = .561 \text{ inch}$$

In this way, the blade lengths for all stages throughout the turbine may be easily determined. Fig. 25 shows the curve of blade lengths for the example here considered. The ordinates represent the blade lengths, in inches, to the scale on the right.

The blades are shown in section in Fig. 24. The exit angle a must be $22\frac{1}{2}°$ for all stages. The entrance angle a' can be readily found from a velocity triangle similar to the one shown in Fig. 14. The pitch of the blades may vary from .5 to .875 inch.

ECONOMICAL CONSIDERATIONS

42. Economy of the Steam Turbine.—The economy of a steam engine is indicated by the pounds of steam consumed by the engine per indicated horsepower per hour. In the steam turbine, however, it is not practicable to get the indicated horsepower, as indicator cards cannot be taken; hence, the economy is usually expressed in pounds of steam consumed per brake horsepower per hour. The published reports of tests of the steam consumption of turbines are as yet few compared with steam-engine tests, but the best turbine performance is about equal to the best compound condensing steam-engine performance, and, under most working conditions, the turbine compares very favorably with the reciprocating engine. So far, however, the best turbine performances have not yet equaled those of the best triple-expansion reciprocating engine.

Theoretically, it seems that in the future the steam turbine will show a better economy of steam than does the reciprocating engine, since, with the same initial pressure, it is practicable to expand the steam to a much lower pressure in the turbine than in the engine. By actual tests, some types of turbines show better average economy under variable loads than reciprocating engines.

43. Fig. 26 shows the steam consumption of an 800-horsepower Curtis turbine at various loads. The turbine ran at a speed of 1,500 revolutions per minute and was used with a condenser. Steam turbines, like ordinary engines, can be used either with or without condensers, but their economy is much better when used with condensers, because, if a high vacuum can be maintained it is possible to work with much higher ratios of expansion than are practicable with ordinary engines. In Fig. 26, curve *A*, it will be noted that the steam consumption at the full load of 800 horsepower is about

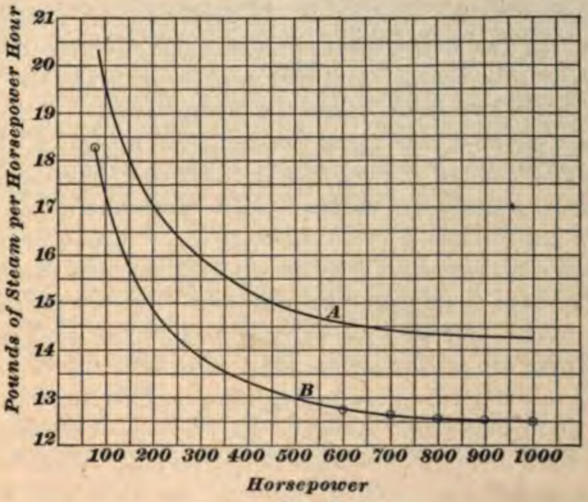


FIG. 26

14.33 pounds of steam per horsepower-hour. This, however, is the steam consumption based on the horsepower output of the dynamo. If the efficiency of the generator is assumed to be 95 per cent. and the mechanical efficiency of the turbine 95 per cent. also, the steam consumption per horsepower, which would be comparable with that per indicated horsepower of an ordinary engine, would be about 13 pounds of steam per indicated horsepower per hour. This is better than would ordinarily be obtained with a reciprocating engine of this size. The steam consumption of the turbine can be considerably reduced by using superheated steam. The

power stations where the load is variable. If it becomes necessary to operate a turbine unit at a comparatively light load, say one-fourth or one-half load, the increase in steam consumption per horsepower per hour is not so great as it would be with a reciprocating engine under the same conditions. Also, a turbine unit will work more efficiently on overloads. The forces acting on the turbine wheels are continuous; hence, a uniform rotary motion is secured without the necessity of heavy flywheels.

APPENDIX I

The various curves in the Heat Chart, Fig. 1, are laid off by finding the coordinates of a number of points and drawing the curves through those points. The saturation curve MN is laid off first by finding the coordinates of dry saturated steam at various pressures. The ordinates are the heat units, in British thermal units, of a pound of steam, and are taken directly from the Steam Table. The abscissas are calculated by using the formula for the entropy of saturated steam in *Entropy and Steam*,

$$N = n + \frac{r}{T} \qquad (1)$$

in which n = entropy of liquid, as given in the Steam Table;
r = heat of vaporization, as given in the Steam Table;
T = absolute temperature $(t + 460°)$ at which steam is vaporized;
N = entropy of 1 pound of dry saturated steam.

Points below the curve MN represent moist steam, that is, a mixture of steam and water. In order to plot the curves below MN, the pressures and qualities for the different points must be assumed, and the corresponding total heats and entropies of the mixtures calculated. To plot the curve for a given pressure, it is necessary to assume different qualities of steam at that pressure and calculate the total heats of the mixtures by the following formula, from *Entropy and Steam:*

$$H_1 = q + xr \qquad (2)$$

in which H_1 = heat contents, in British thermal units;
q = heat of the liquid, in British thermal units;
x = quality of mixture;
r = heat of vaporization, in British thermal units.

The values thus obtained are taken as the ordinates of that curve.

The abscissas being taken in entropy units are determined by the following formula for the entropy of a mixture, from *Entropy and Steam.*

$$N = n + \frac{xr}{T} \qquad (3)$$

For the curves above MN in the superheated region, the ordinates are found by calculating the total heat of superheated steam by the following formula from *Entropy and Steam:*

$$Q = q + r + .48(t_s - t) \qquad (4)$$

in which Q = total heat of 1 pound of superheated steam, in British thermal units;

t_s = temperature of superheated steam, in degrees Fahrenheit;

t = temperature of vaporization, in degrees Fahrenheit.

The abscissas are entropy units found from the entropy of superheated steam by the use of the following formula from *Entropy and Steam:*

$$N_s = n + \frac{r}{T} + 1.105 \log \frac{T_s}{T} \qquad (5)$$

in which N_s = entropy of superheated steam;

T_s = absolute temperature to which steam is superheated;

T = absolute temperature at vaporization;

and the other terms are as already explained.

The following examples will show how the formulas are used to determine the coordinates of points on the curves.

EXAMPLE 1.—Find: (*a*) the heat contents and (*b*) the entropy of 1 pound of dry saturated steam at an absolute pressure of 300 pounds per square inch.

SOLUTION.—(*a*) From the Steam Table, H = 1,209.3 B. T. U. Ans.

(*b*) Apply formula **1**. From the Steam Table, $n = .5863$; $r = 817.4$; and $T = t + 460 = 417.42 + 460 = 877.42$. Then,

$$N = n + \frac{r}{T} = .5863 + \frac{817.4}{877.42} = 1.518, \text{ nearly. Ans.}$$

NOTE.—It will be apparent, on examining the Heat Chart, that, if the entire length of these coordinates were laid off from the axes, the Chart would be so large as to become unwieldy. Therefore, only that portion of the Chart is shown that is necessary to give the location of points that are liable to be used.

EXAMPLE 2.—Find: (*a*) the heat contents and (*b*) the entropy of 1 pound of steam at an absolute pressure of 50 pounds per square inch and a quality of .90.

SOLUTION.—(*a*) From the Steam Table, q = 250.2 B. T. U. and r = 917.4 B. T. U. Hence, applying formula **2**,

$$H_1 = q + xr = 250.2 + .9 \times 917.4 = 1{,}075.86 \text{ B. T. U. Ans.}$$

(*b*) From the Steam Table, $n = .4109$ and $T = t + 460 = 280.85 + 460 = 740.85$. Hence, applying formula **3**,

$$N = n + \frac{xr}{T} = .4109 + \frac{.9 \times 917.4}{740.85} = 1.525, \text{ nearly. Ans.}$$

EXAMPLE 3.—Find: (*a*) the heat contents and (*b*) the entropy of 1 pound of steam at an absolute pressure of 4 pounds per square inch and a quality of .98.

SOLUTION.—(*a*) From the Steam Table, $q = 121.4$ B. T. U. and $r = 1{,}007.2$ B. T. U. Then, by formula **2**,

$H_1 = q + x\,r = 121.4 + .98 \times 1{,}007.2 = 1{,}108$ B. T. U., nearly. Ans.

(*b*) From the Steam Table, $n = .2203$ and $T = t + 460 = 153.09 + 460 = 613.09$. Hence, applying formula **3**,

$$N = n + \frac{x\,r}{T} = .2203 + \frac{.98 \times 1{,}007.2}{613.09} = 1.83, \text{ nearly. Ans.}$$

EXAMPLE 4.—Find: (*a*) the heat contents and (*b*) the entropy of 1 pound of steam at an absolute pressure of 300 pounds per square inch and superheated to 600° F.

SOLUTION.—(*a*) From the Steam Table, $q = 391.9$; $r = 817.4$; and $t = 417.42$. From the statement of the example, $t_s = 600$. Hence, from formula **4**,

$$Q = q + r + .48(t_s - t) = 391.9 + 817.4 + .48(600 - 417.42)$$
$$= 1{,}297 \text{ B. T. U., nearly. Ans.}$$

(*b*) From the Steam Table, $n = .5863$. $T = t + 460 = 417.42 + 460 = 877.42$; $T_s = 600 + 460 = 1{,}060$. Hence, from formula **5**,

$$N_s = n + \frac{r}{T} + 1.105 \log \frac{T_s}{T} = .5863 + \frac{817.4}{877.42} + 1.105(3.02531 - 2.94321)$$
$$= 1.609, \text{ nearly. Ans.}$$

APPENDIX II

A graphic solution was given for finding f for formula **1** in Art. **34**, which is

$$W_1 = \frac{f^2}{g}$$

in which W_1 = work, in foot-pounds, per pound of steam used in any stage of a pressure turbine;

$g = 32.16$.

The derivation of this formula is comparatively simple, though not apparent from the construction. In Fig. 20, which is here reproduced with some additions, c represents the absolute velocity of the steam as it enters this wheel; u is the velocity of the wheel; and a, the angle between c and u. Then, w represents the velocity of the steam at entrance relative to the wheel, making the angle e with the plane of the wheel. If there were no expansion of the steam in the buckets, the relative velocity would be the same on leaving as at the entrance to the buckets. The expansion is divided as equally as possible between the guide and the moving vanes, but not quite all the energy is given up to the moving vanes; hence, the velocity on entering the next set of moving buckets will be slightly greater than in the set under consideration. The relative velocity on leaving any set of moving vanes should, however, about equal the absolute velocity of

entrance, because of the expansion of the steam. Hence, the velocity of the steam relative to the wheel will be $w_2 = c$ on leaving, and, by combining this with u, the absolute velocity on leaving is c_2. The angle e_2 = the angle a, and the angle a_2 = the angle e. The components of c and c_2 in the direction of motion of the wheel are c' and c_2'. But, as c' and c_2' represent the change in velocity, in feet per second, they also represent the negative acceleration of the steam. Now, force is equal to the product of the mass and acceleration. The mass of 1 pound is $\frac{1}{g}$, and the pressure exerted by 1 pound of steam therefore equals $\frac{c' + c_2'}{g}$. When a force acts through a distance, it does work, and in the case considered this distance is u feet per second. Hence, the formula for the work done by 1 pound of steam in 1 second is

$$W_1 = \frac{u(c' + c_2')}{g}$$

In Fig. 27, OY is an axis at right angles to the plane of the wheel. Now, by producing AB to F and describing the semicircle AYF, the

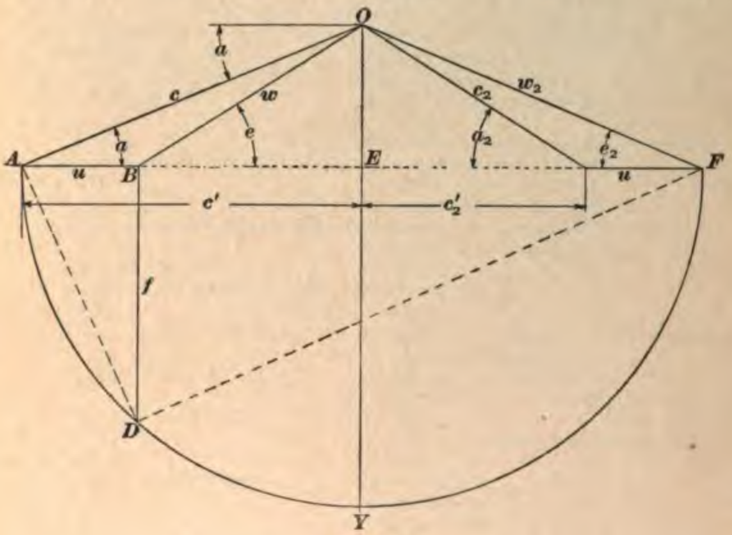


FIG. 27

length of f is obtained by drawing BD perpendicular to AE. Draw AD and DF; then ADF is a right triangle, and it may be proved by geometry that B divides AF into two such parts that $AB \times BF = BD^2$; that is, BD is a mean proportional between AB and BF.

But $AB = u$, $BF = c' + c_2'$, and $BD = f$. Hence, $u(c' + c_2') = f^2$, and, by substituting in the formula, it becomes

$$W_1 = \frac{f^2}{g}$$

Hence, the graphic construction in Fig. 20 gives a value for f in formula 1 of Art. 34 that is nearly enough correct for practical purposes.

INDEX

NOTE.—All items in this index refer first to the section and then to the page of the section. Thus, "Addition, §1, p5," means that addition will be found on page 5 of section 1.

Q

R